

This is what you should know after this lecture + own work:

1. Def. of ionising radiation
2. Basic principles of transmission imaging
3. Fundamentals of interaction of radiation with matter of interest for transmission imaging (kinds of interaction, energy dependence)
4. Attenuation of photons: primary, attenuated, absorbed photons
5. Beer's law and how to apply it
6. Definition of linear and mass attenuation coefficient
7. Definition of object contrast
8. Image contrast vs object contrast
9. Signal Difference to Noise Ratio (SDNR)
10. Influence of Compton on contrast
11. Influence of x-ray energy on contrast
12. Energy dependence of attenuation, Photo Effect, Compton and Pair Production

Transmission imaging I

HLI 202

m colarieti tosti, mct@kth.se

Office hours (för frågor) denna vecka: <https://kth-se.zoom.us/j/63298493978>

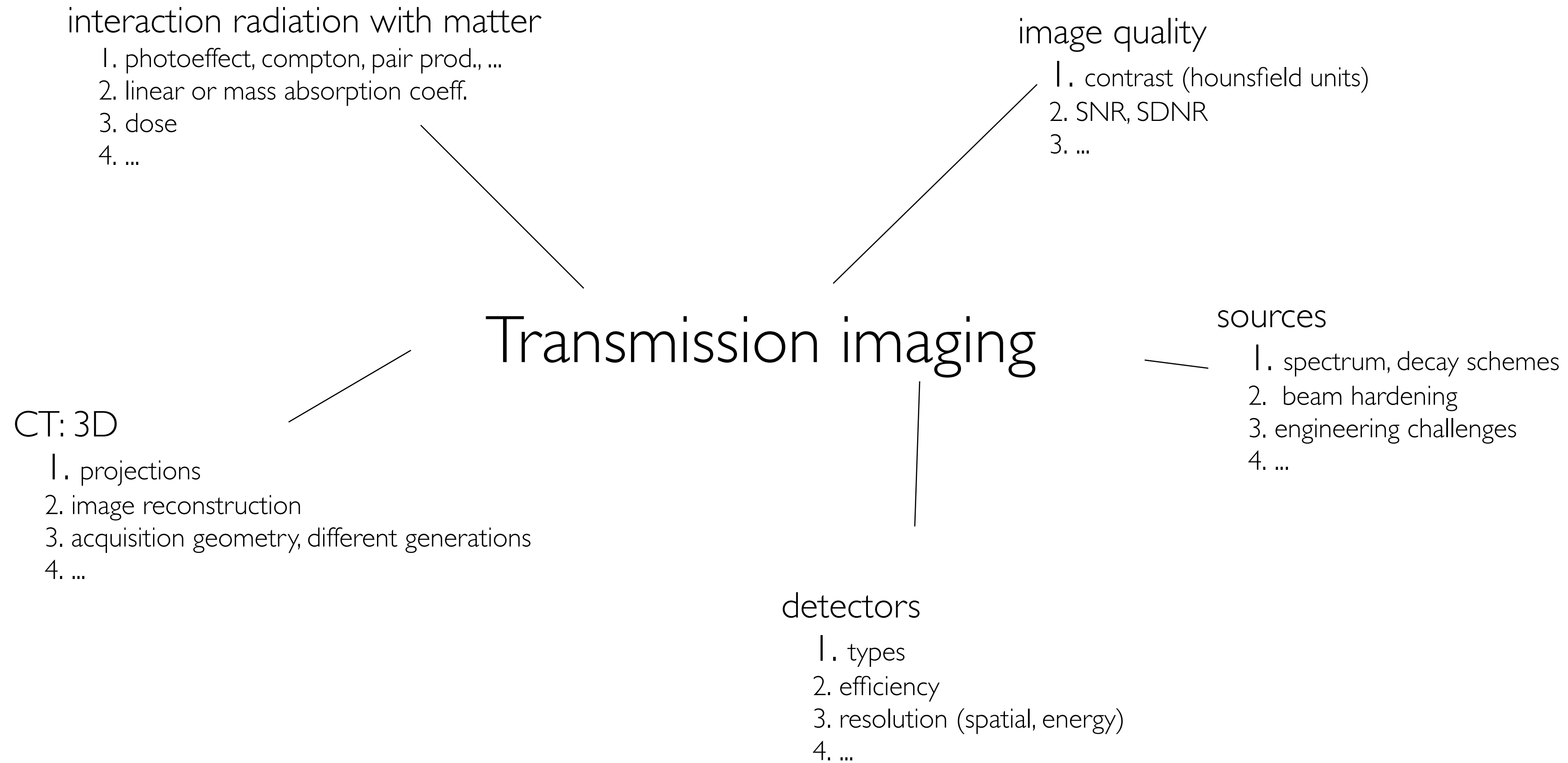
Onsdag 8 feb 12:00-13:00

Fredag 10 feb 13:00-14:00

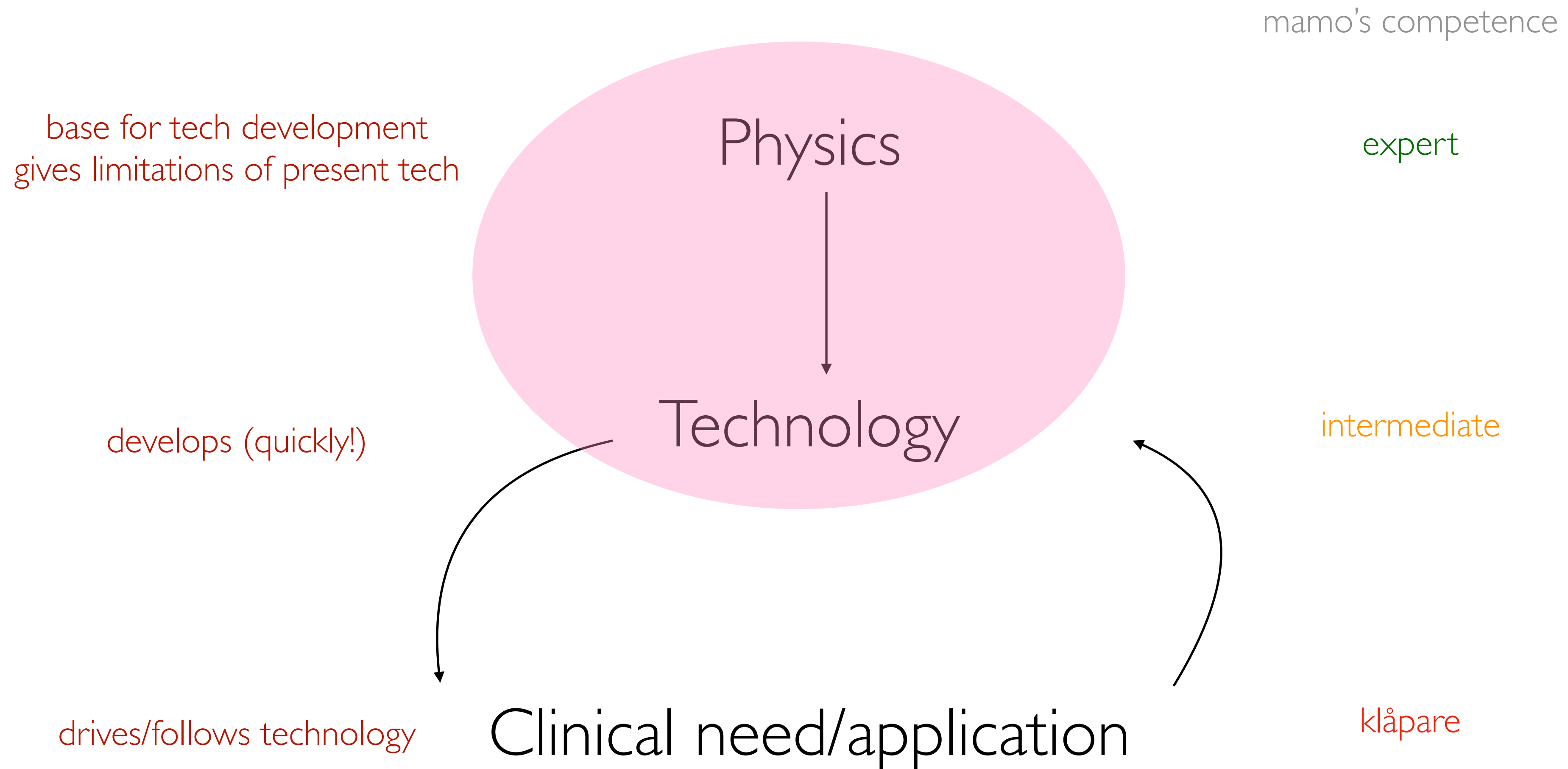
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Intro: orientation chart & lecture organisation



