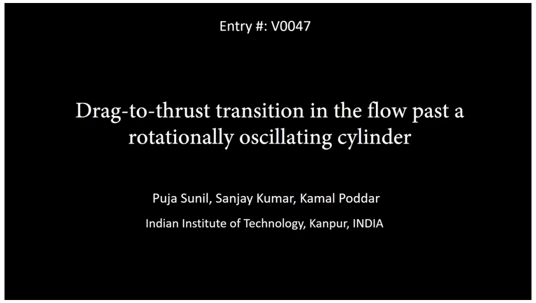
DD2365/2022 – lecture 5
Fluid-structure interaction

Johan Hoffman

Fluid-structure interaction



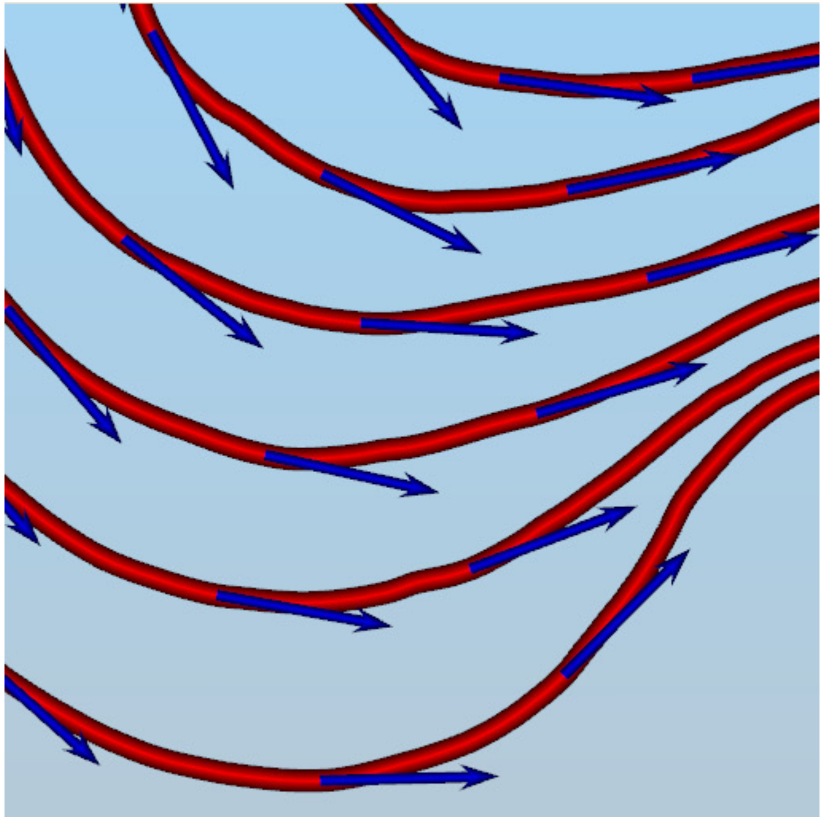
- https://gfm.aps.org/meetings/dfd-2020/5f5f0056199e4c091e67bd9e

[Water and air bubbles.]

Flow visualization: pathlines

[https://en.wikipedia.org/wiki/Streamlines,_streaklines,_and_pathlines#/media/File:Kaberneeme_campfire_site.jpg]

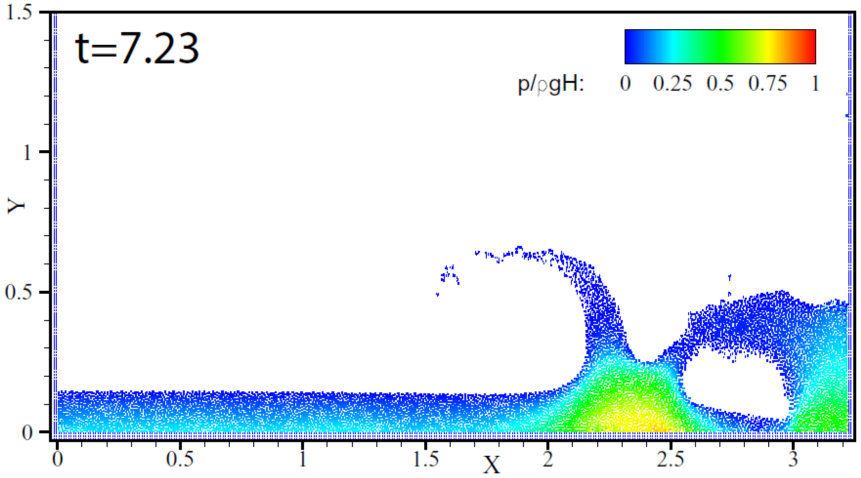
Flow visualization: streamlines



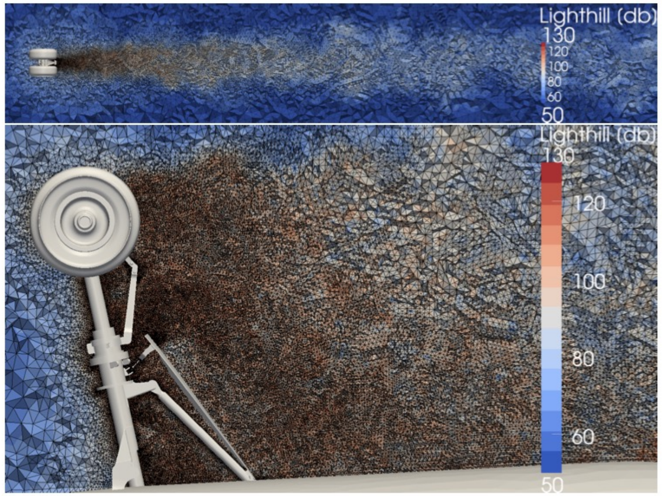
https://www3.nd.edu/~cwang11/2dflowvis.html

Representation of fluid flow

- Pathlines vs Streamlines
- Particles vs mesh/fixed coordinate system
- Lagrangian vs Eulerian representation
- Smooth particle hydrodynamics vs Finite element method

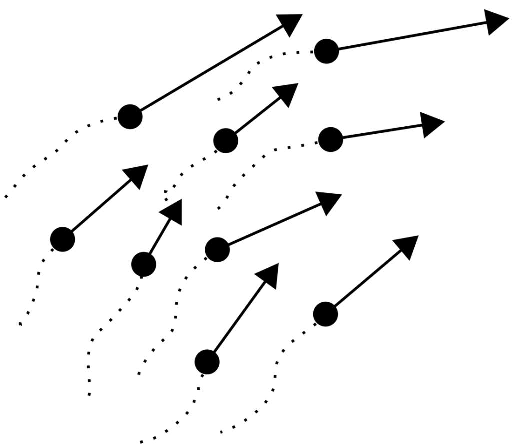


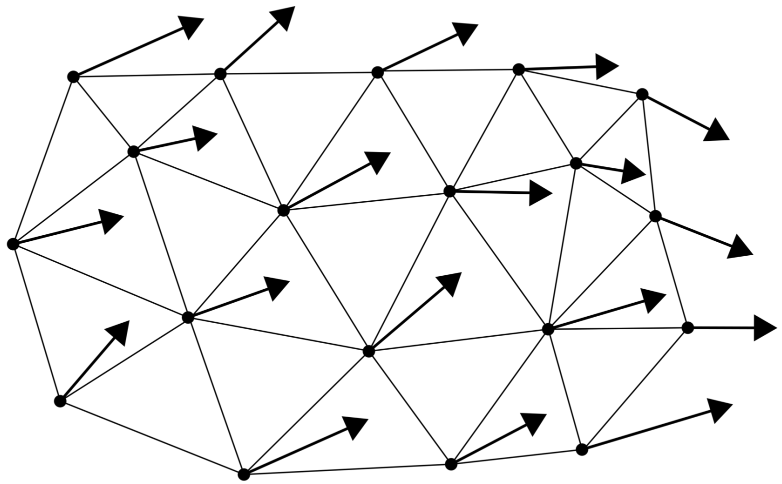
[https://www3.nd.edu/~cwang11/2dflowvis.html]



Representation of fluid flow

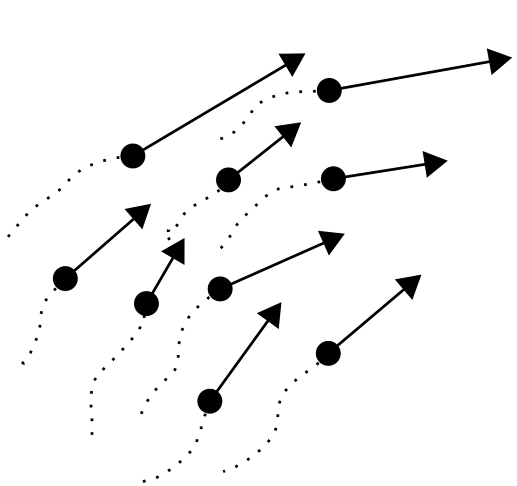
- Pathlines vs Streamlines
- Particles vs mesh/fixed coordinate system
- Lagrangian vs Eulerian representation
- Smooth particle hydrodynamics vs Finite element method

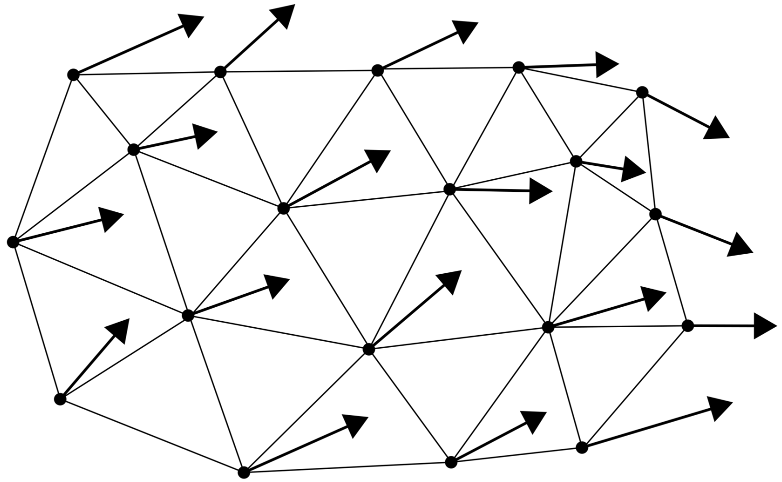




Representation of fluid flow

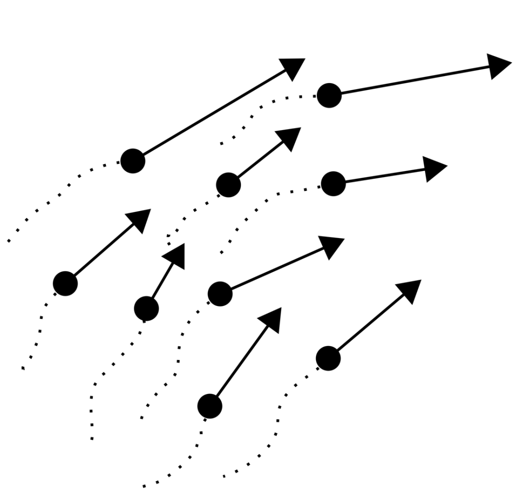
- Lagrangian representation: moving particle position $X(t)$, $X(0) = X_0$
- Eulerian representation: fixed position x, velocity $u(x, t)$

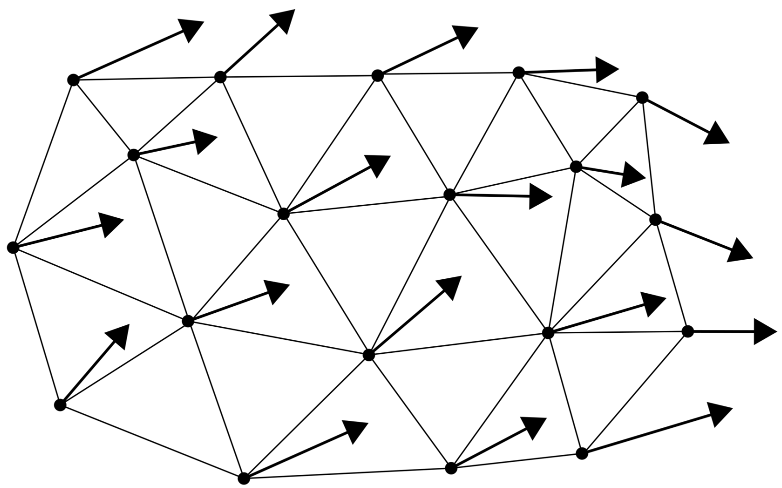




Representation of fluid flow

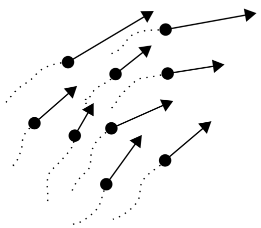
- Lagrangian representation: moving particle position $X(t)$, $X(0) = X_0$
- Eulerian representation: fixed position x, velocity $u(x, t)$
- $u(X(t), t) = \frac{dX}{dt}$

