ADVANCED COURSE

# Distributed Systems

## Consensus

### "The Paxos Protocol"

Paris Carbone

# Course Topics

- Intro to Distributed Systems
- Fundamental Abstractions and Failure Detectors
- Reliable and Causal Order Broadcast
- Distributed Shared Memory-CRDTs
- $\boxed{\text{Consensus (Paxos)}}$
- Replicated State Machines (OmniPaxos, Raft, Zab etc.)
- Time Abstractions and Interval Clocks (Spanner etc.)
- Consistent Snapshotting (Stream Data Management)
- Distributed ACID Transactions (Cloud DBs)

# CONSENSUS

- In consensus, the processes propose values
  - they all have to agree on one of these values
- Solving consensus is key to solving many problems in distributed computing
  - Total order broadcast (aka Atomic broadcast)
  - Terminating reliable broadcast
  - Dynamic group membership
  - Stronger shared store models

# Consensus Interface

***Events***

**Request**: ⟨c Propose | v⟩

**Indication**: ⟨c Decide | v⟩

***Properties:***

***C1, C2, C3, C4***

# Single Value Consensus Properties

C1. **Validity**

Any value decided is a value proposed

C2. **Agreement**

No two correct processes decide differently

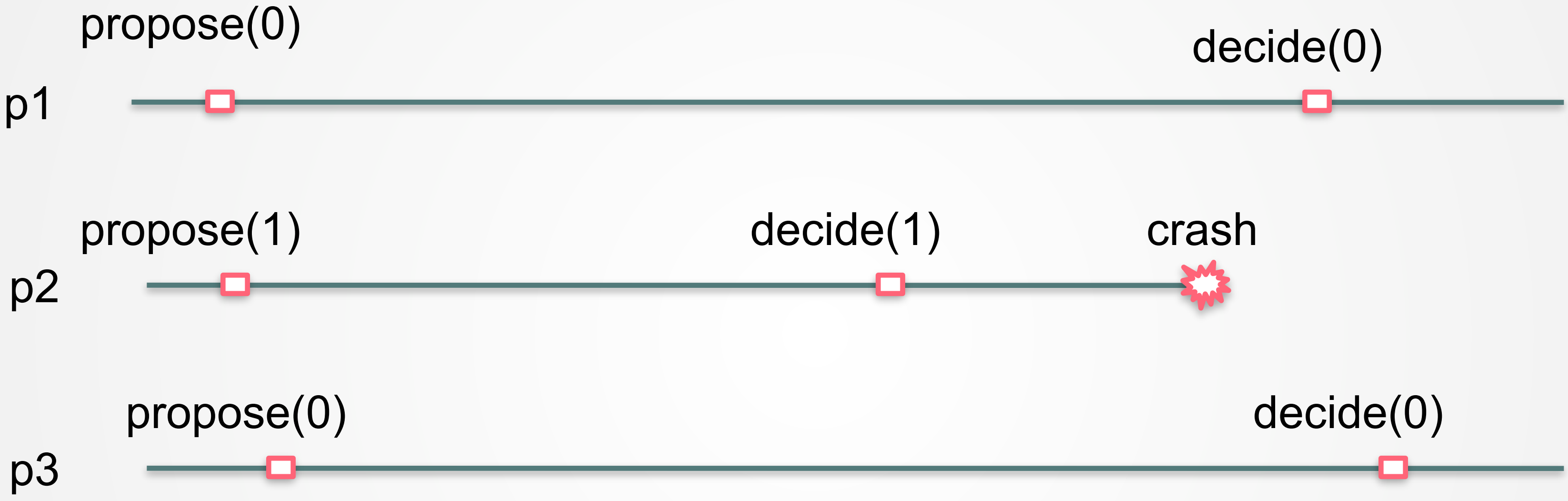
C3. **Termination**

Every correct process eventually decides

C4. **Integrity**

A process decides at most once

# SAMPLE EXECUTION




Does it satisfy consensus? yes

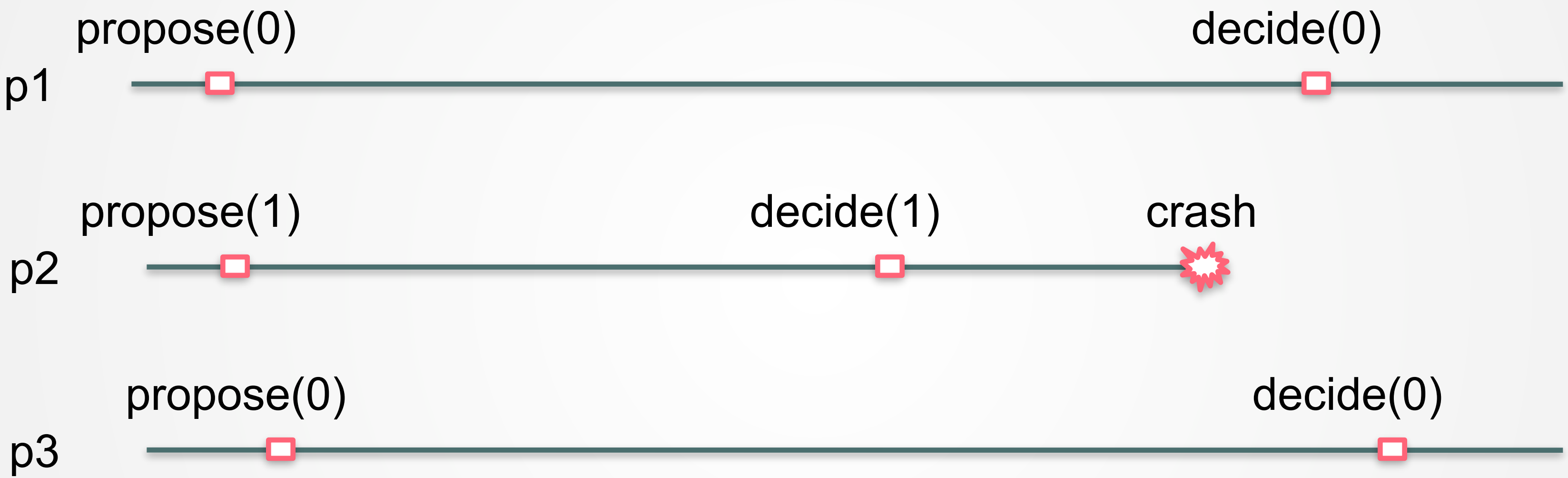
# Fail-Stop Model Algorithm

- **Hierarchical Consensus**
  - Rely on **P + BEB**
  - Round per process p1, …pn. Pi is leader of round i.
  - Each leader broadcasts and decides value
  - First correct process commits the decided value.
  - Each future leader adopts that value.