Philosophy of the course and motivation for it



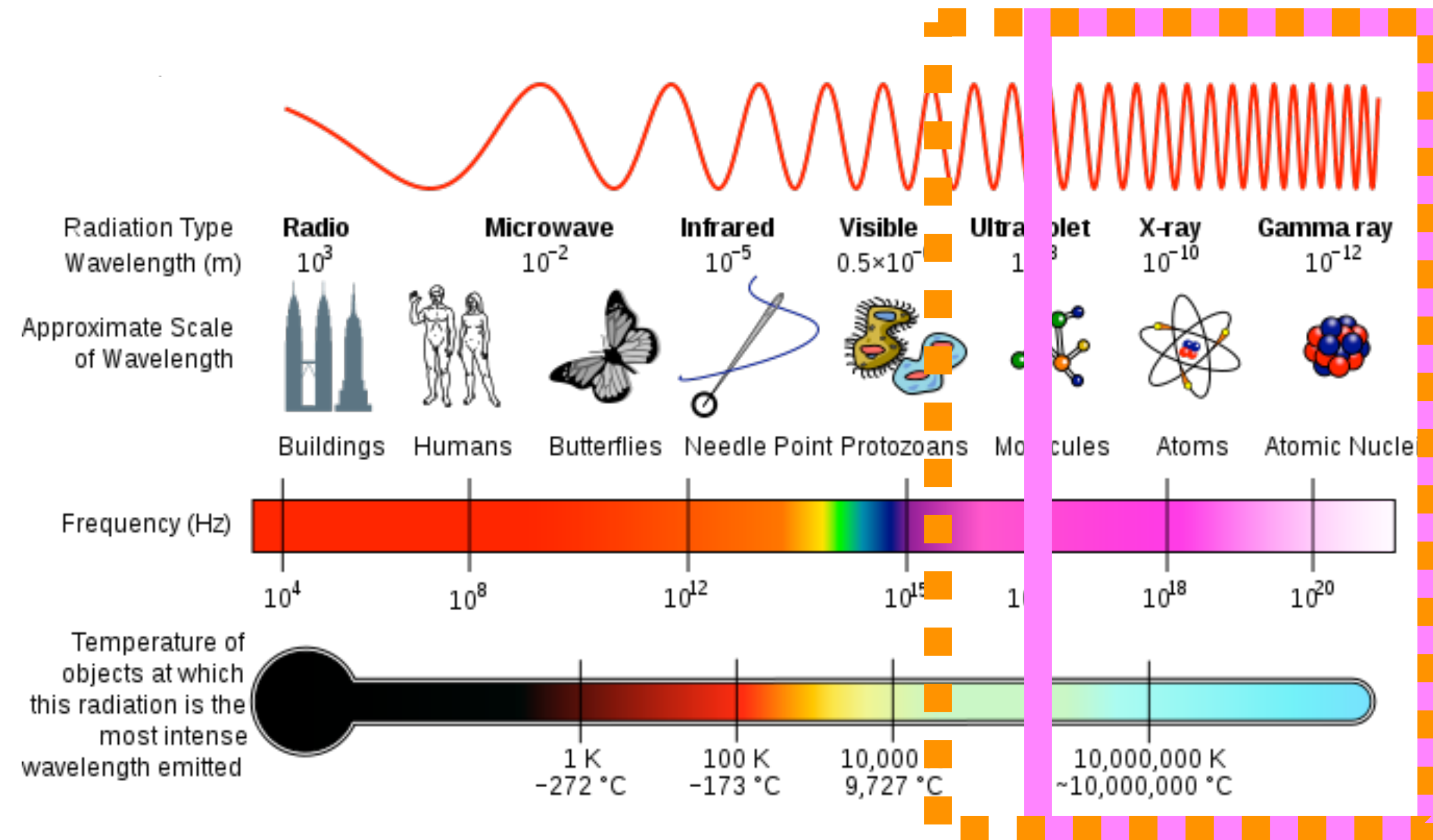
... but we will jump back and forth among these 3

Warning: the material in this lecture is at exactly the same level as you should know it in order to pass the exam (that is, I am not presenting things that are “just so you know it and have a vague idea”)

Let's get started: panorama
view of 2D transmission
imaging with ionising radiation

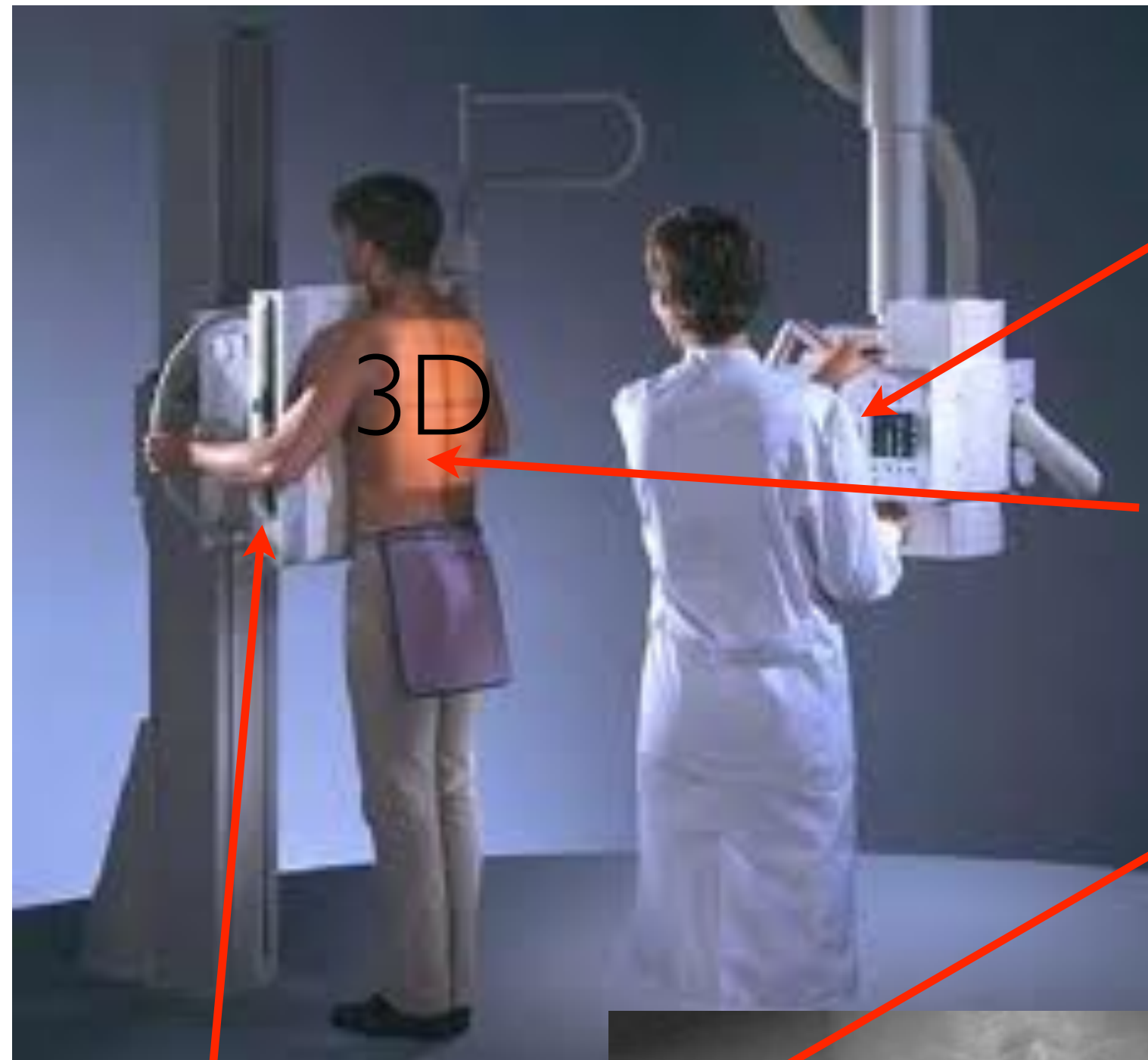
ionising radiation?