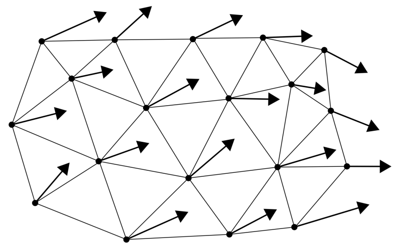




Representation of fluid flow

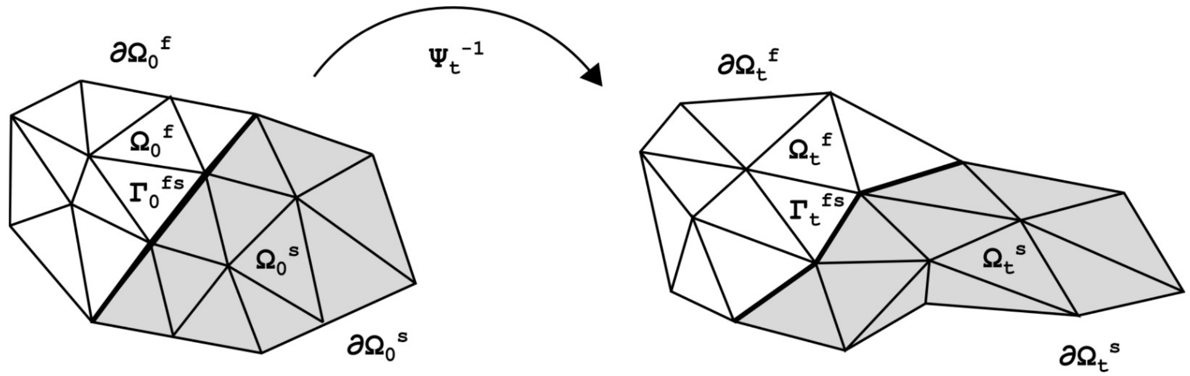
- Lagrangian representation: moving particle position $X(t), X(0) = X_0$
- Eulerian representation: fixed position x, velocity $u(x,t)$
- $u(X(t),t) = \frac{dX}{dt}$
- $\frac{Du}{Dt} = \left(\frac{dX}{dt} \cdot \nabla\right) u + \frac{\partial u}{\partial t} = \frac{\partial u}{\partial t} + (u \cdot \nabla) u$
- Acceleration along particle path



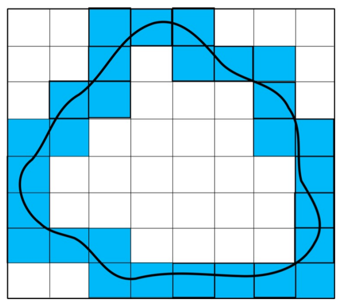


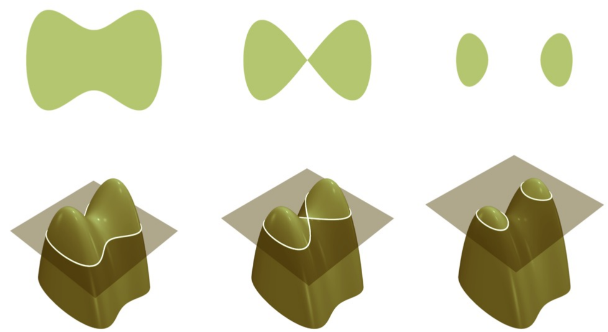
Deforming domains

Interface tracking
- Mesh follow deformation
- Explicit interface representation



Interface capturing
- Mesh fixed
- Implicit interface representation





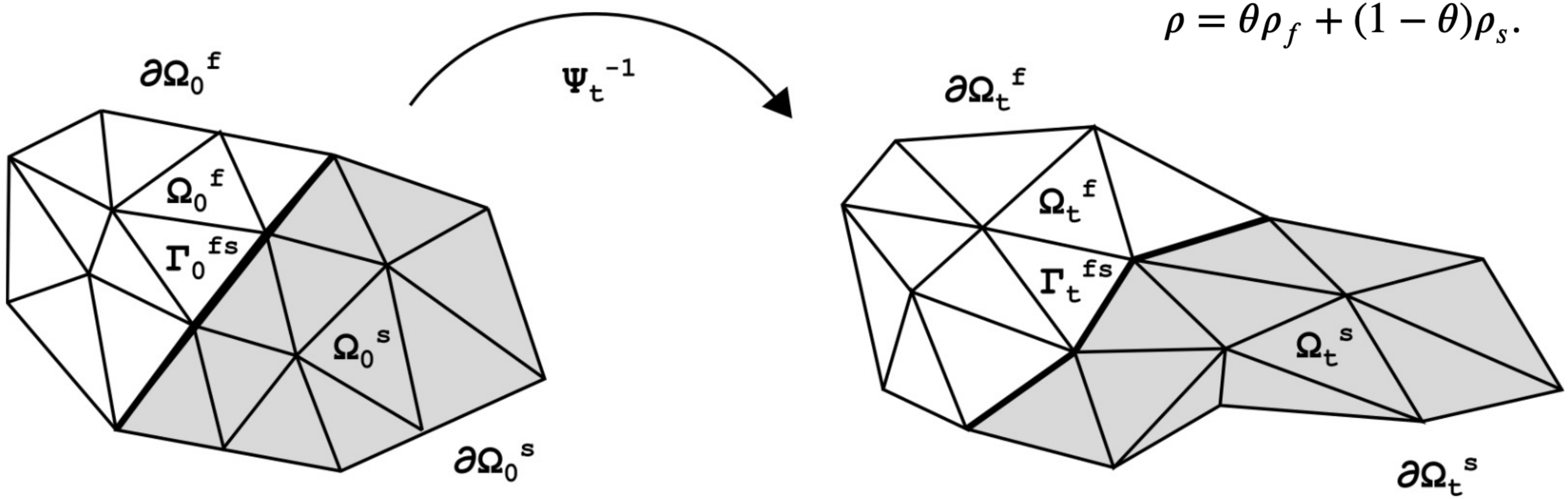
[https://en.wikipedia.org/wiki/Level-set_method#/media/File:Level_set_method.png]

Unified continuum fluid-structure interaction

$$\rho(\dot{\mathbf{u}} + ((\mathbf{u} - \mathbf{m}) \cdot \nabla)\mathbf{u}) + \nabla p - \nabla \cdot \boldsymbol{\tau} = \rho \mathbf{f}, \qquad (\mathbf{x}, t) \in \Omega_t \times I$$
$$\nabla \cdot \mathbf{u} = 0, \qquad (\mathbf{x}, t) \in \Omega_t \times I$$

$$\theta(\mathbf{x}, t) = \begin{cases} 1, & x \in \Omega_t^f, \\ 0, & x \in \Omega_t^s, \end{cases}$$

$$\boldsymbol{\tau} = \theta \boldsymbol{\tau}_f + (1 - \theta)\boldsymbol{\tau}_s,$$
$$\rho = \theta \rho_f + (1 - \theta)\rho_s.$$



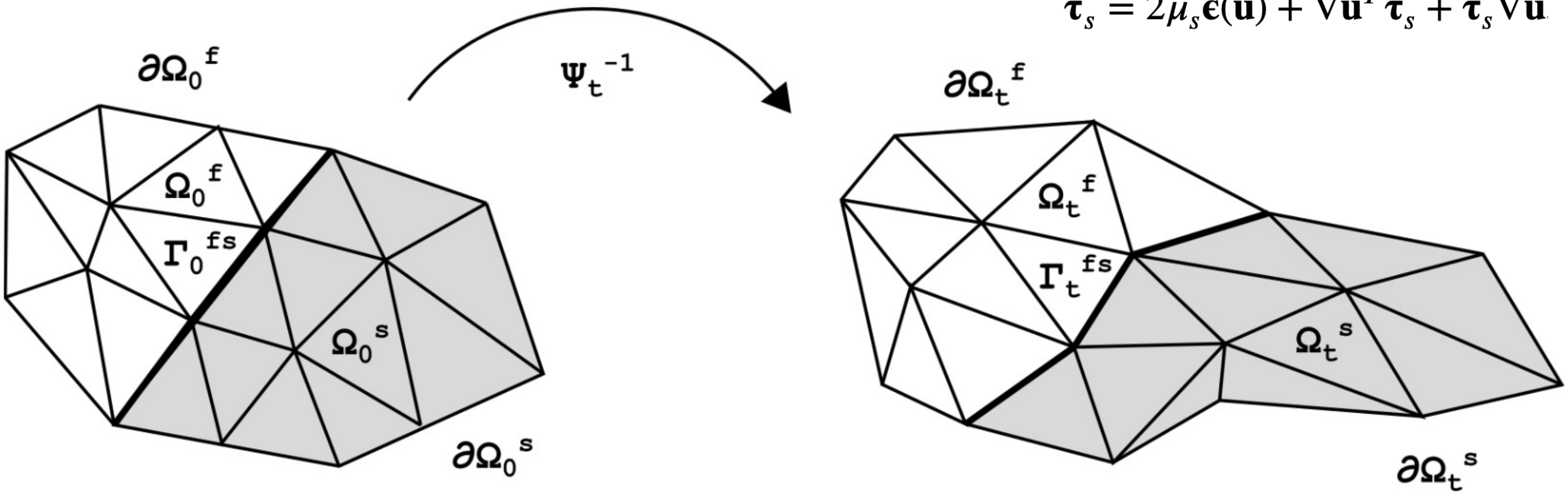
Unified continuum fluid-structure interaction

$$\rho(\dot{\mathbf{u}} + ((\mathbf{u} - \mathbf{m}) \cdot \nabla)\mathbf{u}) + \nabla p - \nabla \cdot \boldsymbol{\tau} = \rho \mathbf{f}, \qquad (\mathbf{x}, t) \in \Omega_t \times I$$

$$\nabla \cdot \mathbf{u} = 0, \qquad (\mathbf{x}, t) \in \Omega_t \times I$$

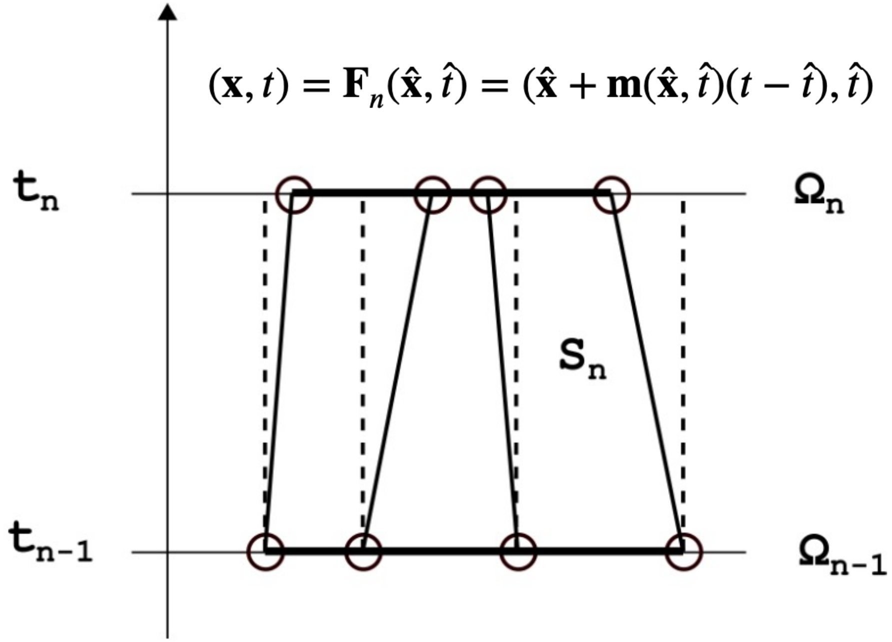
$$\boldsymbol{\tau}_f = 2\mu_f \boldsymbol{\epsilon}(\mathbf{u}), \quad \boldsymbol{\epsilon}(\mathbf{u}) = \frac{1}{2}(\nabla u + \nabla u^T)$$

$$\dot{\boldsymbol{\tau}}_s = 2\mu_s \boldsymbol{\epsilon}(\mathbf{u}) + \nabla \mathbf{u}^T \boldsymbol{\tau}_s + \boldsymbol{\tau}_s \nabla \mathbf{u}$$