# Single Value Uniform Consensus

- Validity
  - Only proposed values may be decided
- Uniform Agreement
  - No two processes decide different values
- Integrity
  - Each processes can decide a value at most once
- Termination
  - Every process eventually decides a value

# SAMPLE EXECUTION




Does it satisfy uniform consensus? no

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# Single Value Uniform Consensus

- Solvable in Fail-Stop model (decide on last round) with strong FD
- **Not** solvable in the Fail-Silent model 😔 (asynchronous system model)
- Given a fixed set of deterministic processes there is no algorithm that solves consensus in the asynchronous model if one process may crash and stop
- There are some infinite executions that where processes are not able to decide on a single value
- Fischer, Lynch and Patterson FLP result

# ASSUMPTIONS

- Partially synchronous system
- Fail-noisy model
- Message duplication, loss, re-ordering

# Importance

- Paxos is arguably the most important algorithm in distributed computing
- This presentation follows the paper

  **"Paxos Made Simple"**

  (Lamport, 2001)

# High Level View of Paxos

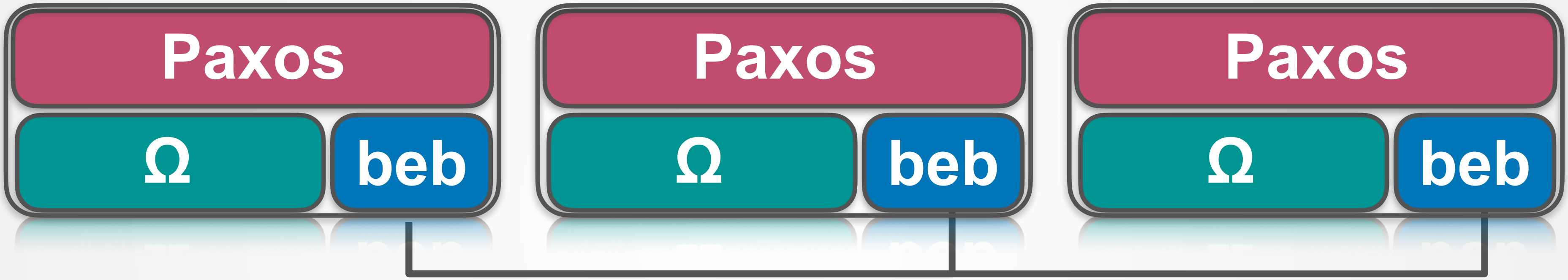
- Elect a single proposer using $\Omega$
  - Proposer imposes its proposal to everyone
  - Everyone decides

- Problem with $\Omega$
  - Several processes might initially be proposers (contention)

# High Level View of Paxos

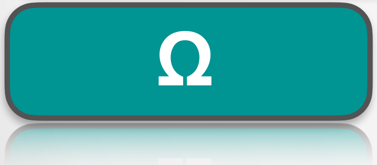
- Abortable Consensus (Paxos) saves the day
  - Processes attempt to impose their proposals
  - Might abort if there is contention (safety) (multiple proposers)
  - Ω ensures eventually 1 proposer succeeds (liveness)

# Typical Usage





Ensures correctness (safety)




Ensures termination (liveness)

(Leader ~ Paxos Proposer)

# The Paxos Algorithm

# TERMINOLOGY

- Proposers
  - Will attempt imposing their proposal to set of acceptors
- Acceptors
  - May accept values issued by proposers
- Learners
  - Will decide depending on acceptors acceptances

- Acceptors **cannot** communicate with each other.
- Proposers **cannot** communicate with each other either.
- Each process plays all 3 roles in classic setting

# STRAWMAN SOLUTION

- Centralized solution
  - Proposer sends value to a central acceptor
  - Acceptor decides first value it gets
- Problem
  - Acceptor is a single-point of failure

# Abortable Consensus

- Decentralises acceptors, i.e. proposers talks to set of acceptors
- Tolerate failures, i.e. acceptors might fail (needs only a majority of acceptors surviving)
- Proposers might fail to impose their proposals (aborts)

# Decentralization & Fault-tolerance

- Quorum approach
  - Each proposer tries to impose its value v on the set of acceptors
  - If majority of acceptors accept v, then v is chosen
  - Learners try to decide the chosen value

# Ballot (round) Array (table)

- Describes the state of the acceptors at various rounds
- Each row describes one round
- Each acceptor's state of $a_i$ initially $\perp$

| Round | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| n = 5 | | | |
| ... | | | |
| n=2 | | | |
| n=1 | | | |
| n=0 | $\perp$ | $\perp$ | $\perp$ |

... Learners
... can query/read acceptor states at any round

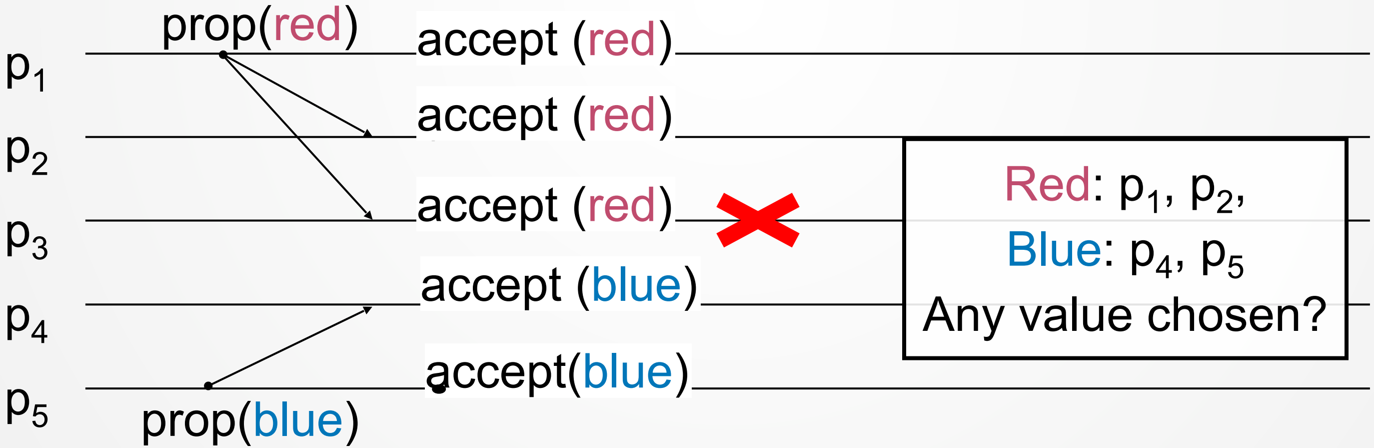
# When to accept

- Ideally, there will be a single proposer
  - Should at least provide obstruction-free progress
    - Obstruction-free = if a single proposer executes without interference (contention) it makes progress

- Suggested invariant
  - P1. An acceptor accepts first proposal it receives

# ATTEMPT

- P1. An acceptor accepts first proposal it receives
- Problem
  - Impossible to later tell what was chosen
  - Forced to allow restarting! Let acceptors change their minds!