EM-strålning med tillräckligt hög energi för att jonisera atomer



Let's jump to clinical
practice

Transmission imaging fundamental elements



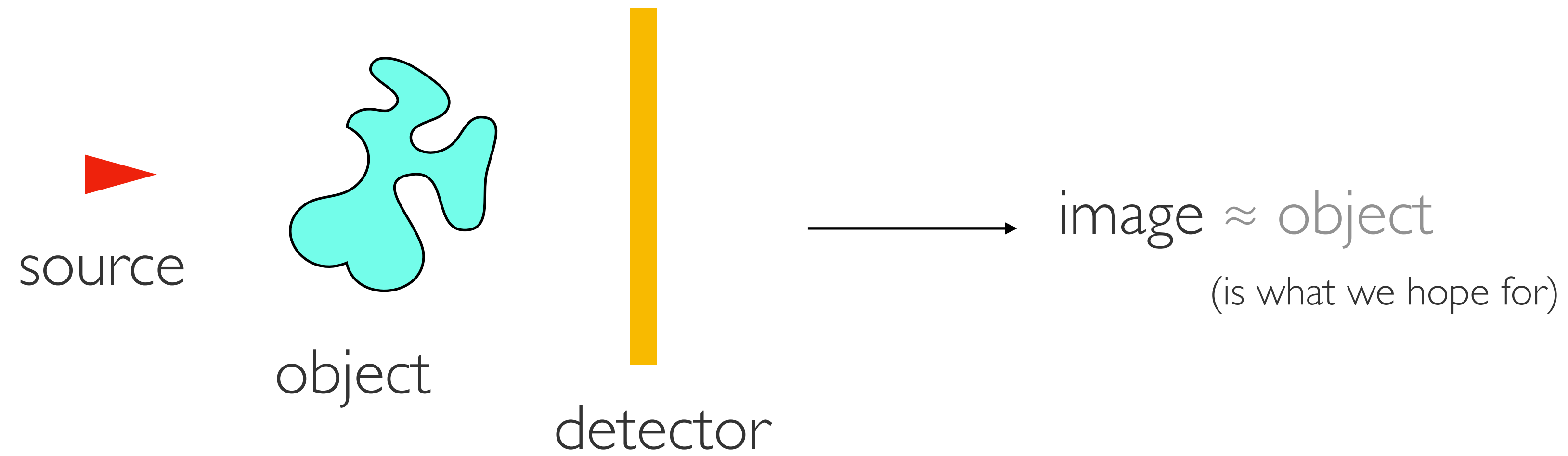
What is absolutely necessary for obtaining the kind of x-ray of the chest shown in the picture?:



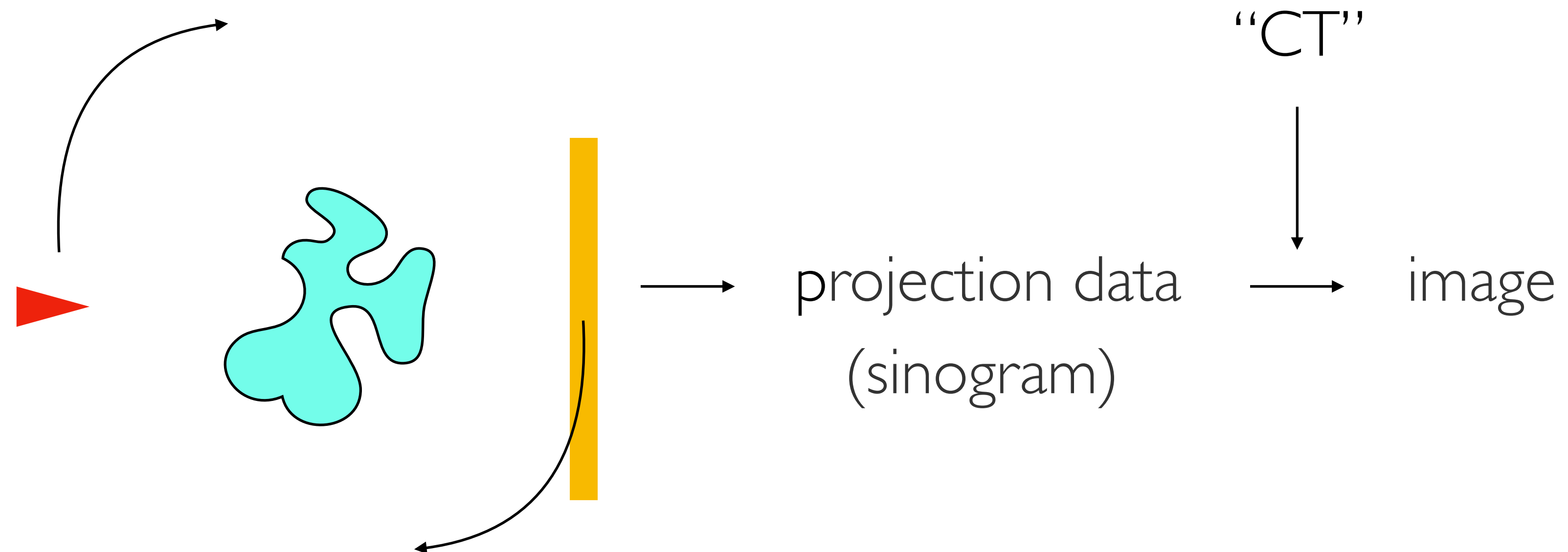
1. All of the transmitted photons must be detected by the detector
2. Bones and soft tissues must transmit x-ray in a different way
3. Some of the photons must be absorbed in the object
4. The patient must be alive

Still clinical, but simplified
(hint that we'll be heading
towards Physics soon!)