Space-time finite element approximation

$$\hat{V}_{h,n}^{(k,m)}(\hat{S}_n) = \{\hat{v}(\mathbf{x},t) : \hat{v} \in V_h^{(k)}(\Omega_{n-1}) \text{ for each } t \in I_n, \text{ and } \hat{v} \in P^m(I_n) \text{ for each } \mathbf{x}_{n-1} \in \Omega_{n-1}\}$$



$$V_{h,n}^{(k,m)}(S_n) = \{v(\mathbf{x},t) = \hat{v}(\mathbf{F}_n^{-1}(\mathbf{x},t),t) : \hat{v} \in \hat{V}_{h,n}^{(k,m)}(\hat{S}_n)\}$$

$$W_h^{(k,m)}(Q) = \{v : v|_{S_n} \in V_{h,n}^{(k,m)}(S_n)\}$$

$$V_h^{(k,m)}(Q) = \{v \in W_h^{(k,m)}(Q) : v \in C(\bar{Q})\}$$

Space-time finite element approximation

find $\mathbf{U} \in [V_{h,g_D}^{(1,1)}(Q)]^d$ and $P \in W_h^{(1,0)}(Q)$, such that

$$(\rho(\dot{\mathbf{U}} + ((\mathbf{U} - \mathbf{M}) \cdot \nabla)\mathbf{U}), \mathbf{v})_Q + (\mu_f \boldsymbol{\epsilon}(\mathbf{U}), \boldsymbol{\epsilon}(\mathbf{v}))_{Q_f} + (\mathbf{T}, \nabla \mathbf{v})_{Q_s} - (P, \nabla \cdot \mathbf{v})_Q + (q, \nabla \cdot \mathbf{U})_Q$$
$$+(\mathbf{g}_N, \mathbf{v})_{\partial Q_N} + SD_{\boldsymbol{\delta}}(\mathbf{U}, \mathbf{M}, P; \mathbf{v}, q) = (\rho \mathbf{f}, \mathbf{v})_Q,$$

for all $\mathbf{v} \in [W_{h,0}^{(1,0)}(Q)]^d$ and $q \in W_h^{(1,0)}(Q)$.

$$W_h^{(k,m)}(Q) = \{v : v|_{S_n} \in V_{h,n}^{(k,m)}(S_n)\}$$

$$V_h^{(k,m)}(Q) = \{v \in W_h^{(k,m)}(Q) : v \in C(\bar{Q})\}$$

Demo ALE

Mesh smoothing – elastic analogy

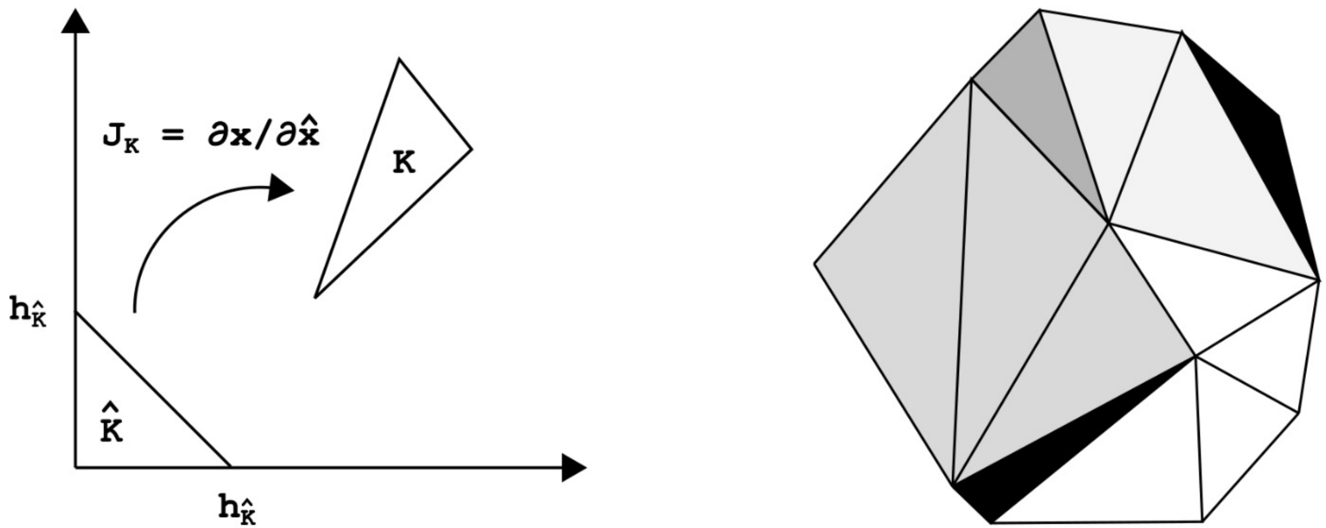
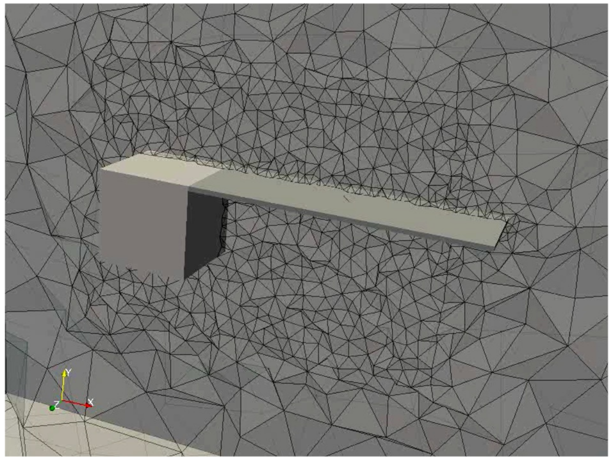
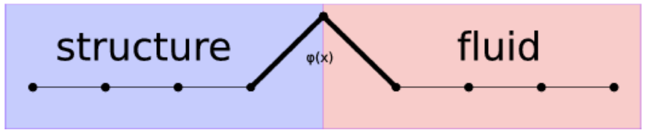


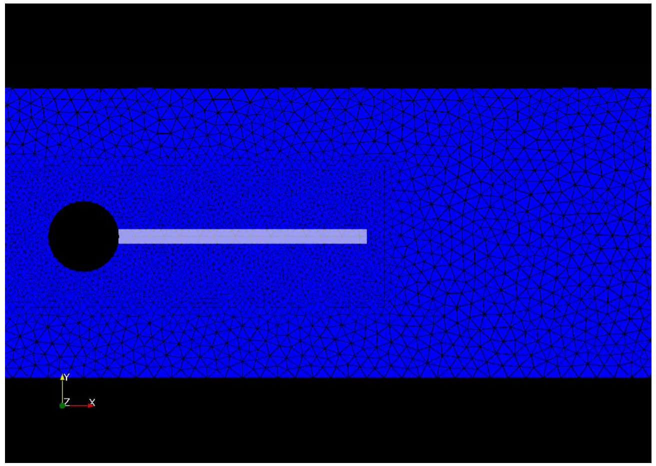
FIGURE 3 Map from a reference triangle $\hat{K}$ to an element K and its associated Jacobian $\mathbf{J}_K$ (left), and a triangle mesh with each element K coloured based on its quality measure $Q(K)$, with darker colours for higher values (right).

Unified Continuum fluid-structure interaction

[J. Hoffman, J. Jansson, M. Stöckli, M3AS, Vol.21(3), 2011.]







ALE-FEM method

- Conforming fluid-solid mesh
- Mesh smoothing

Contact model

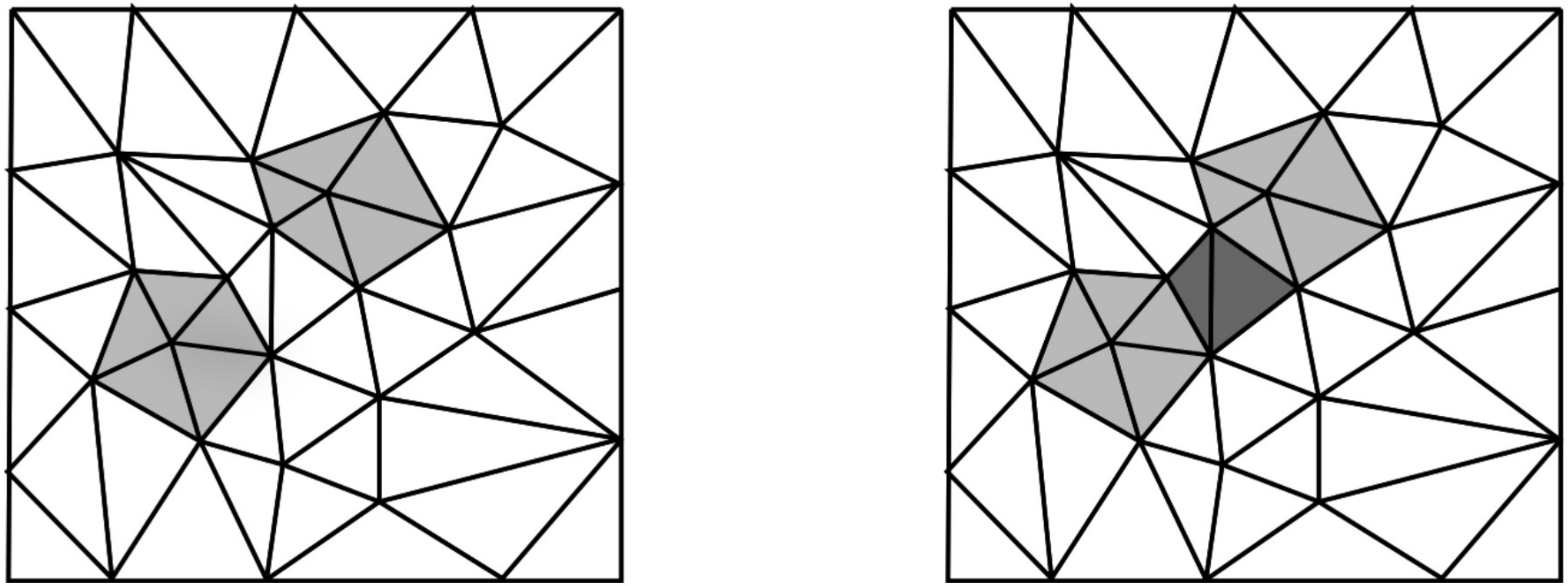
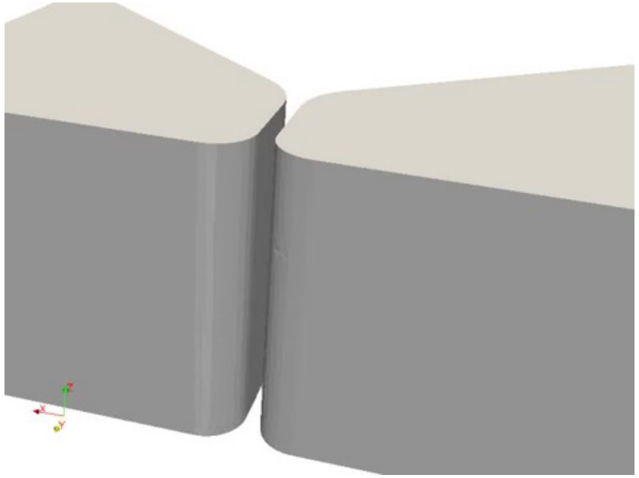


FIGURE 4 Illustration of the UC-FSI contact model, where collision is detected between two structure domains marked by a light shade of grey (left), which then activates a phase change in the contact region marked in a darker shade of grey (right).