# BALLOT (ROUND) ARRAY (TABLE)

Two proposers p1 and p2 that propose red and blue

But $a_3$ crashes

| Round | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| n = 5 | | | | | |
| ... | | | | | |
| n=2 | | | | | |
| n=1 | red | red | red | blue | blue |
| n=0 | $\perp$ | | | $\perp$ | $\perp$ |

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# BALLOT (ROUND) ARRAY (TABLE)

Two proposers p1 and p2 that propose red and blue

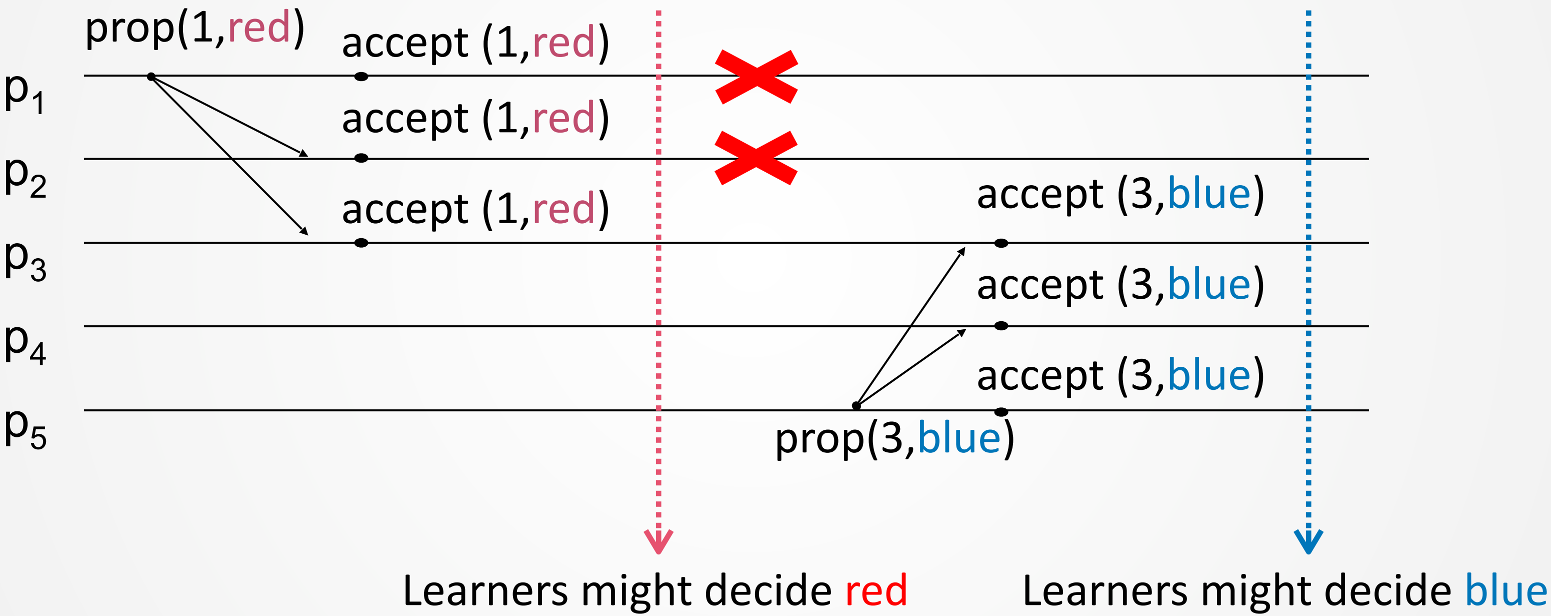
But $a_3$ crashes

| Round | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| n = 5 | | | | | |
| ... | | | | | |
| n=2 | | | | | |
| n=1 | red | red | | blue | blue |
| n=0 | $\perp$ | $\perp$ | $\perp$ | $\perp$ | $\perp$ |

# ENABLING RESTARTING

- Proposer can try to propose again
  - Distinguish proposals with unique sequence number
  - Often called ballot number
  - Monotonically increasing
- Implementation with n nodes
  - process 1 uses seq: 1, n+1, 2n+1, 3n+1, …
  - process 2 uses seq: 2, n+2, 2n+2, 3n+2, …
  - process 3 uses seq: 3, n+3, 2n+3, 3n+3, …
- or…
  - Pair of values: (local clock or logical clock, local identifier)
  - Lexicographic order: if clock collides, choose highest pid

# PROBLEM WITH RESTART



# Ballot (Round) Array (Table)

p1 proposes (1,red) and p2 proposes (3, blue)

But $a_1$ and $a_2$ crashed

| Round | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| n = 5 | | | | | |
| n = 4 | | | | | |
| n = 3 | | | blue | blue | blue |
| n = 2 | red | red | red | ⊥ | ⊥ |
| n = 1 | red | red | red | ⊥ | ⊥ |
| n = 0 | ⊥ | | | ⊥ | ⊥ |

?

# Ensuring Agreement

- Problem (previous slide):
  - If restarting allowed,
    - Majority may first accept red
    - Majority may later accept blue
- Solve it by enforcing:
  - P2. If proposal (n,v) is chosen, every higher numbered proposal chosen has value v

# BIRDS-EYE VIEW

- Abortable Consensus in a nutshell
  - P1. An acceptor accepts first proposal it receives
  - P2. If v is chosen, every higher proposal chosen has value v
- Handwaving
  - P1 ensures obstruction-free progress and validity
  - P2 ensures agreement
  - Integrity trivial to implement
    - Remember if chosen before, at most choose once