2D X-ray



3D: CT (or X-ray CT)

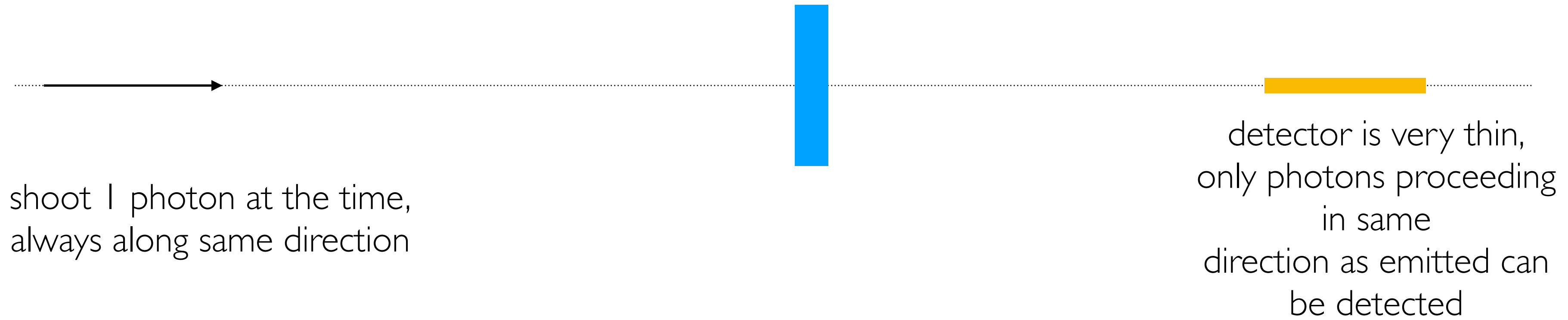


Physics!

So now we're going to look at very simple cases but we will dig very deep into details. The goal is to understand how these details are going to be important for the clinical setting

Let's consider a simple case:

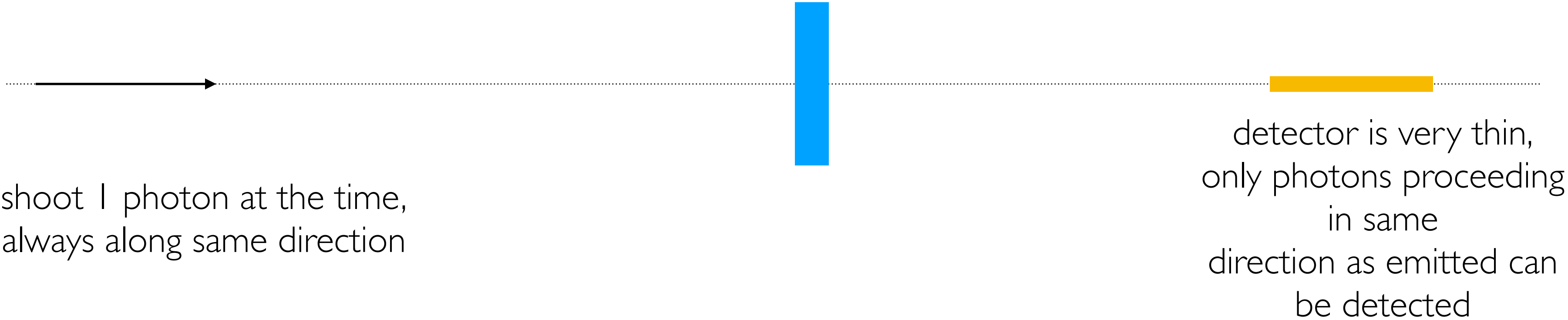
object: homogeneous material, thickness t



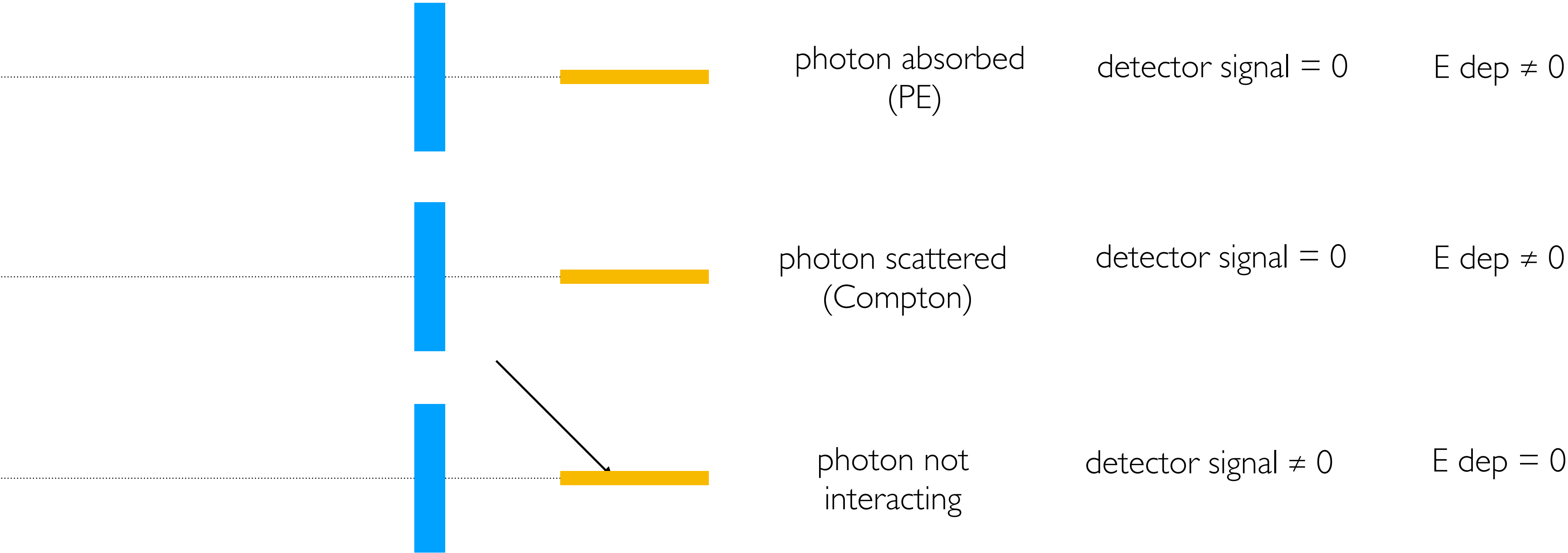
(Keep clinical application in mind: will the detector signal change if I remove or change in any way the object? How? Why? Can I change something in the object without changing the detector output?)

Let's consider a simple case:

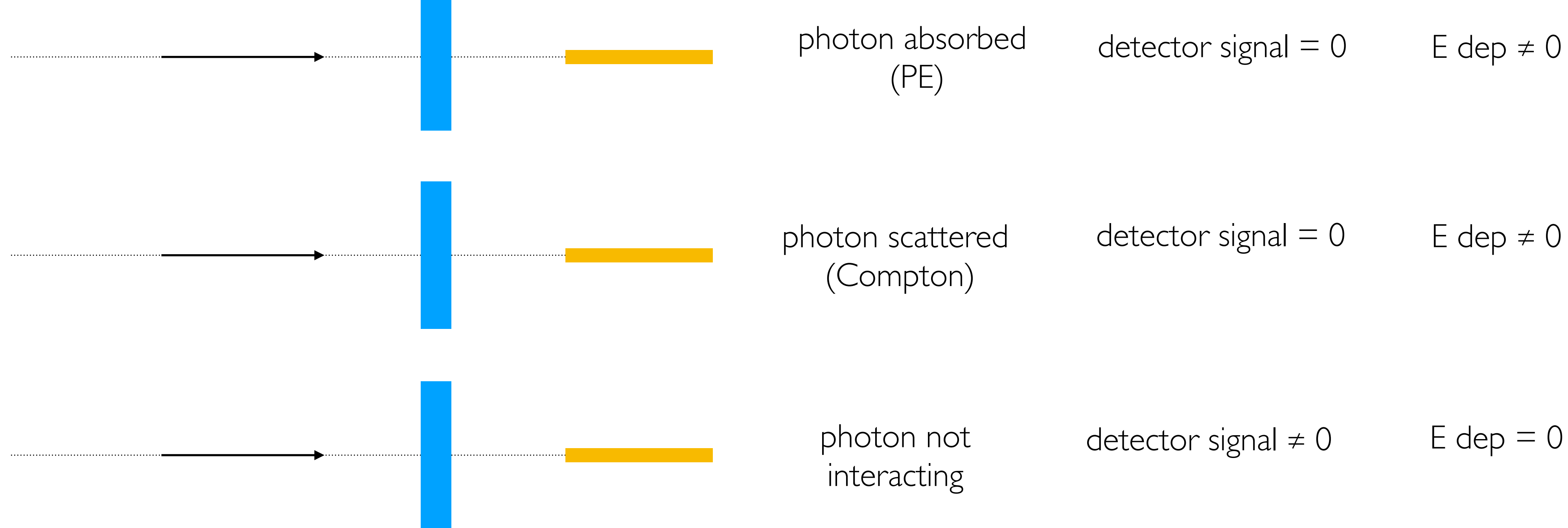
object: homogeneous material, thickness t