Unified Continuum contact model

[Spühler, Degirmenci, Jansson, Hoffman, KTH PhD thesis, 2018]

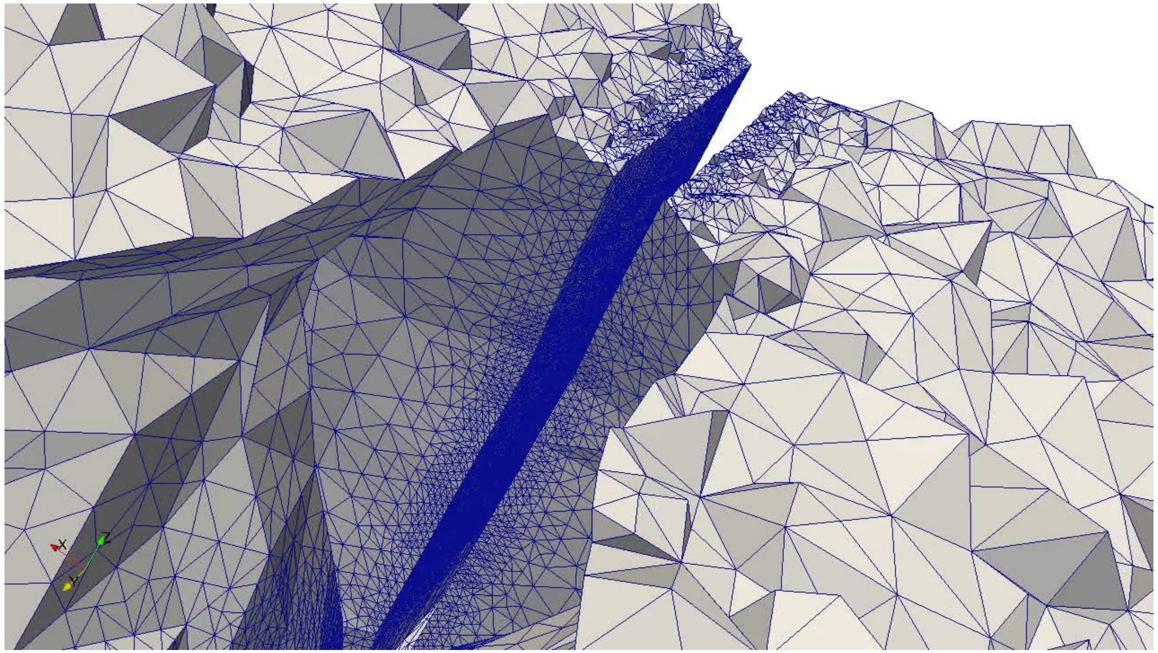


Algorithm 1 *Contact algorithm*

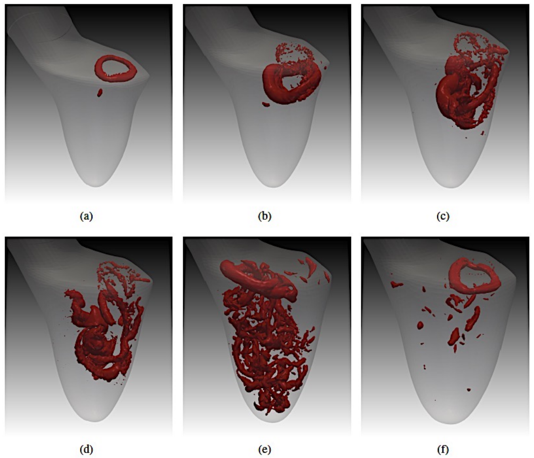
1. *Mark all cells K as non-contact.*
2. *Solve the Eikonal equation $|\nabla D| = 1$ using a artificial viscosity stabilized cG(1) method with $D = 0$ on the boundary Γ and in the structure subdomain Ω_s for the distance $D = D(x)$.*
3. *Compute $|\nabla D|$, and define the medial axis M as: $M = \{x \mid |\nabla D(x)| \leq \gamma\}$, with the threshold parameter $\gamma < 1$.*
4. *Define the contact medial axis: $\hat{M} = \{x \mid x \in M, x \notin \Omega_s, D(x) < \alpha\hat{h}\}$, with $\hat{h}$ the minimum cell size in the mesh.*
5. *Solve the Eikonal equation $|\nabla D_{\hat{M}}| = 1$ using an artificial viscosity stabilized cG(1) method with $D_{\hat{M}} = 0$ on the contact medial axis $\hat{M}$ for the distance from $\hat{M}$ $D_{\hat{M}} = D_{\hat{M}}(x)$.*
6. *Mark all fluid cells as contact which fulfill: $C = \{x \mid D_{\hat{M}} \leq \beta\hat{h}\}$*

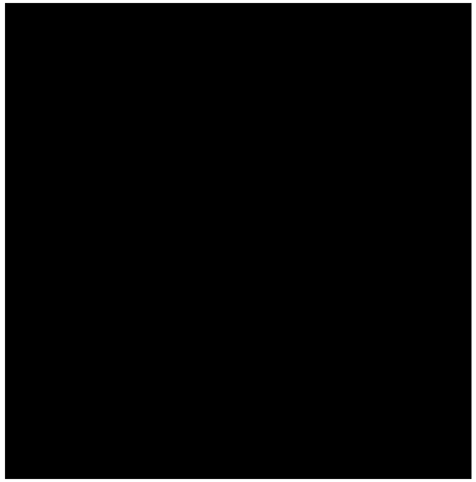
FSI simulation of vocal folds

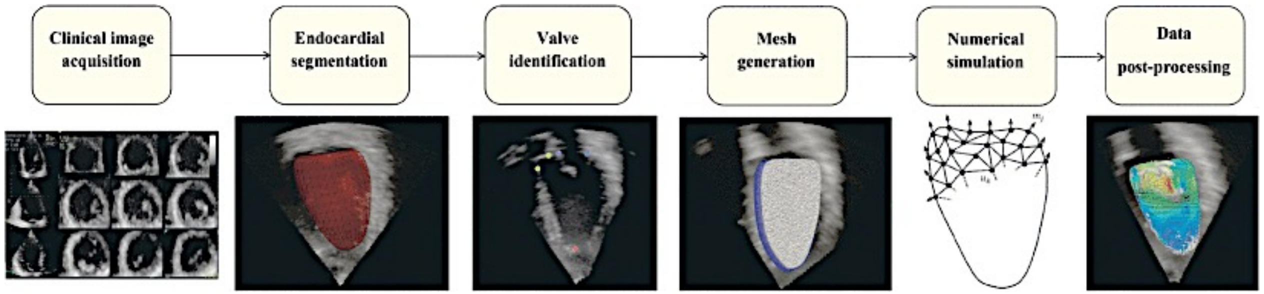
[N.C.Degirmenci, et al, Proc. Interspeech, 2017]



Heart simulation and data analysis



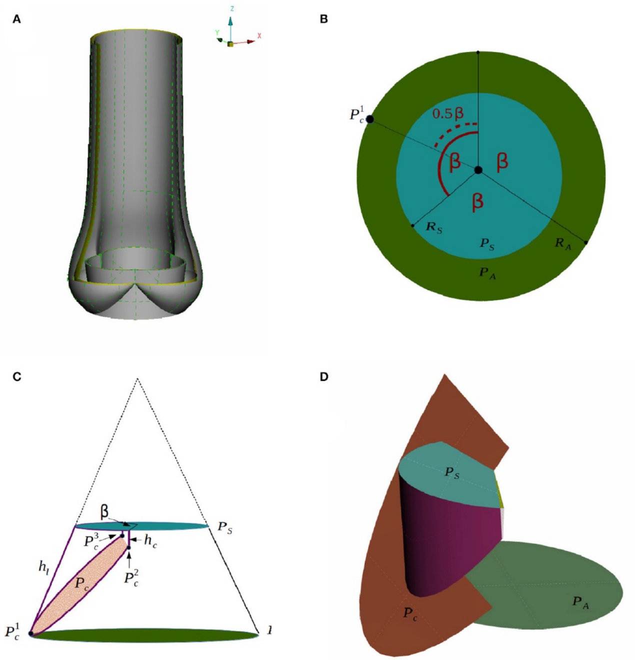


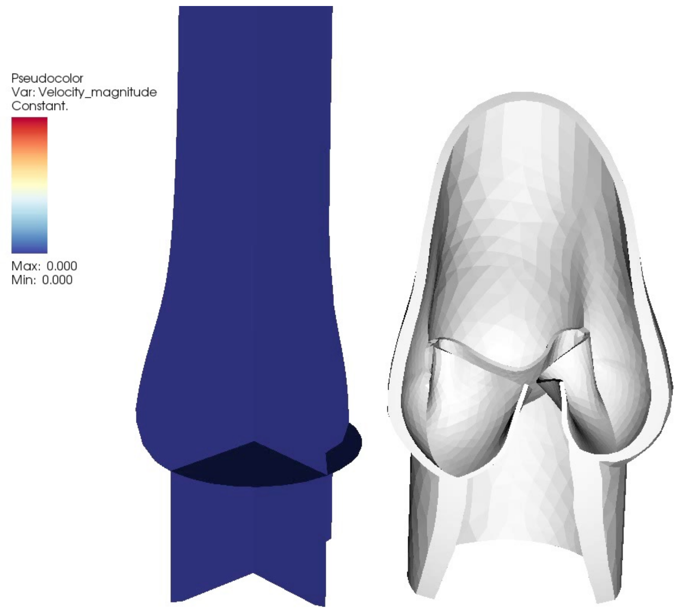


[Spühler et al., 2017, 2020]

FSI simulation of aortic valves

[Spühler et al., Frontiers in Physioloy, Vol.9, 2018]





Navier-Stokes Brinkman model

$$\rho\left(\frac{\partial}{\partial t}\boldsymbol{u}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)-\nabla\cdot\boldsymbol{\sigma}(\boldsymbol{u},p)+\frac{\mu}{K}(\boldsymbol{u}-\boldsymbol{u}_s)=0 \qquad \text{in } \mathbb{R}^+\times\Omega \tag{1}$$

$$\nabla\cdot\boldsymbol{u}=0 \qquad \text{in } \mathbb{R}^+\times\Omega \tag{2}$$

$$\boldsymbol{u}=0 \qquad \text{on } \Gamma_{\text{noslip}} \tag{3}$$

$$\boldsymbol{\sigma}\boldsymbol{n}-\rho\beta(\boldsymbol{u}\cdot\boldsymbol{n})_{-}\boldsymbol{u}=\boldsymbol{h} \qquad \text{on } \Gamma_{\text{outflow}} \tag{4}$$

$$\boldsymbol{u}=\boldsymbol{g} \qquad \text{on } \Gamma_{\text{inflow}} \tag{5}$$

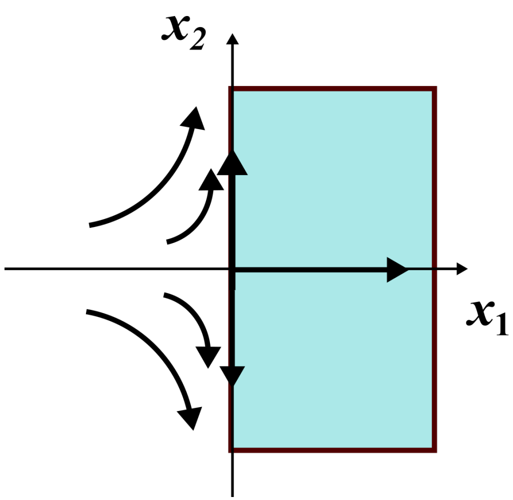
$$\boldsymbol{u}\big|_{t=0}=\boldsymbol{u}_0 \tag{6}$$

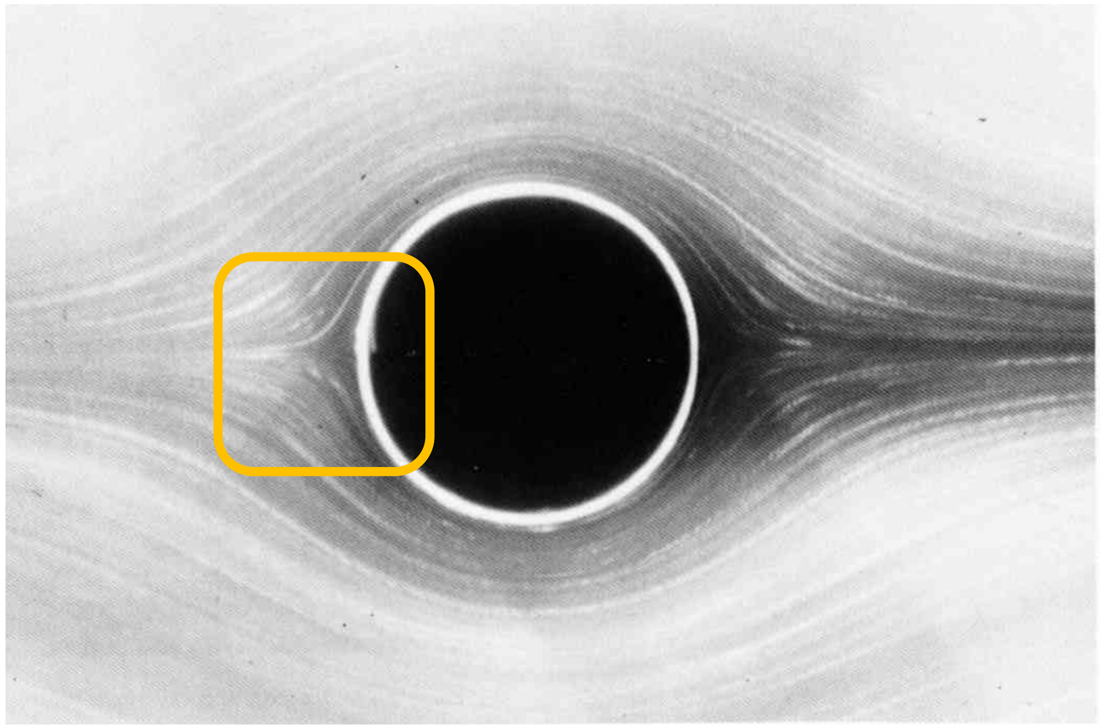
Here $p(\boldsymbol{x},t)$ and $\boldsymbol{u}(\boldsymbol{x},t)$ represent the fluid pressure and the flow velocity respectively, μ is the dynamic viscosity and ρ the density. The volume penalization term $\frac{\mu}{K(t,\boldsymbol{x})}\boldsymbol{u}(t,\boldsymbol{x})$ is commonly known as *Darcy drag* which is characterized by the permeability $K(t,\boldsymbol{x})$.