# ATTEMPT

P2. If v is chosen, every higher proposal chosen has value v

**How to implement it?**

P2a. If v is chosen, every higher proposal accepted has value v

**Lemma**

P2a => P2

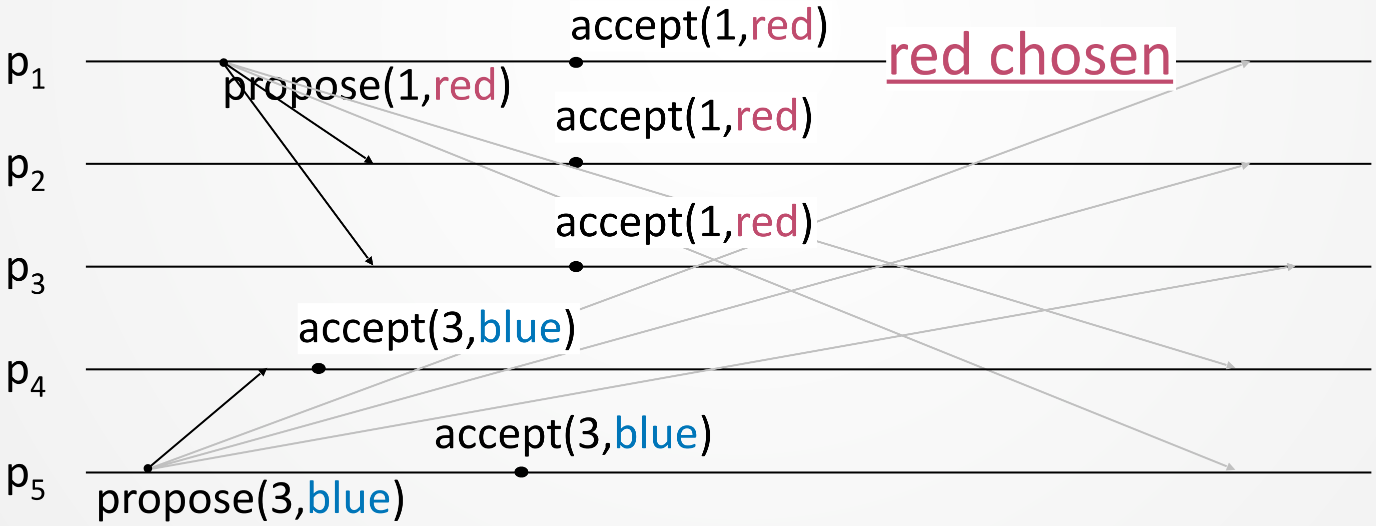
# PROBLEM

Recall

P1. An acceptor accepts first proposal it receives

P2a. If v is chosen, every higher proposal accepted has value v

Problem: we cannot prevent an acceptor from accepting higher value proposal

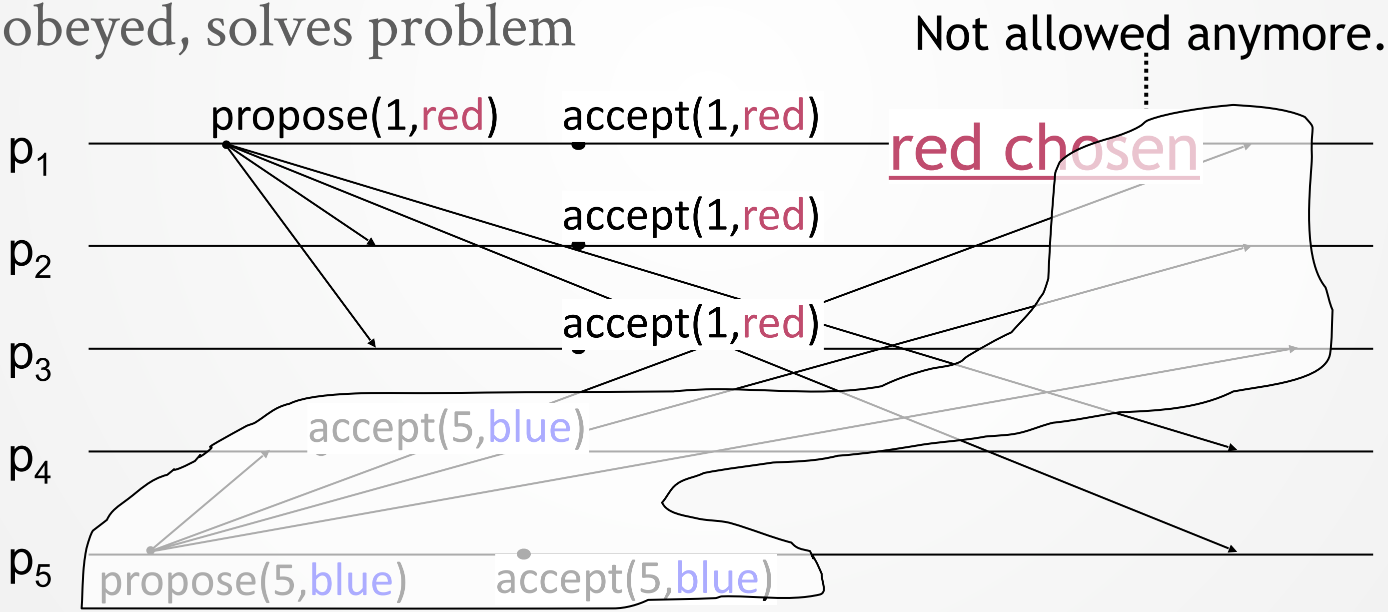


# SOLUTION

Strengthen P2a

P2b. If v is chosen, every higher proposal issued has value v

If obeyed, solves problem



# Ballot (round) Array (table)

p1 proposes (1,red) and p2 proposes (3, blue)

But $a_1$ and $a_2$ crashed before p2 proposes (3, blue)

| Round | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| n = 5 | | | | | |
| n = 4 | | | | | |
| n = 3 | | | red | $\perp$ | $\perp$ |
| n=2 | red | red | red | $\perp$ | $\perp$ |
| n=1 | red | red | red | $\perp$ | $\perp$ |
| n=0 | $\perp$ | | | $\perp$ | $\perp$ |

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# BALLOT (ROUND) ARRAY (TABLE)

p1 proposes (1,red) and p2 proposes (3, blue)

At round 3 p2 **has to issue** (3,red)

| Round | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| n = 5 | | | | | |
| n = 4 | | | | | |
| n = 3 | | | red | red | red |
| n=2 | red | red | red | ⊥ | ⊥ |
| n=1 | red | red | red | ⊥ | ⊥ |
| n=0 | ⊥ | | | ⊥ | ⊥ |

# P2 PRESERVED

- P2. If v is chosen, every higher proposal chosen has value v
- P2a. If v is chosen, every higher proposal accepted has value v
- P2b. If v is chosen, every higher proposal issued has value v

- **Lemma**
  - P2b => P2a
- Recall P2a => P2.
  - Thus P2b => P2

# MAIN LEMMA

- P2c. If any proposal (n,v) is issued, there is a majority set S of acceptors such that either
  - (a) no one in S has accepted any proposal numbered less than n
  - (b) v is the value of the highest proposal among all proposals less than n accepted by acceptors in S

- Lemma: P2c => P2b

# CASE A

(a) no one in S has accepted any proposal number < 3

p2 issues (3, blue) at round 3

| Round | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| n = 5 | | | | | |
| n = 4 | | | | | |
| n = 3 | red | red | blue | blue | blue |
| n=2 | red | red | ⊥ | ⊥ | ⊥ |
| n=1 | red | red | ⊥ | ⊥ | ⊥ |
| n=0 | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |

# Case B

- (b) v is the value of the highest proposal among all proposals less than n accepted by acceptors in S
- red is chosen at round 3, no proposer at round 4
- Proposer at round 5 will always get red querying any majority

| Round | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| n = 5 | | | | | |
| n = 4 | | | | | |
| n = 3 | red | red | red | ? | ? |
| n=2 | red | red | ? | ? | ? |
| n=1 | red | red | ⊥ | ⊥ | ⊥ |
| n=0 | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |

# CASE B

- (b) v is the value of the highest proposal among all proposals less than n accepted by acceptors in S
- red is chosen at round 3, no proposer at round 4
- Proposer at round 5 will always get red querying any majority

| Round | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| n = 5 | | red | red | red | |
| n = 4 | | | | | |
| n = 3 | red | red | red | ? | ? |
| n=2 | red | red | ? | ? | ? |
| n=1 | red | red | ⊥ | ⊥ | ⊥ |
| n=0 | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |

# How to implement P2c

- A proposer at round **n** needs a query phase to get

   1. the value of highest round number
   2. a promise that the state of S does not change <u>until round **n**</u>

| Round | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| n = 5 | | | | | |
| n = 4 | | | | | |
| n = 3 | red | red | red | ? | ? |
| n=2 | red | red | ? | ? | ? |
| n=1 | red | red | ⊥ | ⊥ | ⊥ |
| n=0 | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |

# Prepare Phase

- A proposer issues prop(n, v)
- Guarantee (P2c)?
  - v is the value of the highest proposal among all proposals less than n accepted by acceptors in S
- Need a prepare(n) phase before issuing prop(n, v)
  - Extract a promise from a majority of acceptors not to accept a proposal less than n
  - Acceptor sends back its highest numbered accepted value

# Abortable Consensus in Paxos

**Proposer**

Pick unique sequence n, send prepare(n) to all acceptors

3) Proposer upon majority S of promises:

Pick value v of highest proposal number in S, or if none available pick v freely

Issue accept(n,v) to all acceptors

5) Proposer upon majority S of responses:

If got majority of acks

decide(v) and broadcast decide(v);

Otherwise abort

**Acceptors**

2) Upon prepare(n):

- Promise not accepting proposals numbered less than n
- Send highest numbered proposal accepted with number less than n (promise)

5) Upon accept(n,v):

- If not responded to prepare m>n, accept proposal (ack); otherwise reject (nack)

**abortable consensus satisfies:**

P2c. If (n,v) is issued, there is a majority of acceptors S such that:

a) no one in S has accepted any proposal numbered "<" n, OR

b) v is value of highest proposal among all proposals "<" n accepted by acceptors in S

# Getting Familiar with Paxos

# MESSAGE LOSS AND FAILURES

- Many sources of abort
  - Contention (multiple proposals competing)
  - Message loss (e.g. not getting an ack)
  - Process failure (e.g. proposer dies)
- So Proposers try Abortable Consensus again…
  - Prepare(5), Accept(5,v), Prepare(15), …
  - Eventually the Paxos should terminate (FLP85?)