What can happen?



Definitions ahead

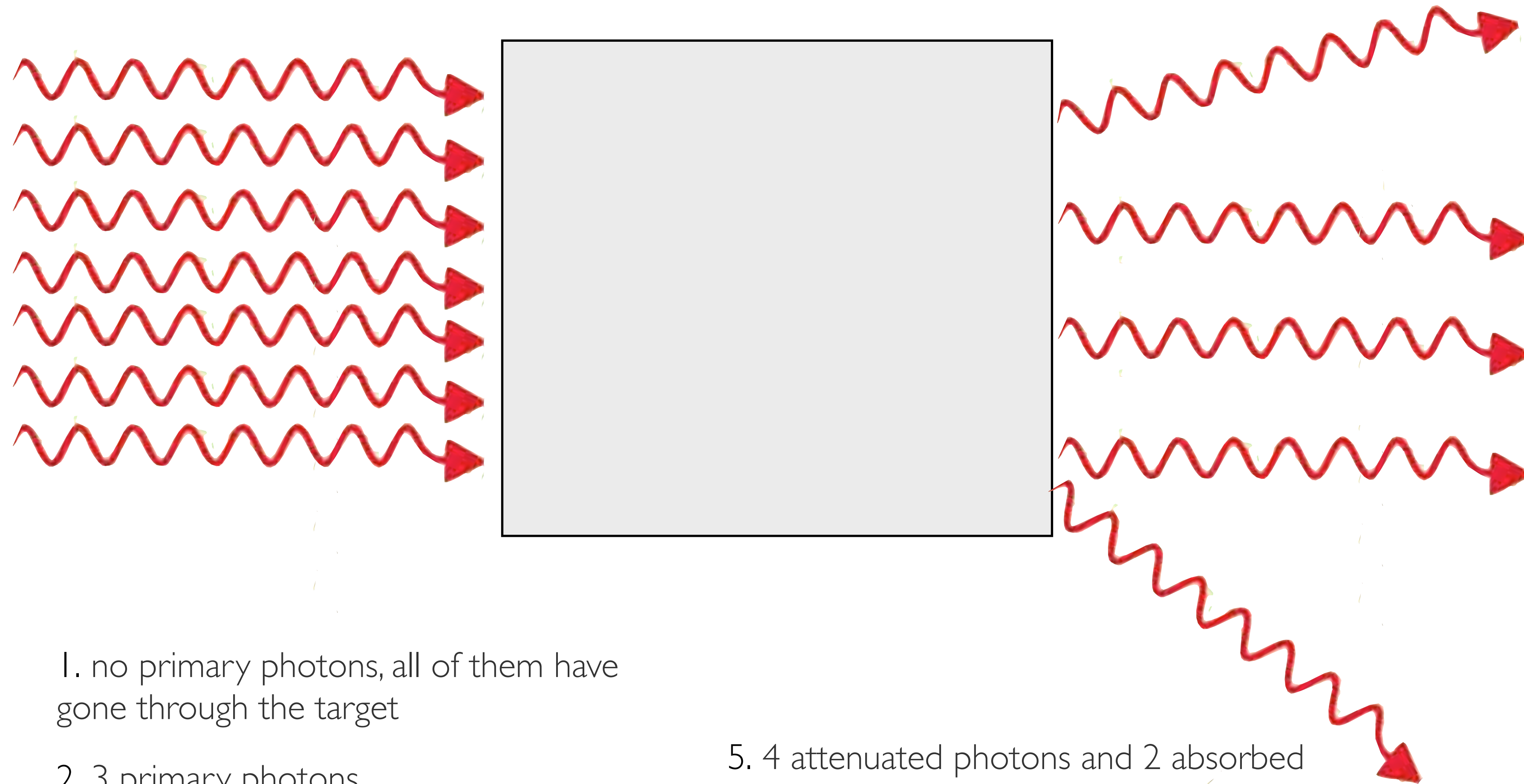


Def 1: Photons interacting in the object, that is photons depositing energy in the object are called *attenuated photons*

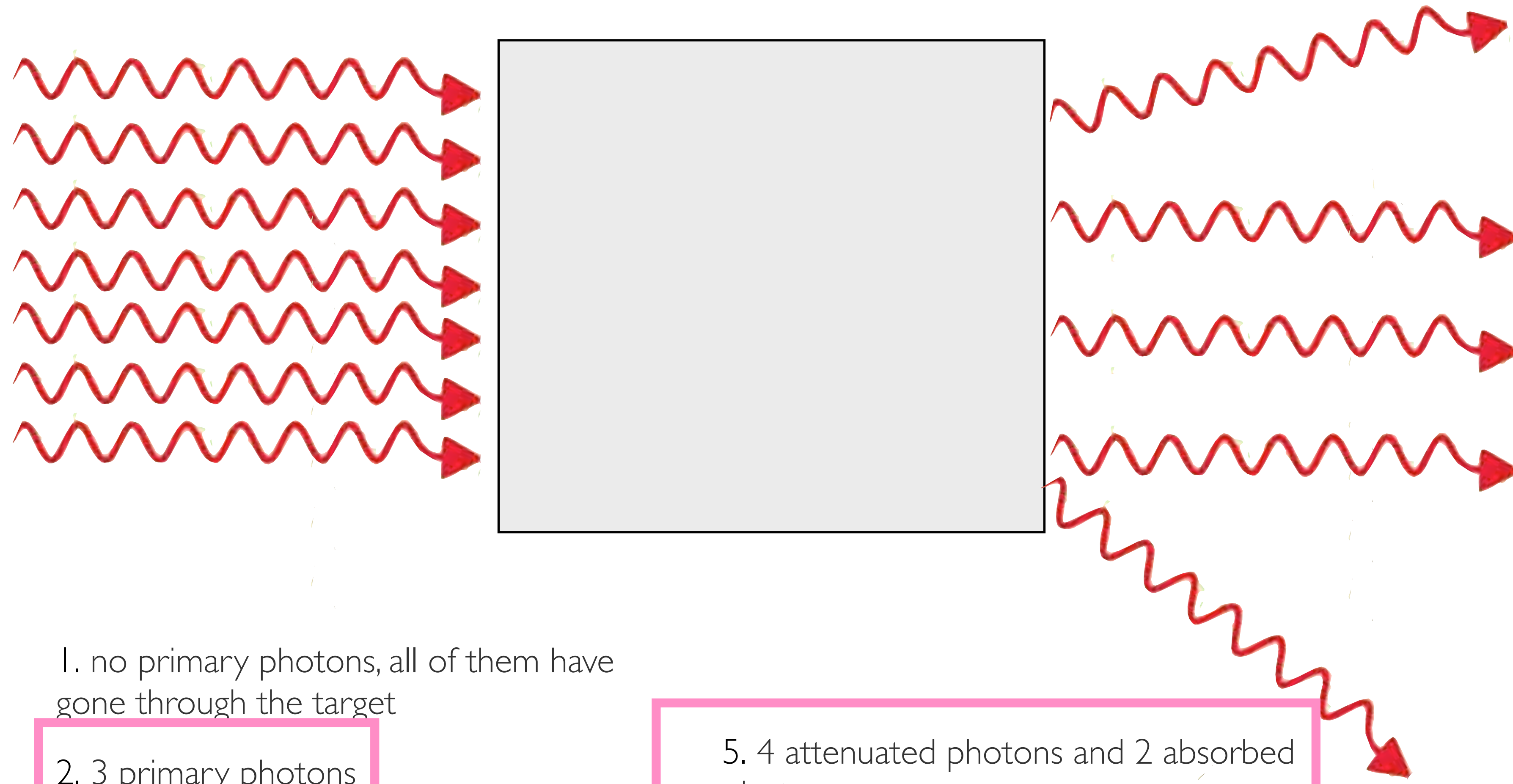
Def 2: Photons not interacting in the object, are called *primary photons*