Demo NS-Brinkman

Incompressible flow – attachment point

- $\nabla \cdot u = 0$
- $\frac{\partial u_2}{\partial x_2} = -\frac{\partial u_1}{\partial x_1}$

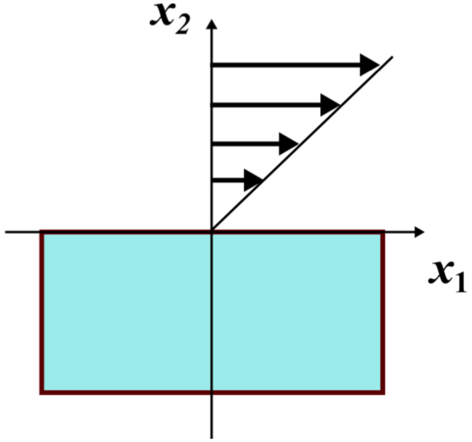


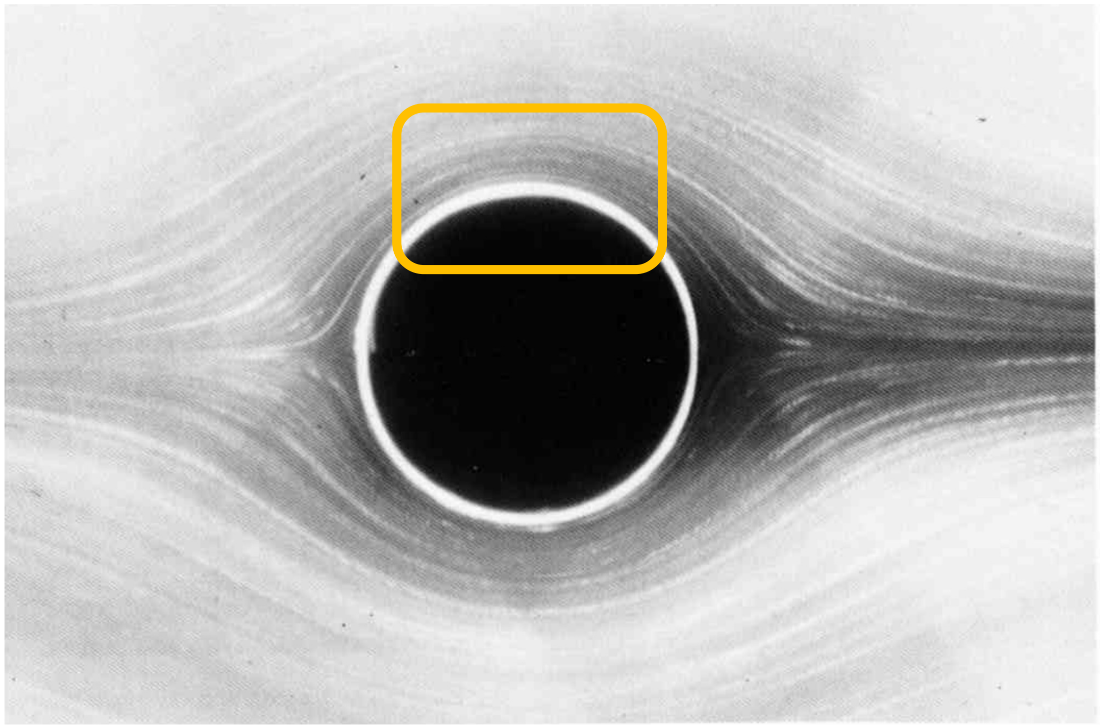


[Water and aluminum dust.]

Incompressible flow – boundary layer

- $\nabla \cdot u = 0$
- $u_1 = f(x_2)$
- $u_2 = 0$

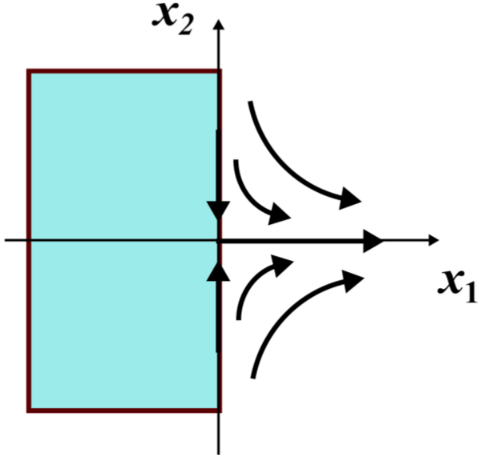


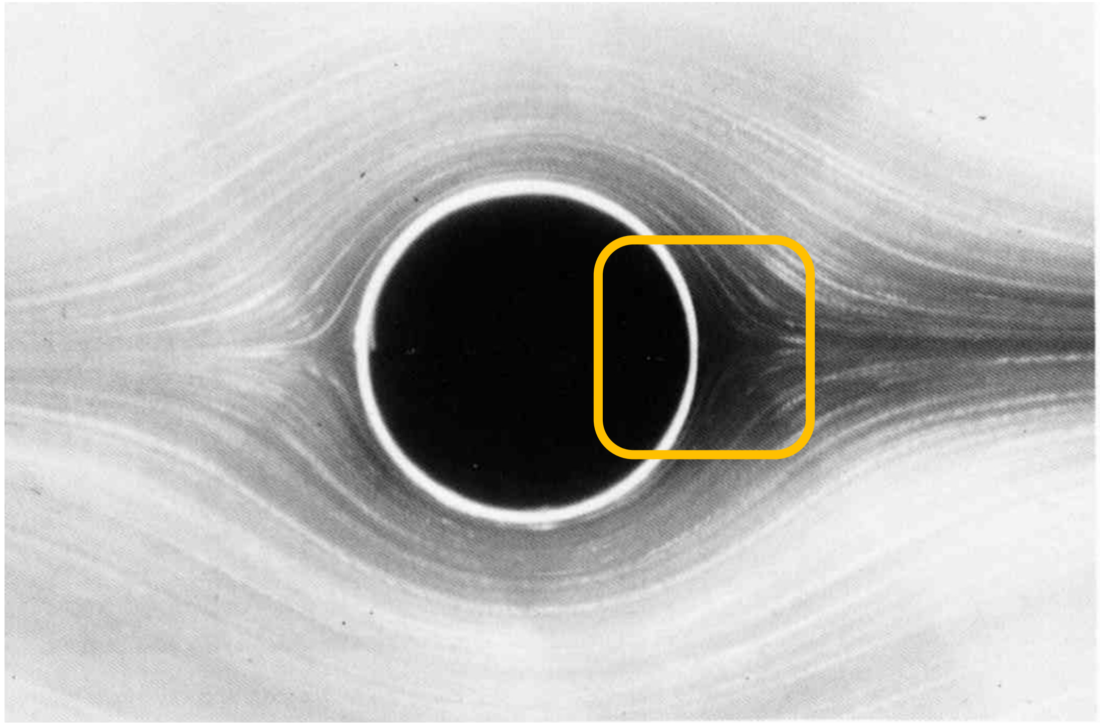


[Water and aluminum dust.]

Cylinder (Re = 0.16) – separation point

- $\nabla \cdot u = 0$
- $\frac{\partial u_1}{\partial x_1} = -\frac{\partial u_2}{\partial x_2}$

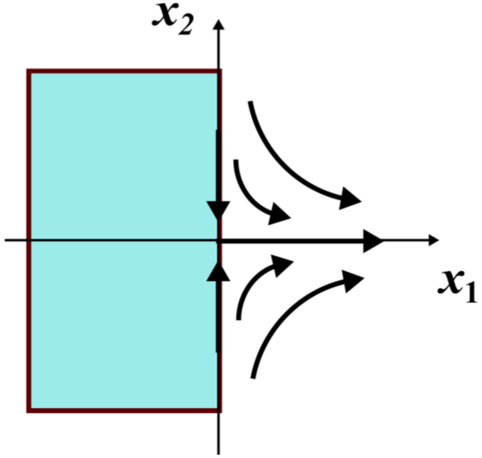


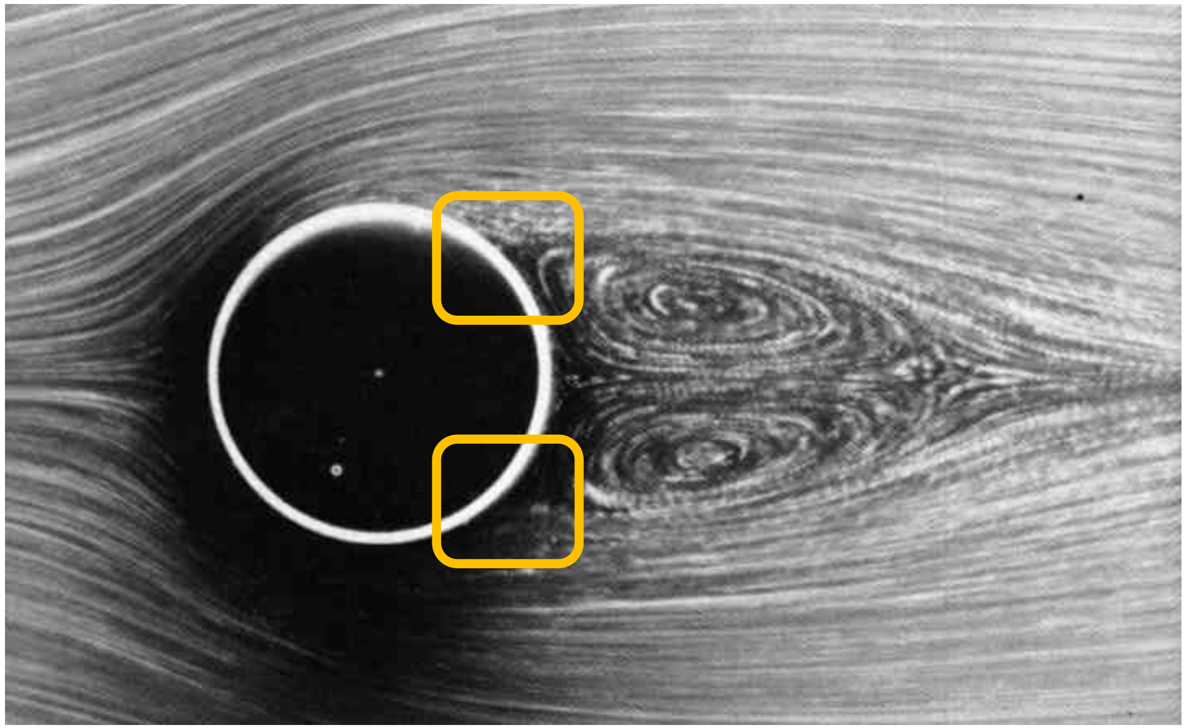


[Water and aluminum dust.]