# FLP GHOST

$p_1$: a.prep(1):ok  b.prep(3):ok  a.acpt(1,v):fail  a.prep(4):ok  b.acpt(3,v):fail

$p_2$: a.prep(1):ok  b.prep(3):ok  a.acpt(1,v):fail  a.prep(4):ok  b.acpt(3,v):fail

$p_3$: a.prep(1):ok  b.prep(3):ok  a.acpt(1,v):fail  a.prep(4):ok  b.acpt(3,v):fail

proposers a and b forever racing…

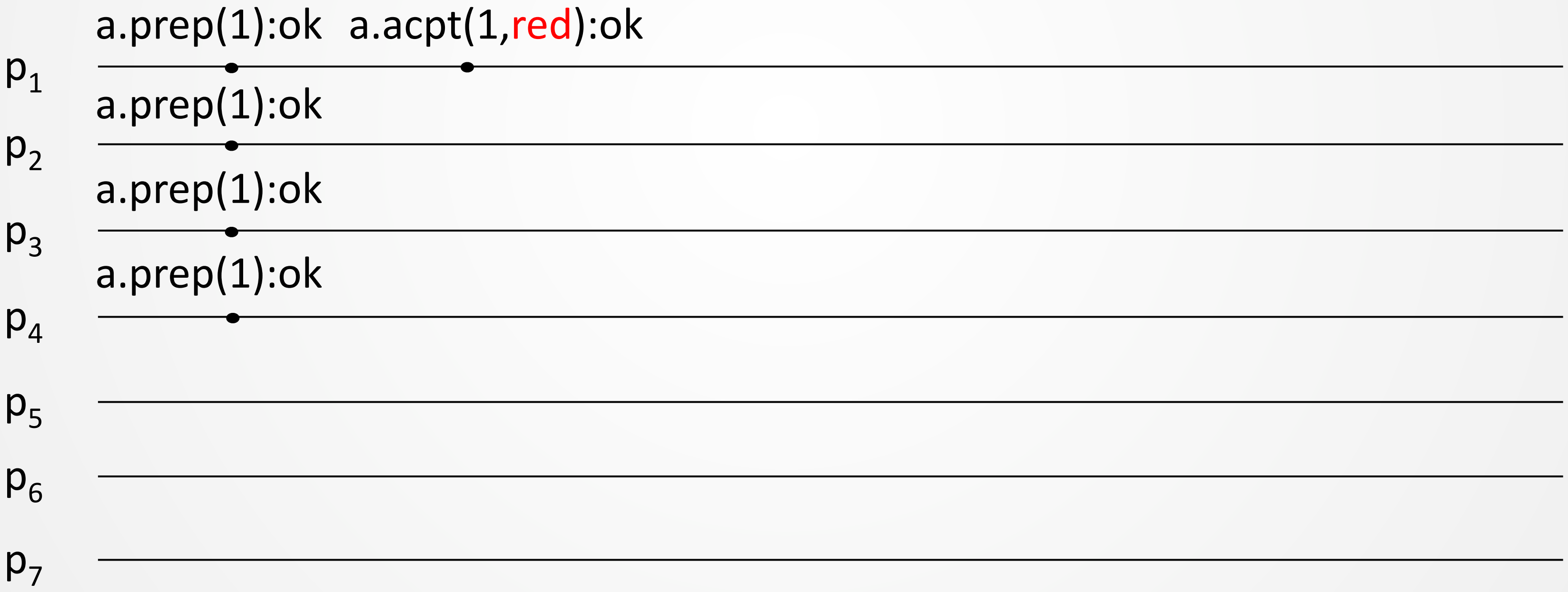
Eventual leader election (Ω) ensures liveness

Eventually only one proposer => termination

# FAMILIARIZING WITH PAXOS (1/4)

Different processes accept different values , same process accepts different values

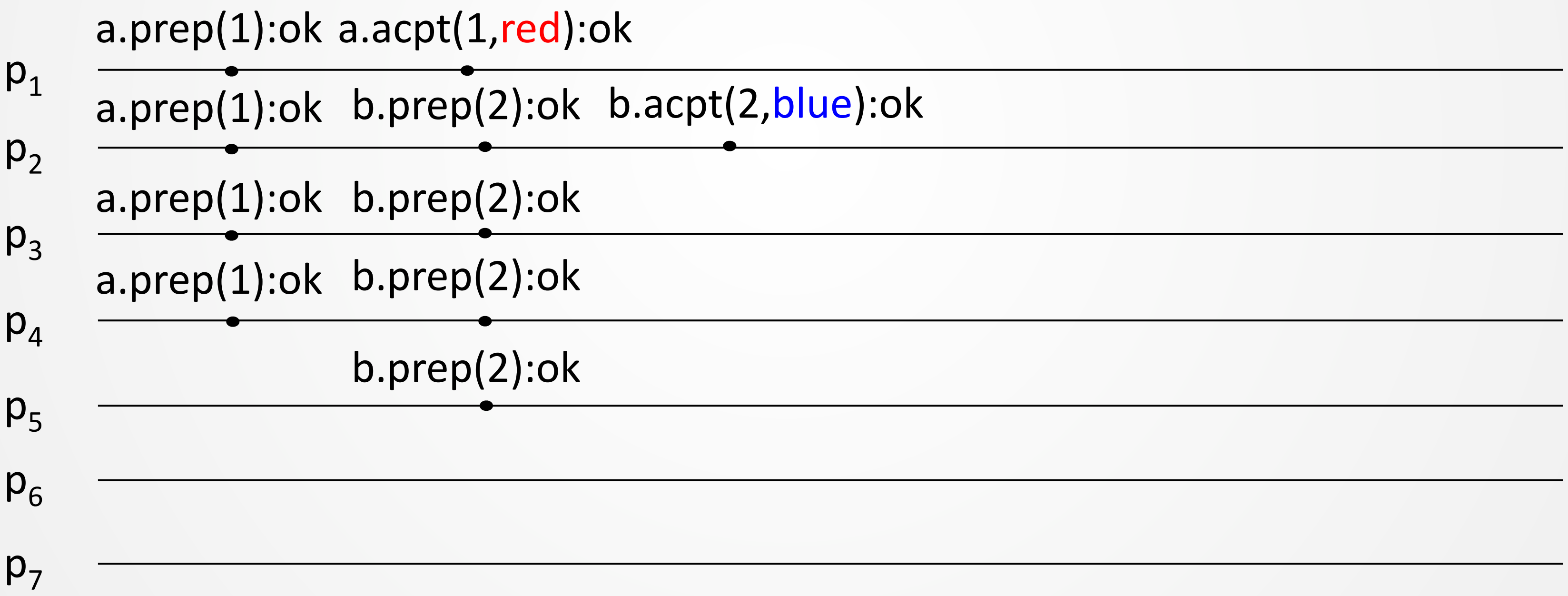
Assume 4 proposers {a,b,c,d}, 7 acceptors {$p_1$,…,$p_7$}



