ID2203 –Distributed Systems, Advanced
Proof Exercise – VT20 P3

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0 Notation

- For any set B, $\bar{B}$ is the complement (or dual) of B, that is U/B for some universal set U.
- $\mathbb{R}$ is the set of real numbers.
- $\mathbb{N}$ is the set of natural numbers (starting at 1).
- $\mathbb{B}$ is the boolean set $\{\bot, \top\}$ or $\{true, false\}$.
- $|X|$ is the size (number of items) of a set X.

1 Timeline

You must solve all the tasks (to the best of your ability) by February 3rd, and we will discuss them during the Exercise session.

2 Tasks

2.1 Dual Reduction

Show that, for all sets A, B, $A \subseteq (A \cap B) \cup (A \cap \bar{B})$.

2.2 Induced Orders

Show that, for any set X and function $f : X \to \mathbb{R}$, if f is *injective*, then $R = \{(a, b) \mid a, b \in X \text{ and } f(a) \leq f(b)\}$ is a total order on X (we say "f induces a total order on X").

Tip: Show *Antisymmetry* $(\forall_{x,y\in X}\ xRy \wedge yRx \Rightarrow x = y)$, *Transitivity* $(\forall_{x,y,z\in X}\ xRy \wedge yRz \Rightarrow xRz)$, and *Totality* $(\forall_{x,y\in X}\ xRy \vee yRx)$ for (X, R).

2.3 Least Elements

Is the following proposition true or false?
For every non-empty, finite set X with a total order $R \subseteq X^2$, we can find an element $l \in X$, such that $\forall_{x\in X}\ lRx$.

Tip: Relate X to an equal sized prefix of $\mathbb{N}$.

2.4 De Morgan

Are the following two propositions (individually) true or false?
For all sets A, B,

1. $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$
2. $\overline{(A \cup B)} = \bar{A} \cup \bar{B}$

Tip: To show $=$, show mutual inclusion $\subseteq$, *and* $\supseteq$.