Cylinder (Re = 26) – 2 separation points

- $\nabla \cdot u = 0$
- $\frac{\partial u_1}{\partial x_1} = -\frac{\partial u_2}{\partial x_2}$

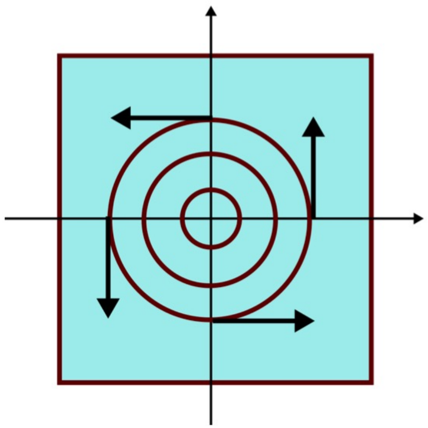


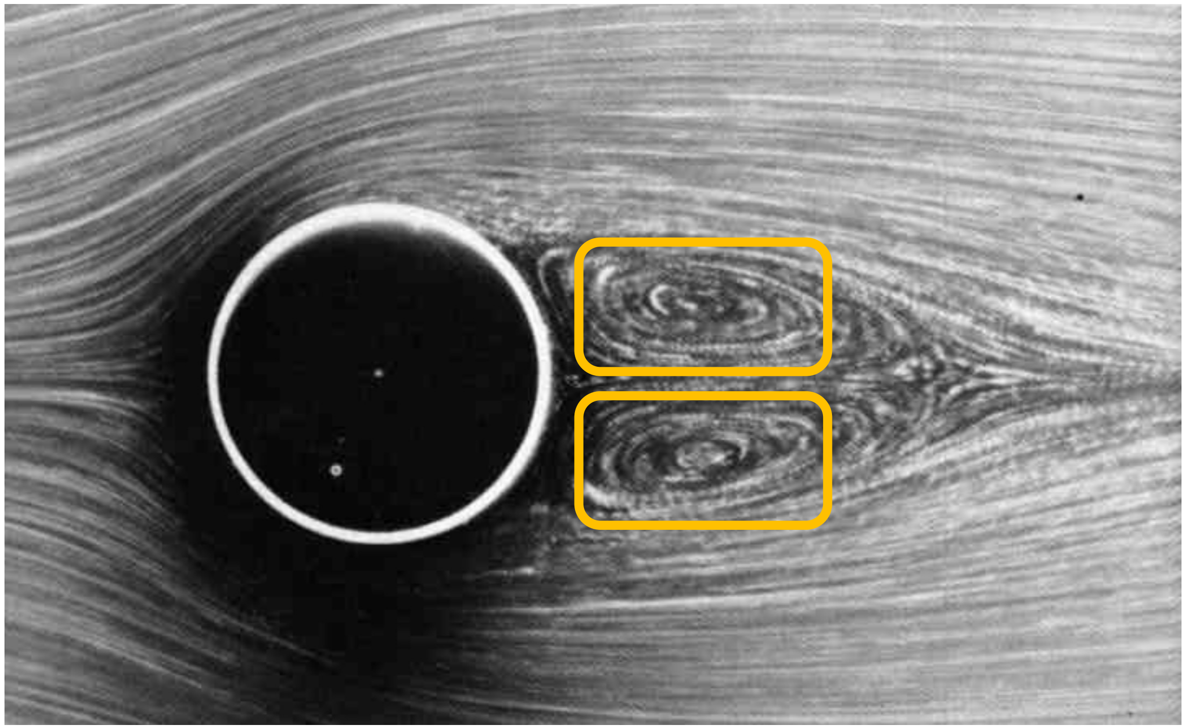


[Oil and magnesium.]

Cylinder (Re = 26) – 2 vortices

- $\nabla \cdot u = 0$
- $u_1 = f(x_2)$
- $u_2 = g(x_1)$



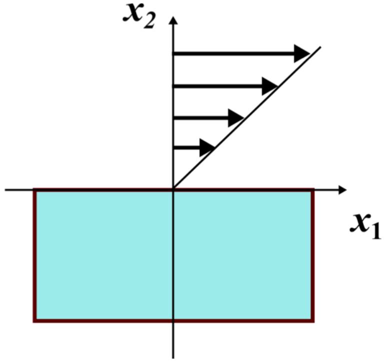


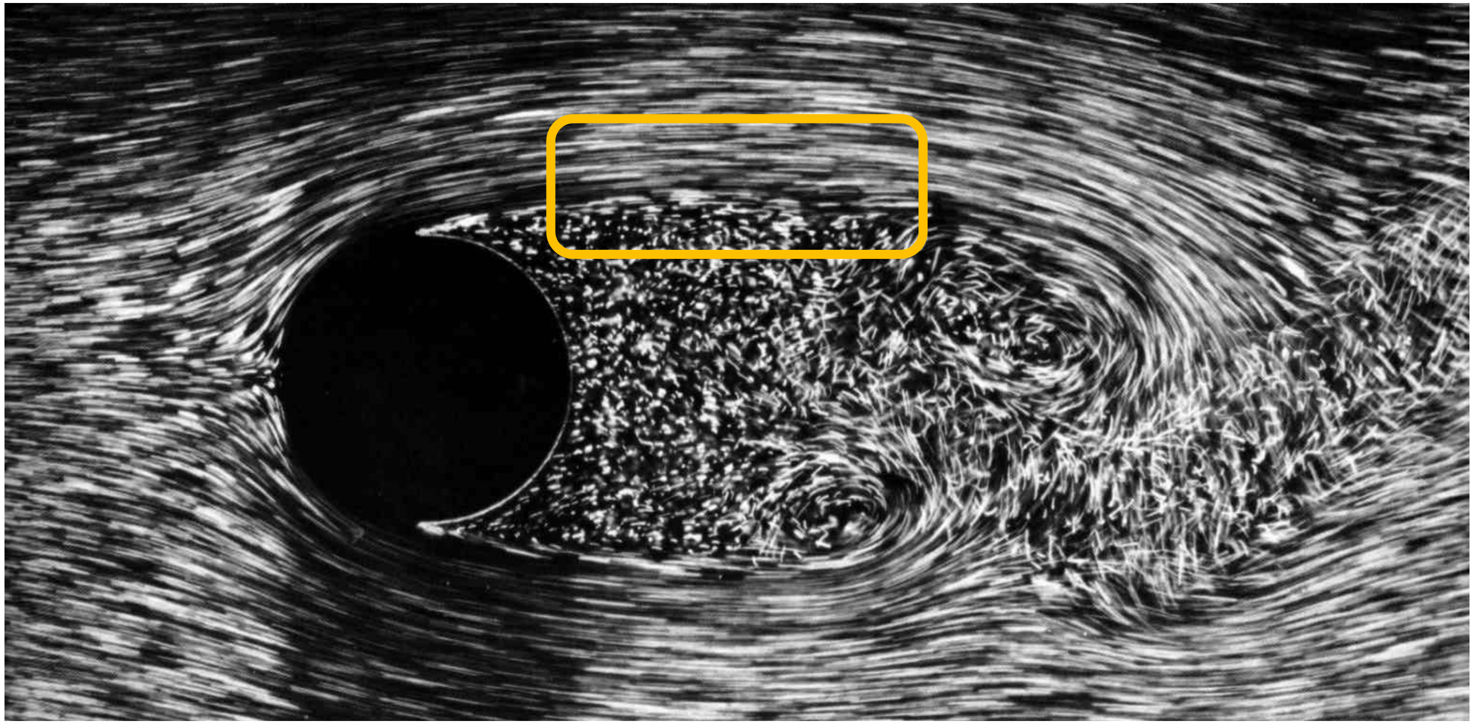
[Oil and magnesium.]

Cylinder (Re = 300) – Karman vortex street

[Wind and smoke.]

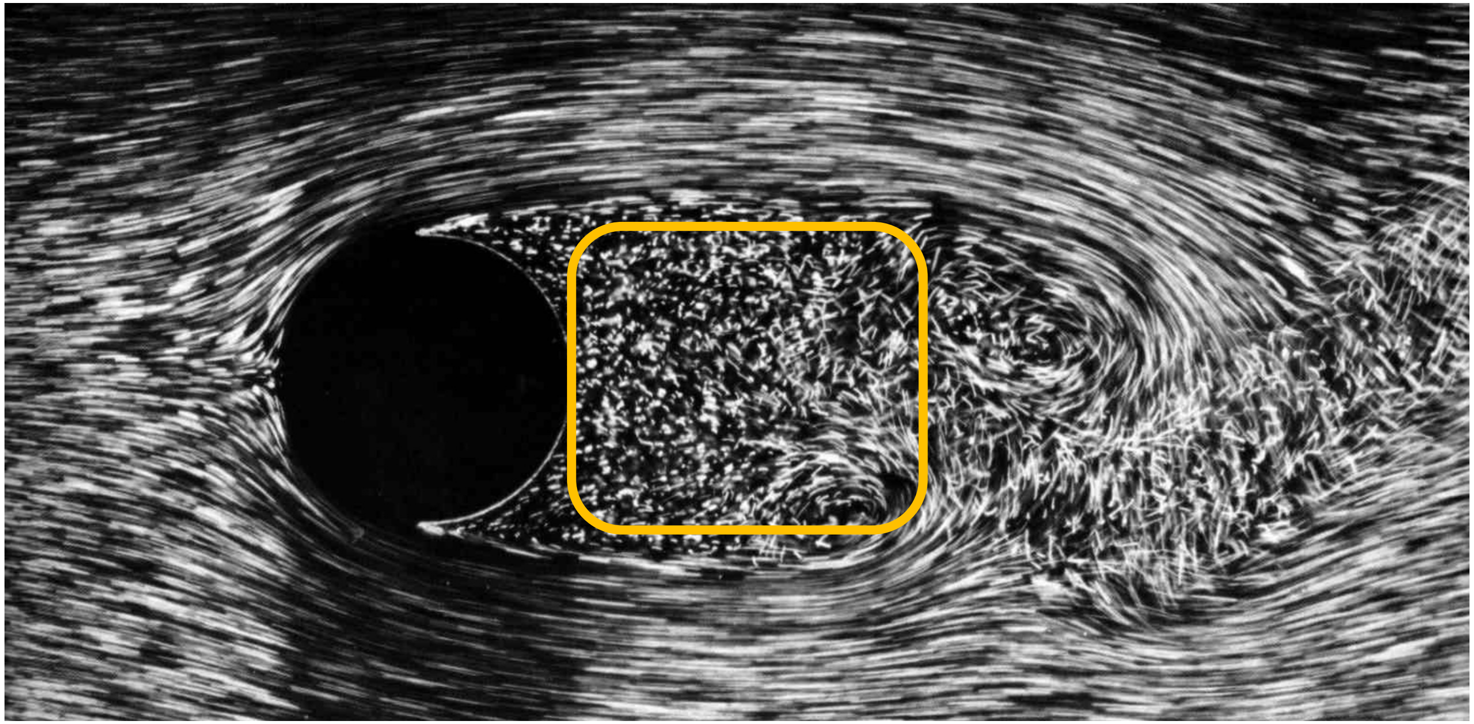
Cylinder (Re = 2000) – shear layer





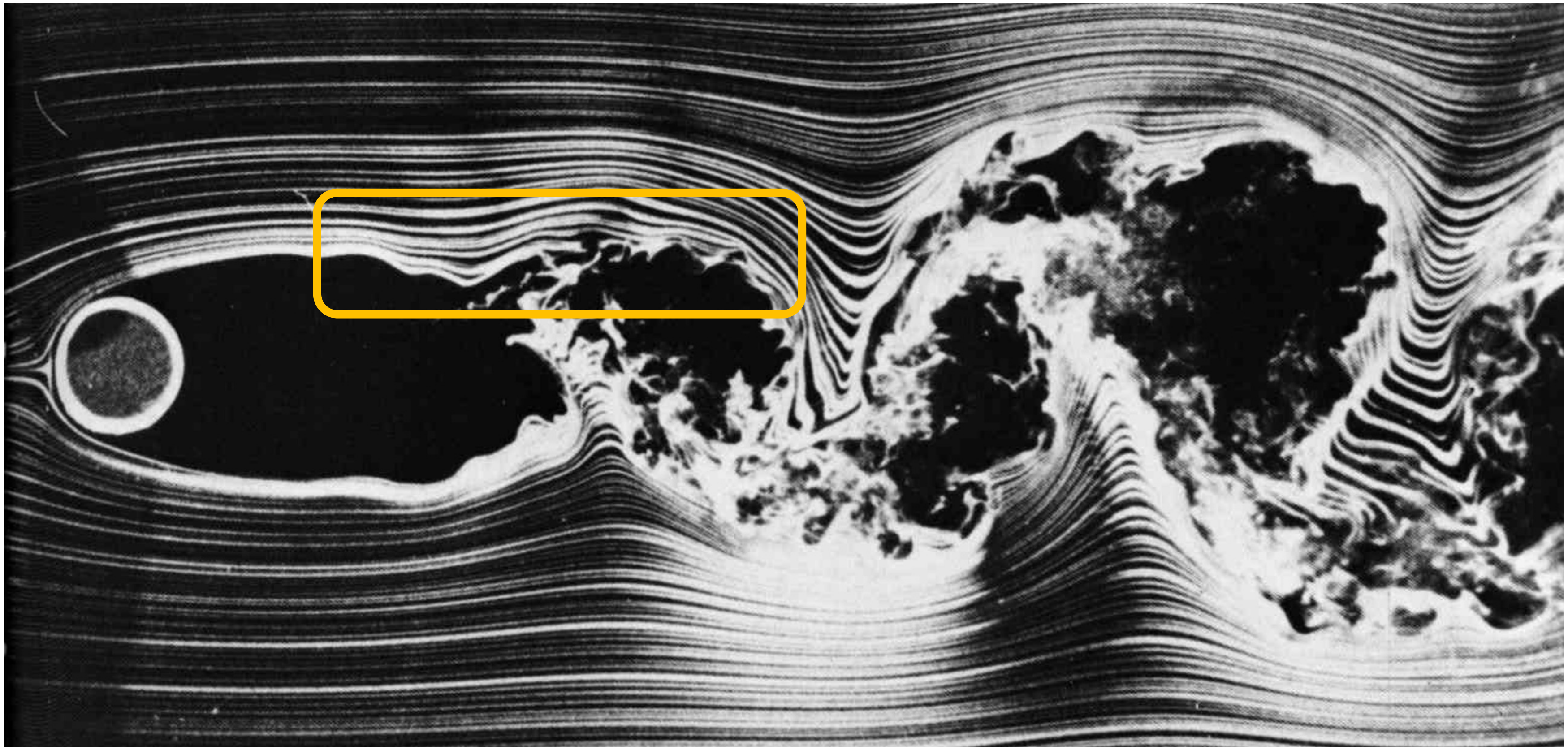
[Water and air bubbles.]