# Familiarizing with Paxos (2/4)

Different processes accept different values , same process accepts different values

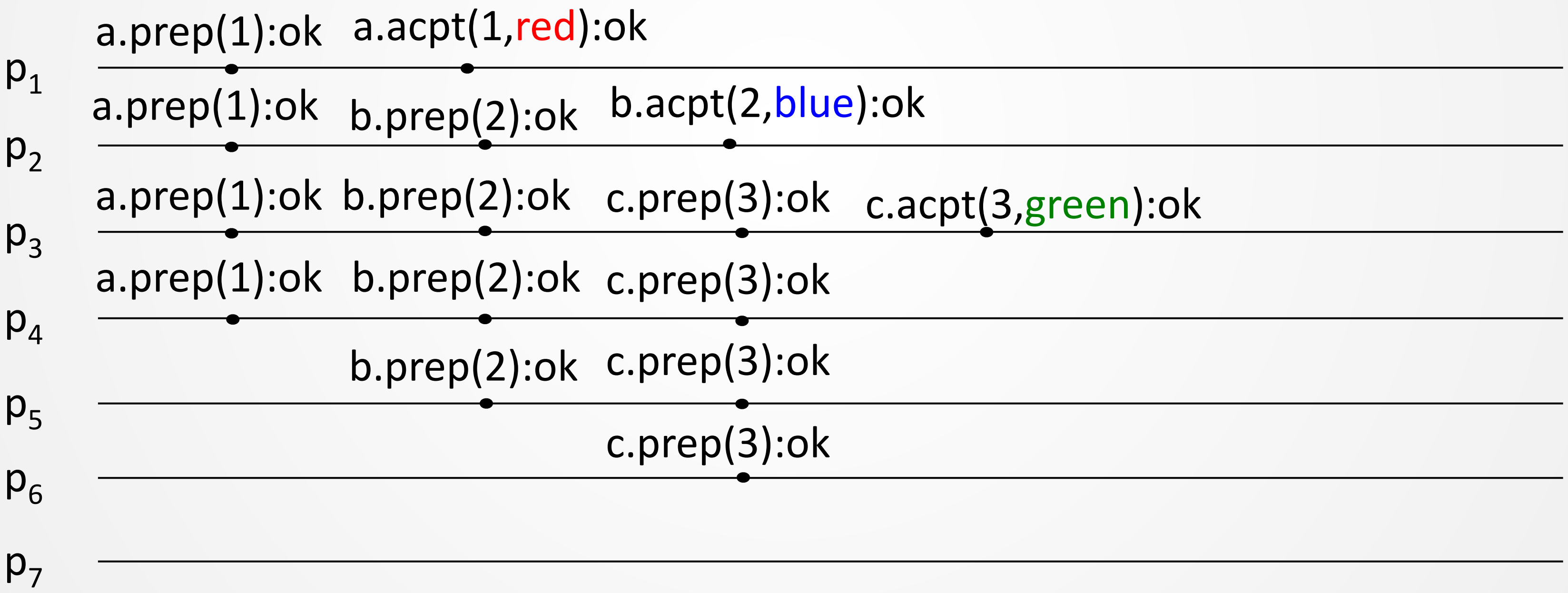
Assume 4 proposers {a,b,c,d}, 7 acceptors $\{p_1,\ldots,p_7\}$



# FAMILIARIZING WITH PAXOS (3/4)

Different processes accept different values , same process accepts different values

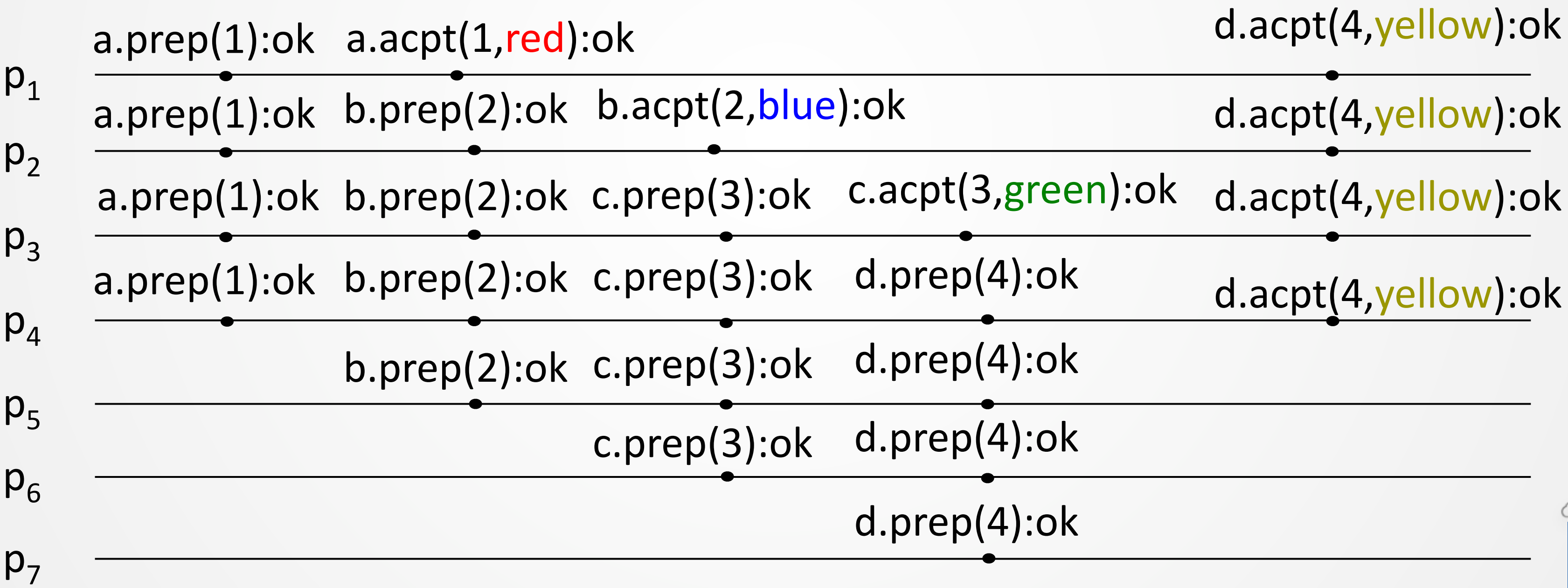
Assume 4 proposers {a,b,c,d}, 7 acceptors $\{p_1,\ldots,p_7\}$



# FAMILIARIZING WITH PAXOS (4/4)

Different processes accept different values , same process accepts different values