Question: Could I equivalently define attenuated photons as photons not depositing energy in the detector? Hint: to answer the question, try to design at least a case in which scattered photons end up interacting in the detector

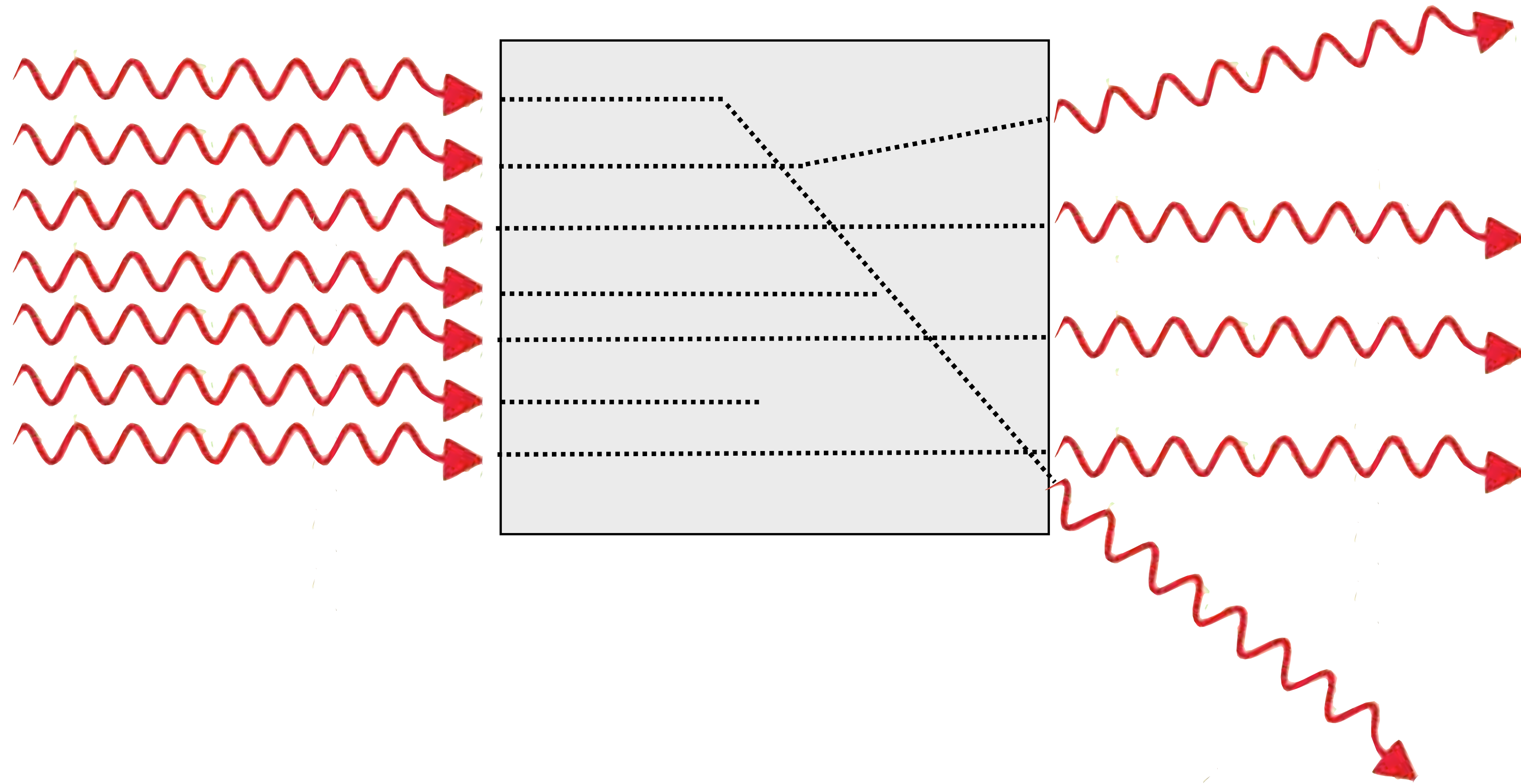
In the situation in the figure there are:



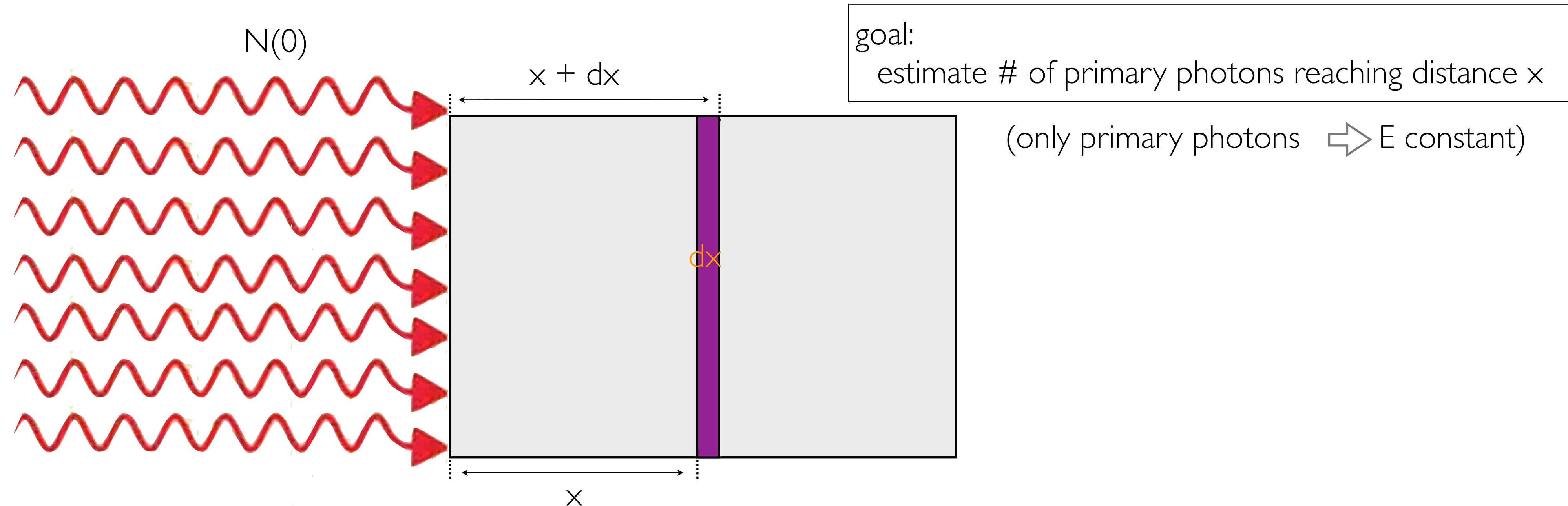
In the situation in the figure there are:



Let us sort out the answer to the previous question



Beer's law



let σ be the probability of interaction with 1 electron in the material

let D be the density of electrons in the material

Q: We have derived the equation:

$$P(x+dx) = P(x)(1 - D\sigma dx)$$

$P(x)$ is found by observing that:

1. $\frac{P(x+dx) - P(x)}{dx} = -P(x)D\sigma$
2. the derivative of $P(x)$ is equal to itself times a constant
3. the probability of interaction for primary photons is independent of the distance travelled
4. none of the above, $P(x)$ is experimentally determined

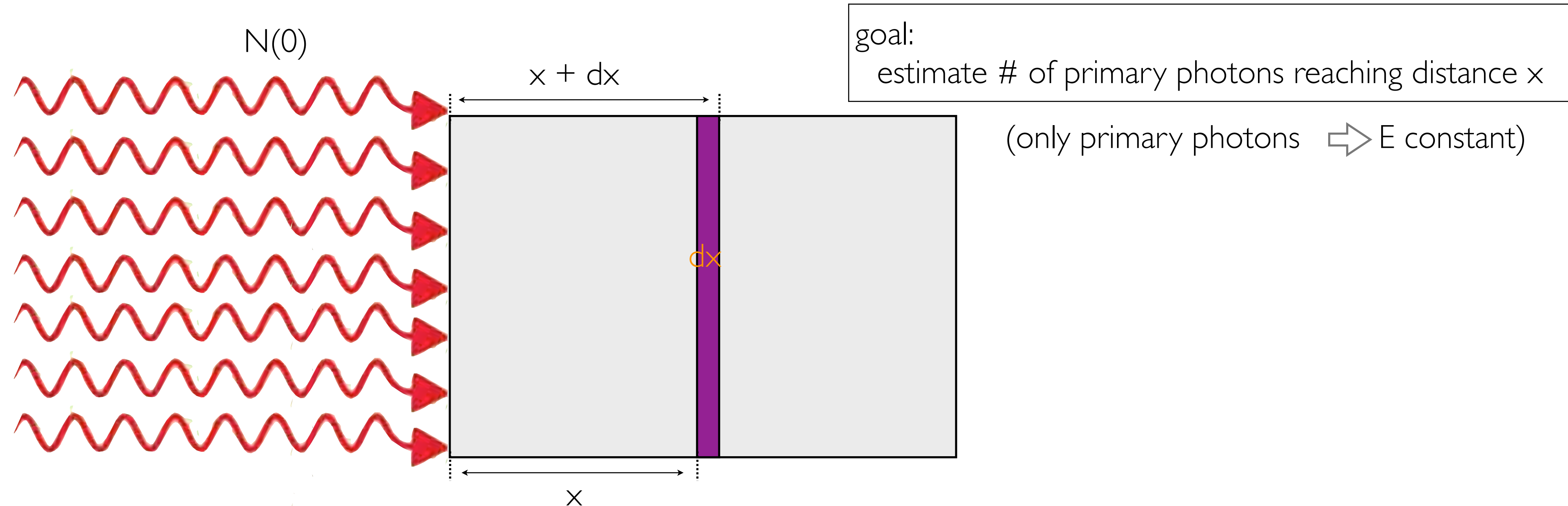
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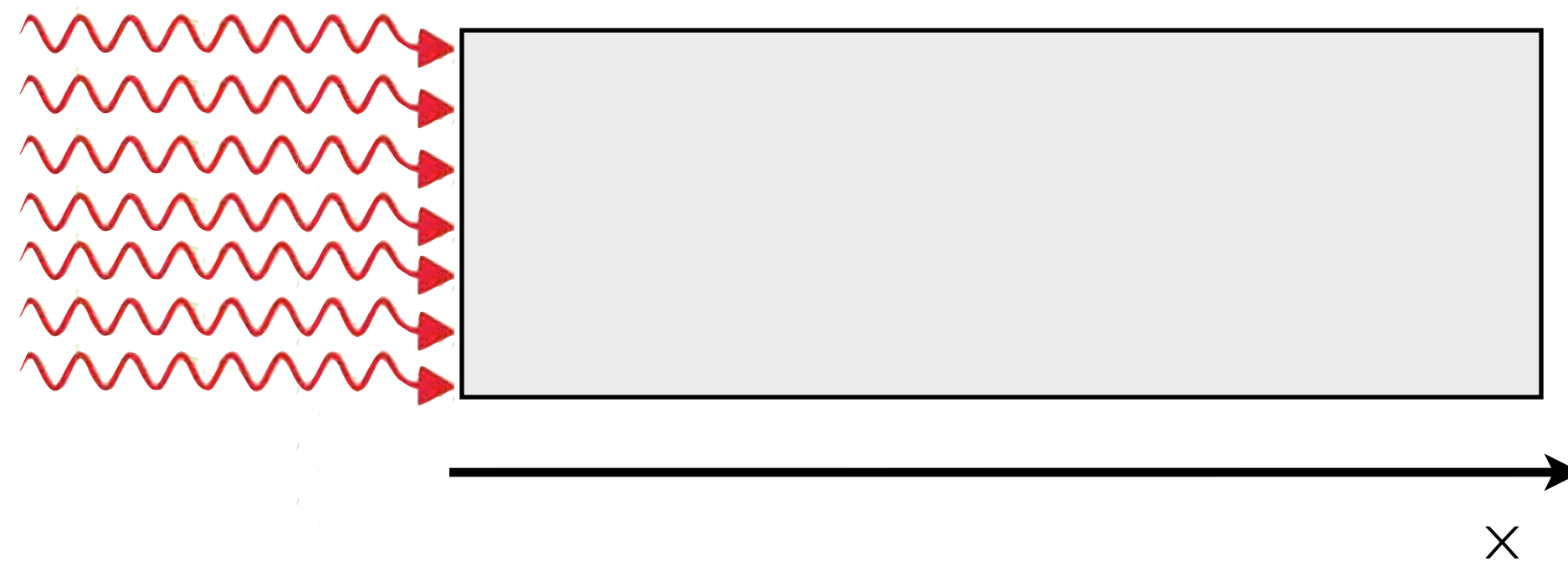
Beer's law