Cylinder (Re = 2000) – 3D turbulent wake



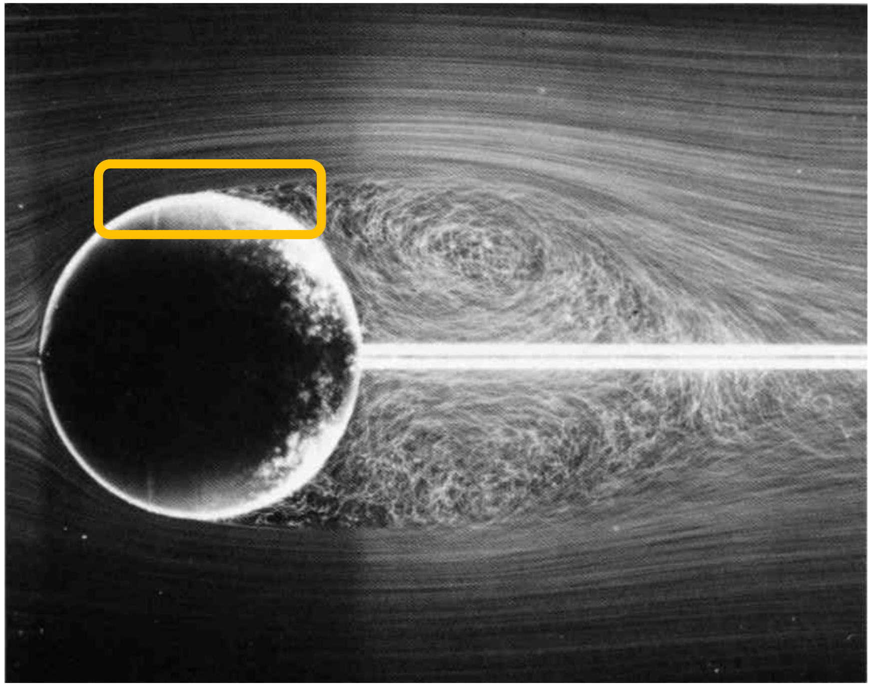
[Water and air bubbles.]

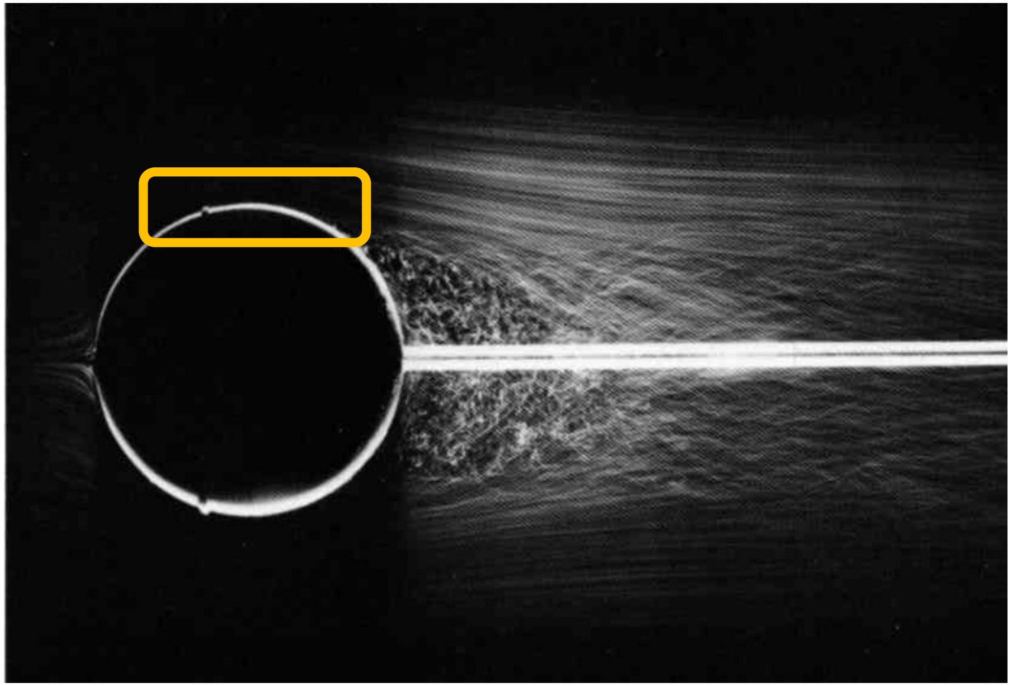
Re = 10 000 – turbulent shear layers



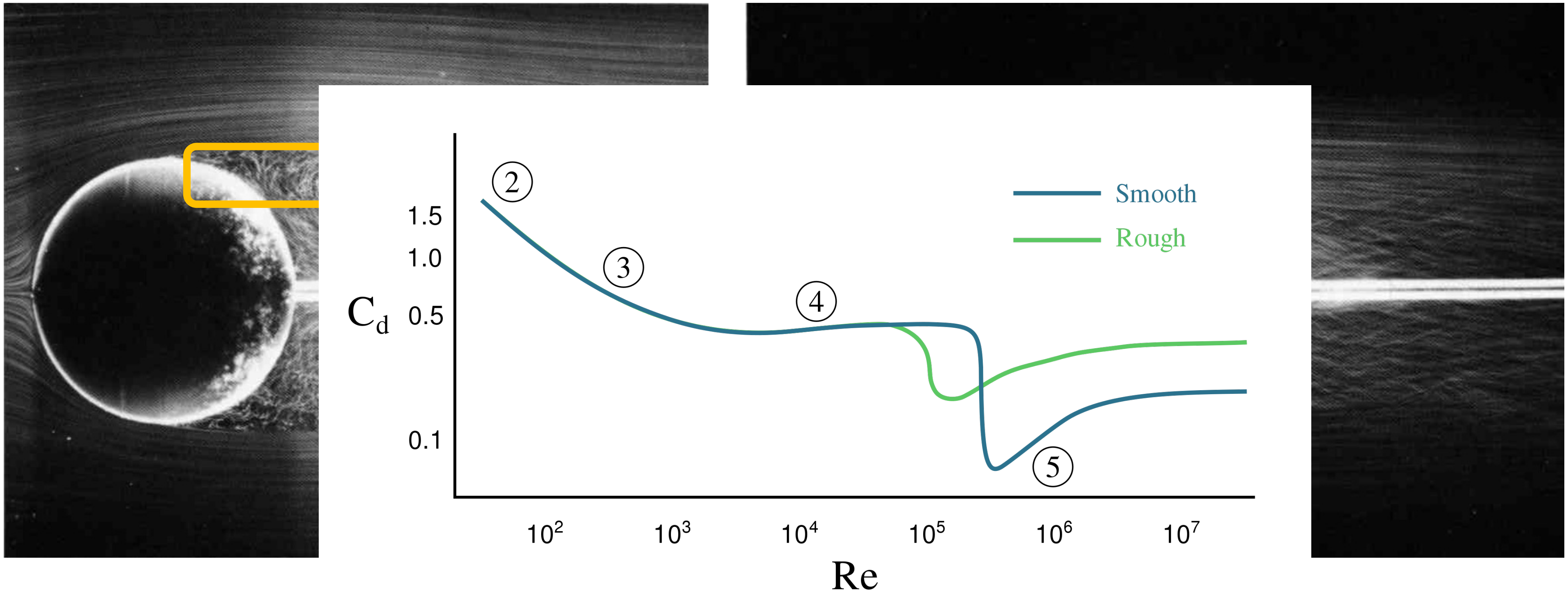
[Water and air bubbles.]

Sphere: Re = 15 000 vs 30 000
turbulent boundary layers (drag crisis)

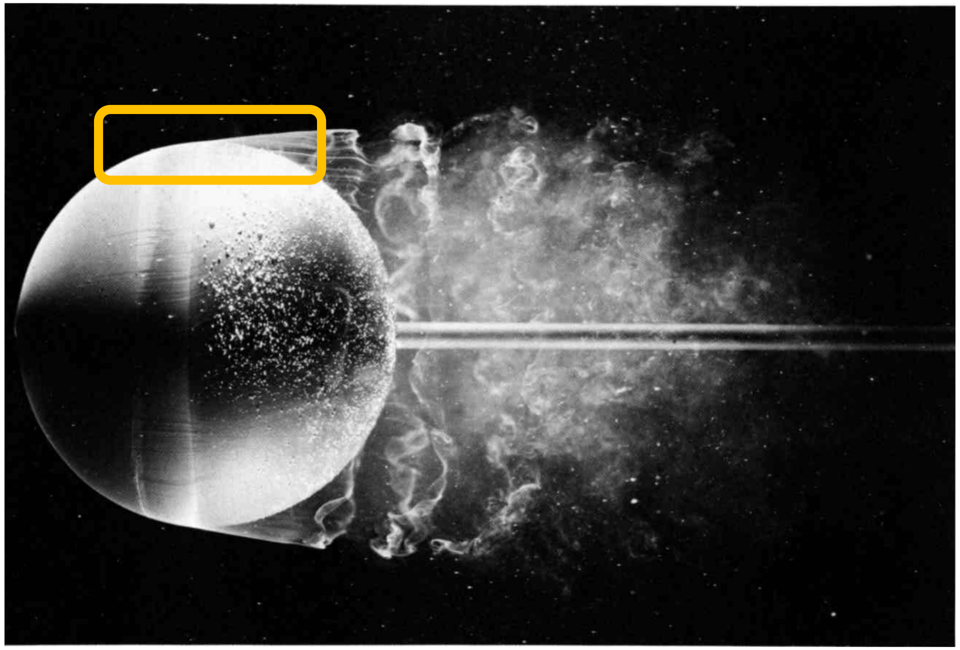


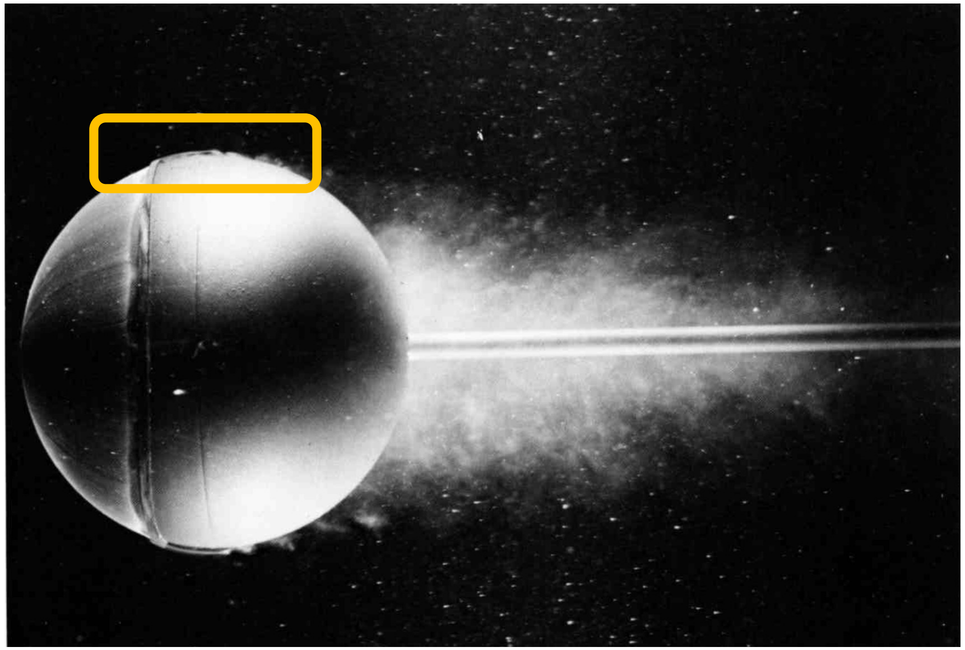


Sphere: Re = 15 000 vs 30 000
turbulent boundary layers (drag crisis)