Assume 4 proposers {a,b,c,d}, 7 acceptors $\{p_1,\ldots,p_7\}$



# Optimizations

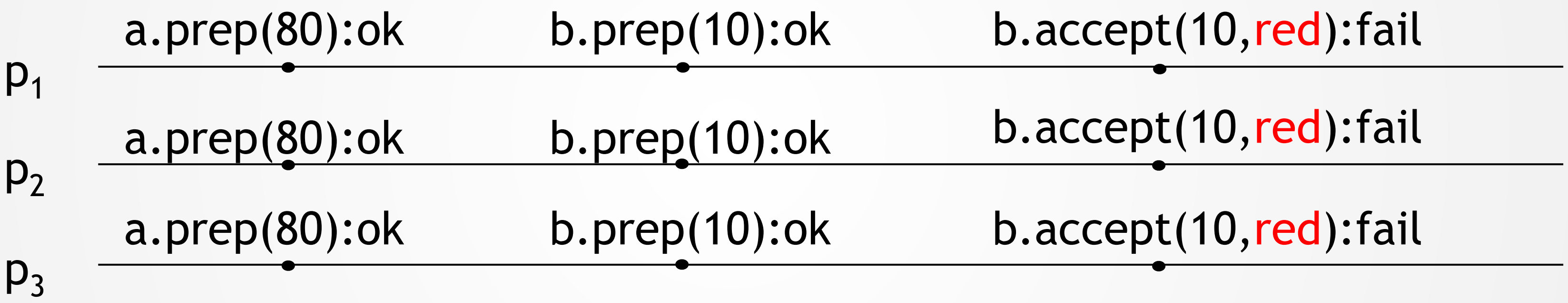
# PAXOS (AC) IN A NUTSHELL

- Necessary
  - Reject accept(n,v) if answered prepare(m) : m>n
    - i.e. prepare extracts promise to reject lower accept

# Possible scenario #1

Caveat

- Proposers {a,b,c}, acceptors $\{p_1,p_2,p_3\}$




- accept(10) will be rejected, why answer prepare(10)?
- No point answering prepare(n) if accept(n,v) will be rejected

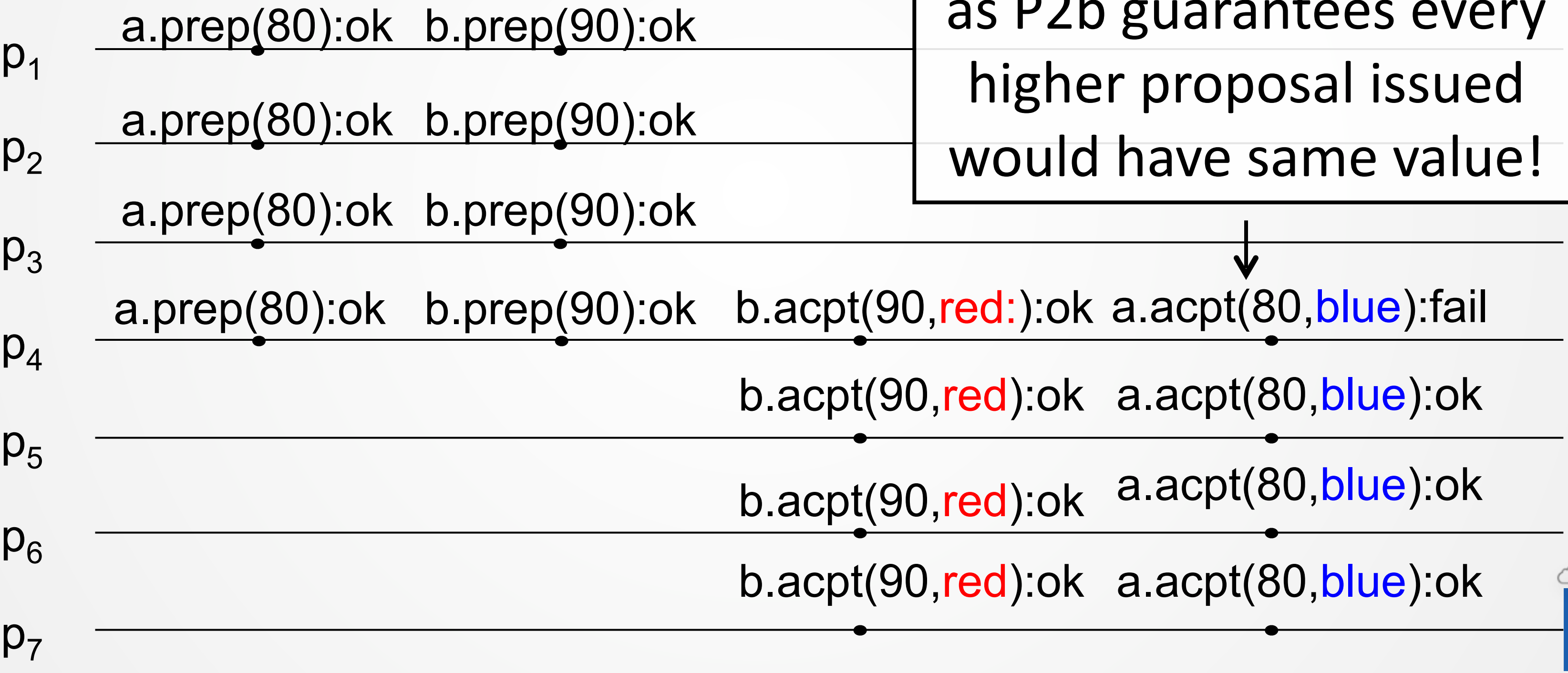
# Summary of Optimizations

- Necessary
  - Reject accept(n,v) if answered prepare(m) : m>n
    - i.e. prepare extracts promise to reject lower accept
- Optimizations
  - a) Reject prepare(n) if answered prepare(m) : m>n
    - i.e. prepare extracts promise to reject lower prepare

# POSSIBLE SCENARIO #2

Caveat

accept(80,blue) can anyway not get majority, as P2b guarantees every higher proposal issued would have same value!



# Summary of Optimizations (2)

- Necessary
  - Reject accept(n,v) if answered prepare(m) : m>n
    - i.e. prepare extracts promise to reject lower accept
- Optimizations

a) Reject prepare(n) if answered prepare(m) : m>n

i.e. prepare extracts promise to reject lower prepare

b) Reject accept(n,v) if answered accept(m,u) : m>n

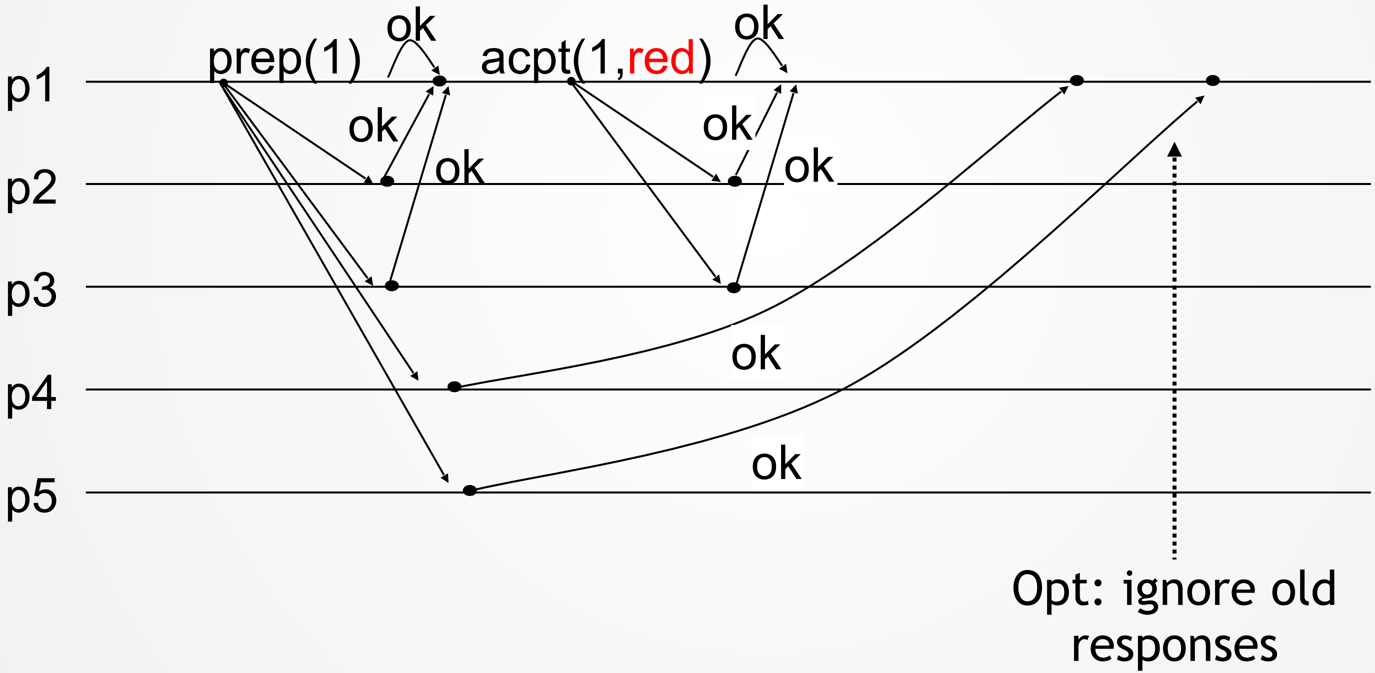
i.e. accept extracts promise to reject lower accept

c) Reject prepare(n) if answered accept(m,u) : m>n

i.e. accept extracts promise to reject lower prepare

# POSSIBLE SCENARIO #3

Caveat



# SUMMARY OF OPTIMIZATIONS (3)

- Necessary
  - Reject accept(n,v) if answered prepare(m) : m>n

    i.e. prepare extracts promise to reject lower accept

- Optimizations

  a) Reject prepare(n) if answered prepare(m) : m>n
  i.e. prepare extracts promise to reject lower prepare

  b) Reject accept(n,v) if answered accept(m,u) : m>n
  i.e. accept extracts promise to reject lower accept

  c) Reject prepare(n) if answered accept(m,u) : m>n
  i.e. accept extracts promise to reject lower prepare

  d) Ignore old messages to proposals that got majority

# State to Remember

- Each acceptor remembers
  - Highest proposal (n,v) accepted
    - Needed when proposers ask prepare(m)
    - Lower prepares anyway ignored (optimization a & c)
  - Highest prepare it has promised
    - It has promised to ignore accept(m) with lower number
- Can be saved to stable storage (recovery)

# OMITTING ACCEPT

- Paxos requires 2 round-trips (with no contention)
  - Prepare(n) : prepare phase (read phase)
  - Accept(n, v): accept phase (write phase)
- P2. If v is chosen, every higher proposal chosen has value v
- Improvement
  - Proposer skips the accept phase if a majority of acceptors return the same value v

# Performance

- Paxos requires 4 messages delays (2 round-trips)
  - Prepare(n) needs 2 delays (Broadcast & Get Majority)
  - Accept(n,v) needs 2 delays (Broadcast & Get Majority)

- In many cases only accept phase is run
  - Paxos only needs 2 delays to terminate
    - (Believed to be) optimal - **more on that later**

# Paxos Correctness

P2b. If v is chosen, every higher proposal issued has value v

P2c. If any prop (n,v) is issued, there is a set S of a majority of acceptors s.t. either

(a) no one in S has accepted any proposal numbered less than n

(b) v is the value of the highest proposal among all proposals less than n accepted by acceptors in S

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## **Lemma**: P2c => P2b

### Proof map:

Prove lemma by assuming P2c, prove P2b follows

Prove P2b follows by assuming v is chosen, prove every higher proposal issued has value v

Thus: if P2c is true, and prop (n,v) chosen

Show by induction every higher proposal issued has value v

- P2b. If v is chosen, every higher proposal issued has value v

- P2c. If any prop (n,v) is issued, there is a set S of a majority of acceptors s.t. either
  - (a) no one in S has accepted any proposal numbered less than n
  - (b) v is the value of the highest proposal among all proposals less than n accepted by acceptors in S

It suffices to show that all proposals (m,u), where m≥n, have value u=v

| Round | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| 5 | | | |
| 4 | | | |
| 3 | | | |
| 2 | v | v | |
| 1 | w | $\perp$ | $\perp$ |
| 0 | $\perp$ | $\perp$ | $\perp$ |

- P2b. If v is chosen, every higher proposal issued has value v

- ~~P2c. If any prop (n,v) is issued, there is a set S of a majority of acceptors s.t. either~~
  - (a) no one in S has accepted any proposal numbered less than n
  - (b) v is the value of the highest proposal among all proposals less than n accepted by acceptors in S

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"All proposals (m,u), where m≥n, have value u=v"

**Induction base**

Inspect proposal (n,u). Since (n,v) chosen & proposals are unique, u=v

| Round | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| 5 | | | |
| 4 | | | |
| 3 | | | |
| 2 | v | v | |
| 1 | w | ⊥ | ⊥ |
| 0 | ⊥ | ⊥ | ⊥ |

# Induction step

- Assume proposals n, n+1, n+2,…, m have value v (ind.hypothesis)
  - Show proposal (m+1,u) has u=v
- u is the value of the highest proposal among all proposals less than m+1 accepted by acceptors in S

- By the induction hypothesis, all proposals n,…,m have value v. Majority of prop m+1 intersects with majority of prop n, thus u=v

| Round | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| 5 | | | |
| 4 | | | v |
| 3 | | v | |
| 2 | v | v | |
| 1 | w | $\perp$ | $\perp$ |
| 0 | $\perp$ | $\perp$ | $\perp$ |

# Agreement Satisfied

This algorithm satisfies P2c

- accept(n,v) only issued if a majority S responded to prepare(n), s.t. for each $p_i$ in S:
  - a) either: $p_i$ hadn't accepted any prop less than n, or
  - b) v is value of highest proposal less than n accepted by $p_i$

- By their promise, a) and b) will not change

- prepare(n) often called read(n)
- accept(n,v) often called write(n,v)

# AGREEMENT

- P2c. If (n,v) is issued, there is a majority of acceptors S s.t.
  - a) no one in S has accepted any proposal numbered less than n, or
  - b) v is the value of the highest proposal among all proposals less than n accepted by acceptors in S
- P2. If (n,v) is chosen, every higher proposal chosen has value v
- We proved that if P2c is satisfied, then P2 is satisfied
  - P2c => P2
- Thus the algorithm satisfies agreement (safety)

# OBSTRUCTION FREEDOM AND VALIDITY

- P1. An acceptor accepts first "proposal" it receives

- P1 is satisfied because we accept
  - if prepare(n) & accept(n,v) received first

- Thus the algorithm satisfies obstruction-free progress (liveness)