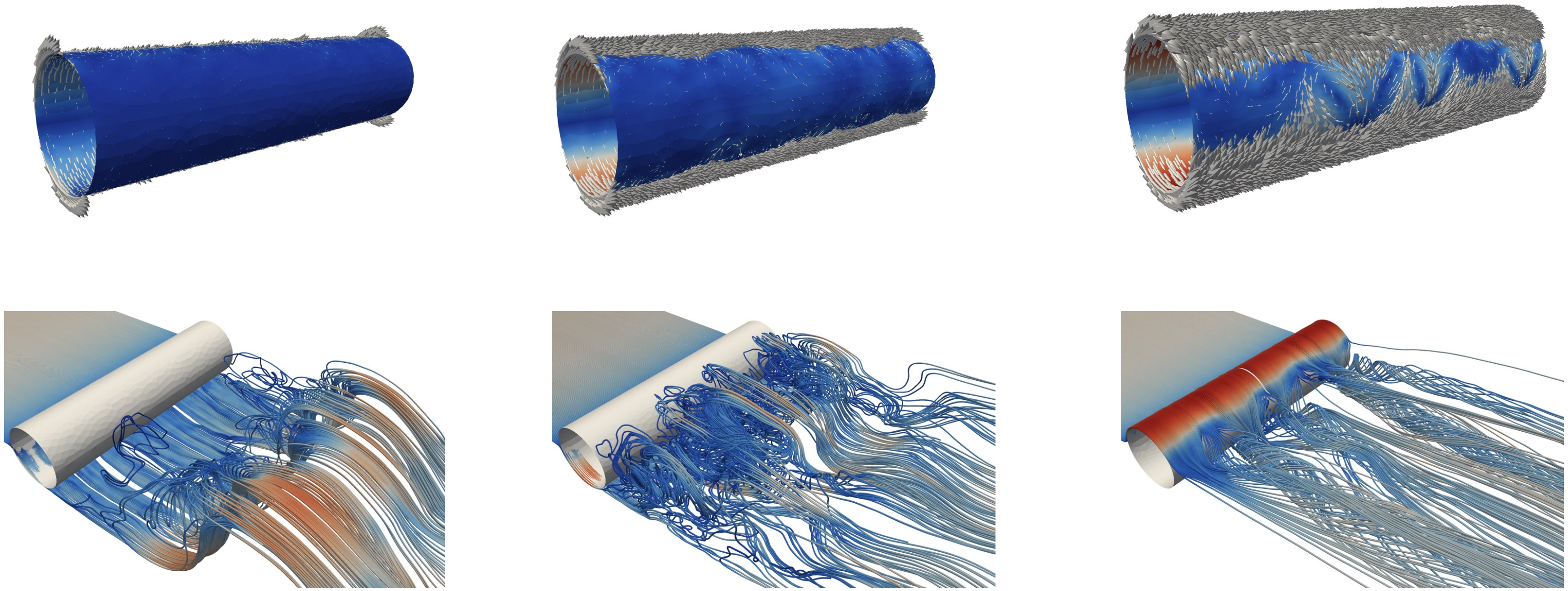


Sphere: Re = 15 000 vs 30 000
trip wire – to trigger turbulent boundary layer





Simulation of drag crisis – slip/friction bc



Demo 2D Navier-Stokes equations for high Re

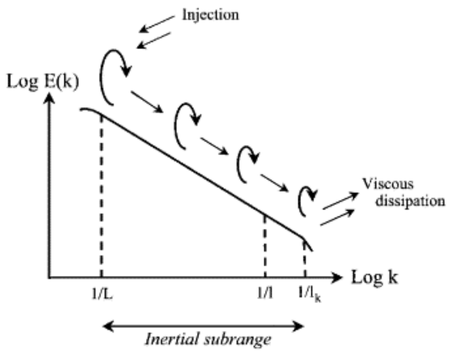
$$
\begin{aligned}
\dot{u} + (u \cdot \nabla)u + \nabla p - \nu \Delta u &= f, \qquad (x,t) \in \Omega \times I \\
\nabla \cdot u &= 0, \qquad (x,t) \in \Omega \times I \\
u(x,0) &= u^0(x) \qquad x \in \Omega
\end{aligned}
$$

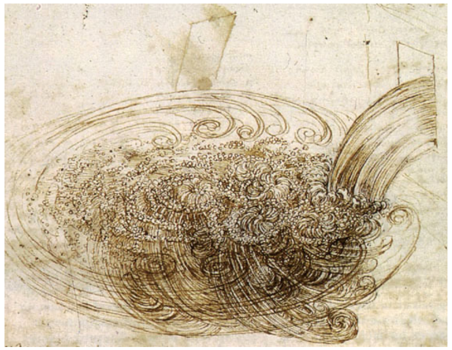
$\Omega \subset \mathbb{R}^3,\ I = (0,T),\ \Gamma = \partial\Omega$, velocity u, pressure p, kinematic viscosity ν, force f

Velocity boundary conditions: $u = u_D \quad (x,t) \in \Gamma_D \times I$

Stress boundary conditions (natural in weak form):
$-pn + \nu(n \cdot \nabla)u = t \quad (x,t) \in \Gamma_N \times I$

Direct Numerical Simulation (DNS)

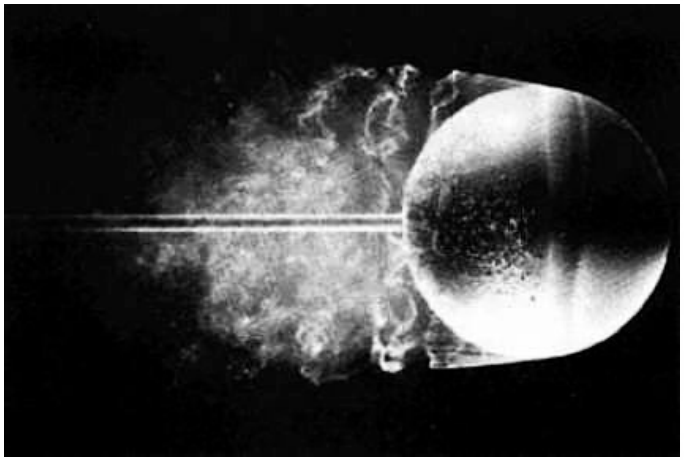
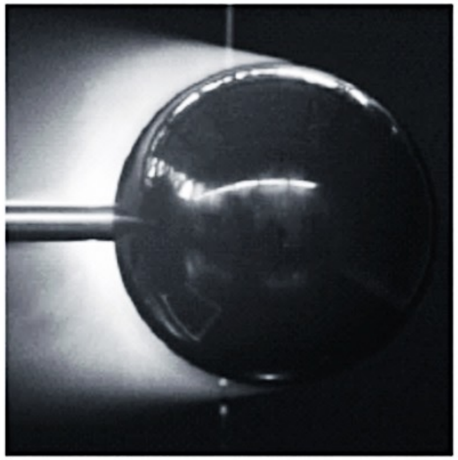





- Energy dissipation to heat at smallest scale in turbulent flow (Kolmogorov scale) ~ $Re^{-3/4}$
- Cost of Direct Numerical Simulation (DNS) ~ $Re^{9/4}$
- $Re > 10^6$: DNS impossible! Need cheaper models!

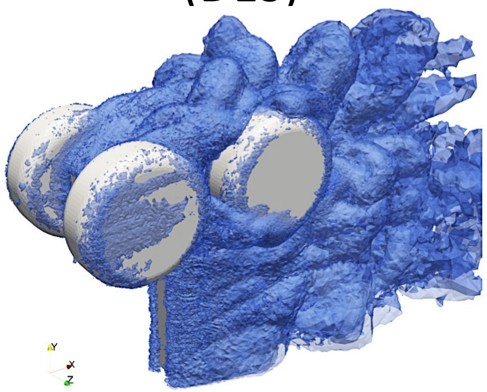
Reynolds Averaged NSE (RANS)

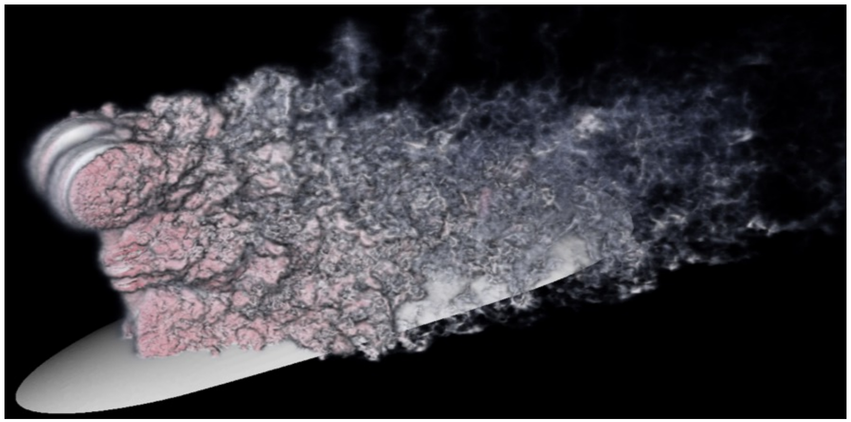
- Compute statistical ensemble average of NSE solutions
- Introduces Reynolds stresses that need to be modeled (turbulent eddy viscosity) : the closure problem
- Calibration of RANS model parameters is a challenge

Large Eddy Simulation (LES)

- Compute only the largest scales of the flow
- Filter NSE (spatial average): model subgrid scales
- Near walls "Resolved LES" impractical for high Re
- Wall-layer models: e.g. assume RANS near the wall (DES)





Direct FEM Simulation of Turbulence (DFS)

- Direct FEM approximation of the Navier-Stokes equations
  - No RANS/LES averaging/filtering (only the mesh scale)
  - No explicit turbulence/subgrid model (no closure problem)
  - Automatic turbulence model based on NSE residual
    (cf. Implicit LES, MILES [Fureby/Grinstein AIAA 99],
    VMM-LES [Bazilevs et.al. 07, Principe/Codina/Henke 10,…])

- Automatic mesh resolution : adaptive algorithm with error control in output of interest (drag, lift,…)
- Cheap wall-layer model : slip (friction) boundary condition
  (no boundary layer resolution : no boundary layer mesh)

Direct FEM Simulation of Turbulence (DFS)

- For (v,q) in W_h : find (U,P) in V_h such that

$$(D_t U + (U\cdot\nabla)U,v) + (\nu\nabla U,\nabla v) - (P,\nabla\cdot v) + (q,\nabla\cdot U) + (\delta R(U,P),R(v,q)) = (f,v)$$

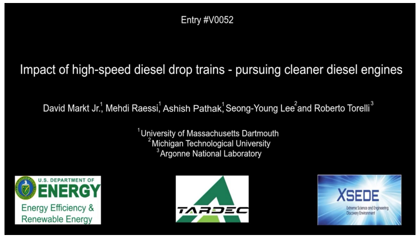
- Slip (no penetration) velocity: $u\cdot n = 0$
- Wall shear stress: $\tau = n^T\sigma t = \beta(u\cdot t)$ (β skin friction coefficient)

- Least squares stabilization of residual: R(U,P), with $\delta \sim h$
- $R(v,q) = [D_t v + (U\cdot\nabla)v + \nabla q - \nu\Delta v - f, \nabla\cdot v]^T$
- No explicit subgrid model of unresolved scales
- Dissipation: $-dK/dt = ||\beta^{1/2}u\cdot t||^2 + ||\nu^{1/2}\nabla U||^2 + ||\delta^{1/2}R(U,P)||^2$

Demo turbulence model

Multiphase flow

- https://gfm.aps.org/meetings/dfd-2020/5f5ec1d2199e4c091e67bd66



- https://gfm.aps.org/meetings/dfd-2020/5f5f6542199e4c091e67be2a

Multiphase flow

- https://gfm.aps.org/meetings/dfd-2020/5f5fc002199e4c091e67bf49

Credits

Gallery of fluid motion (American Physical Society)

- https://gfm.aps.org

Album of fluid flow (Milton Van Dyke)

- https://en.wikipedia.org/wiki/An_Album_of_Fluid_Motion
- https://www.abebooks.com/9780915760022/Album-Fluid-Motion-Milton-Dyke-0915760029/plp