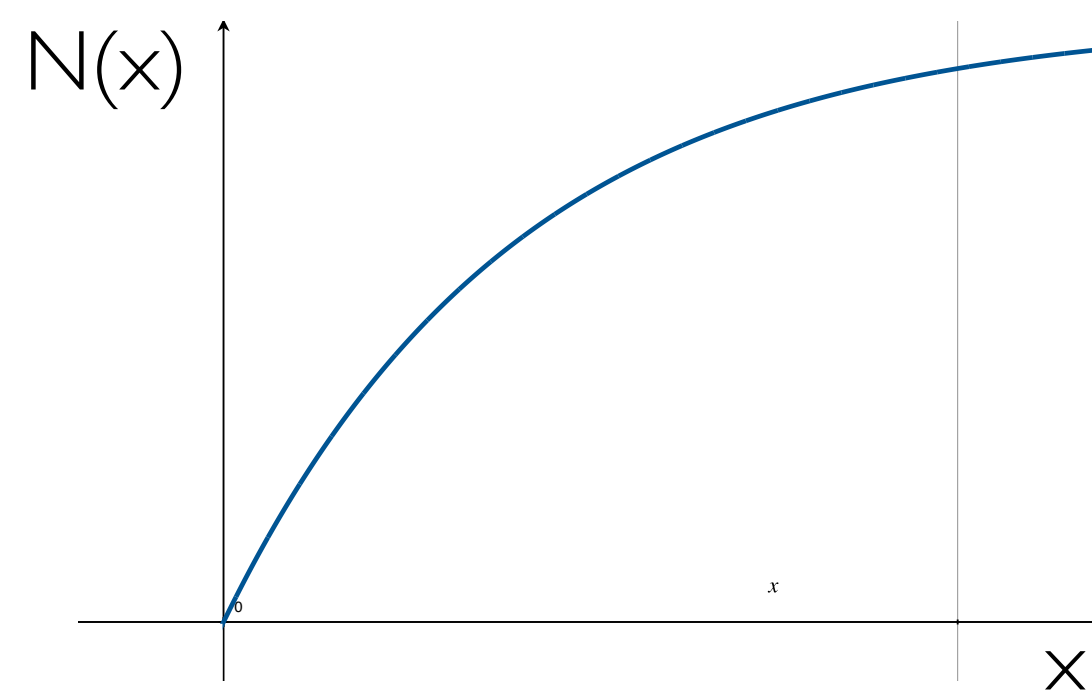
let σ be the probability of interaction with 1 atom of the material

Q:

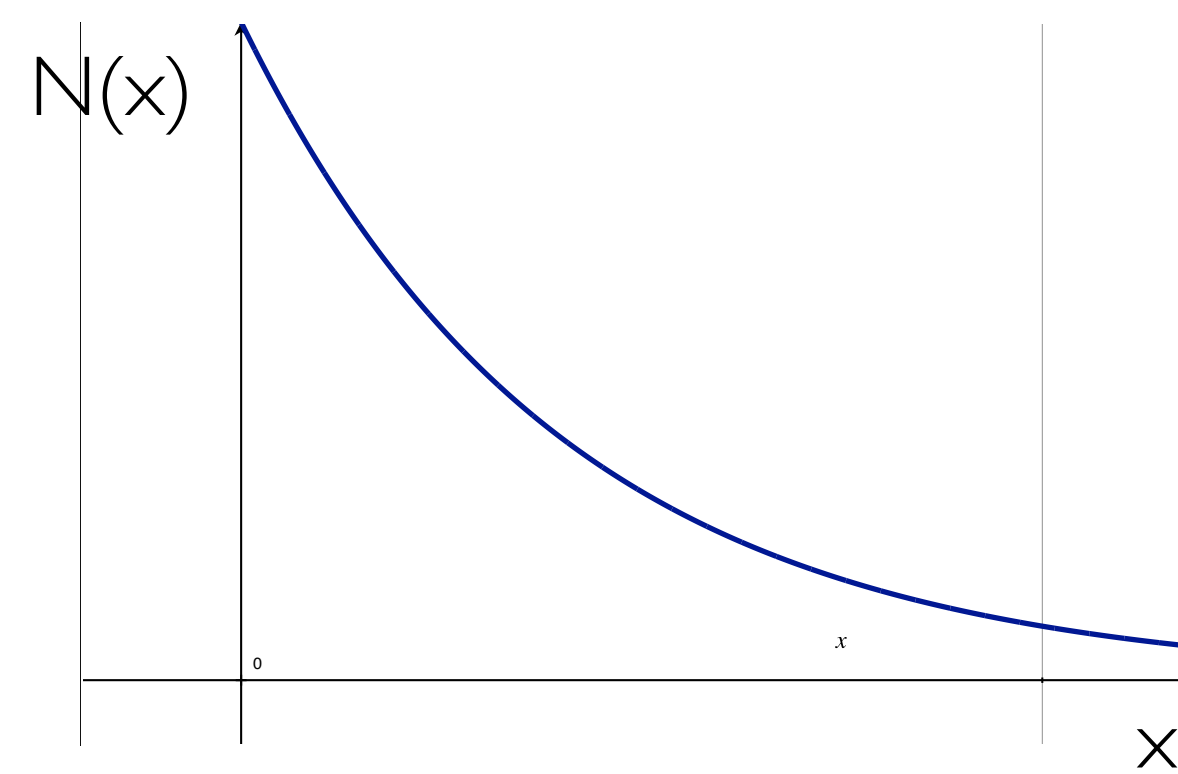
$N(0)$



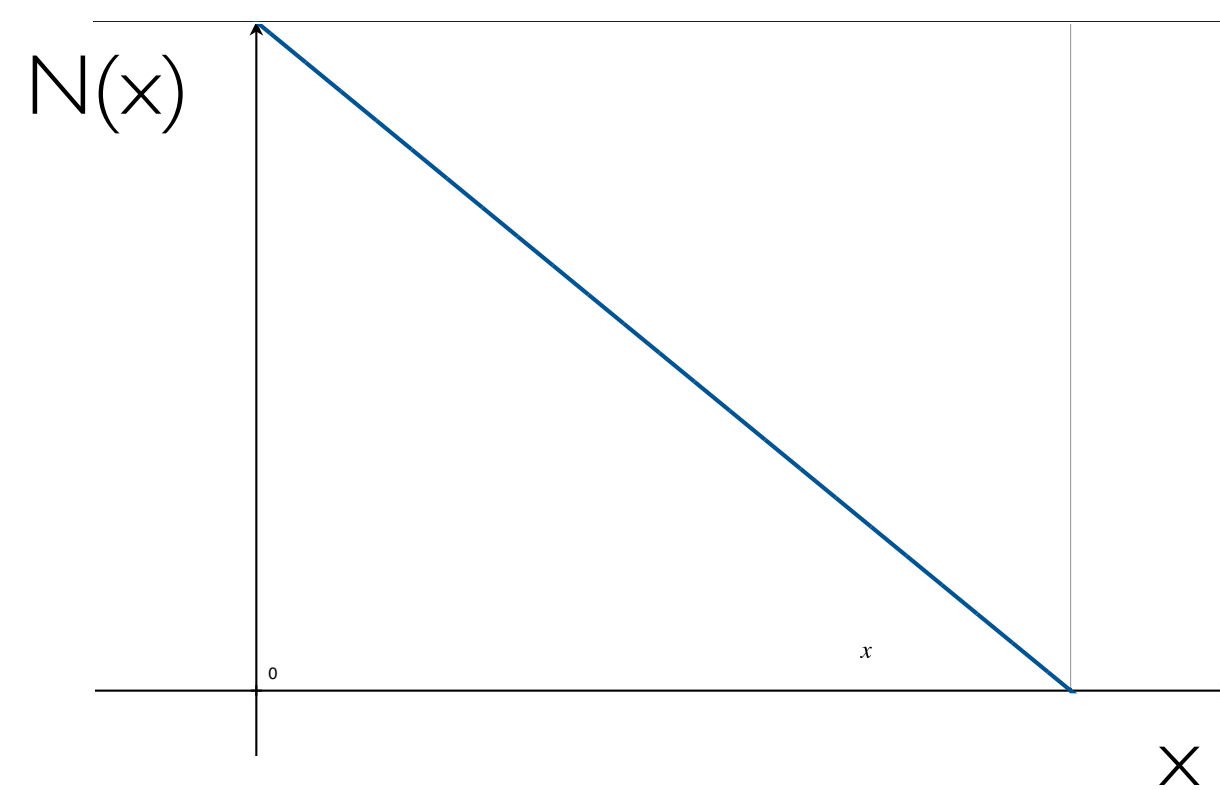
$N(x)$ looks like:



1

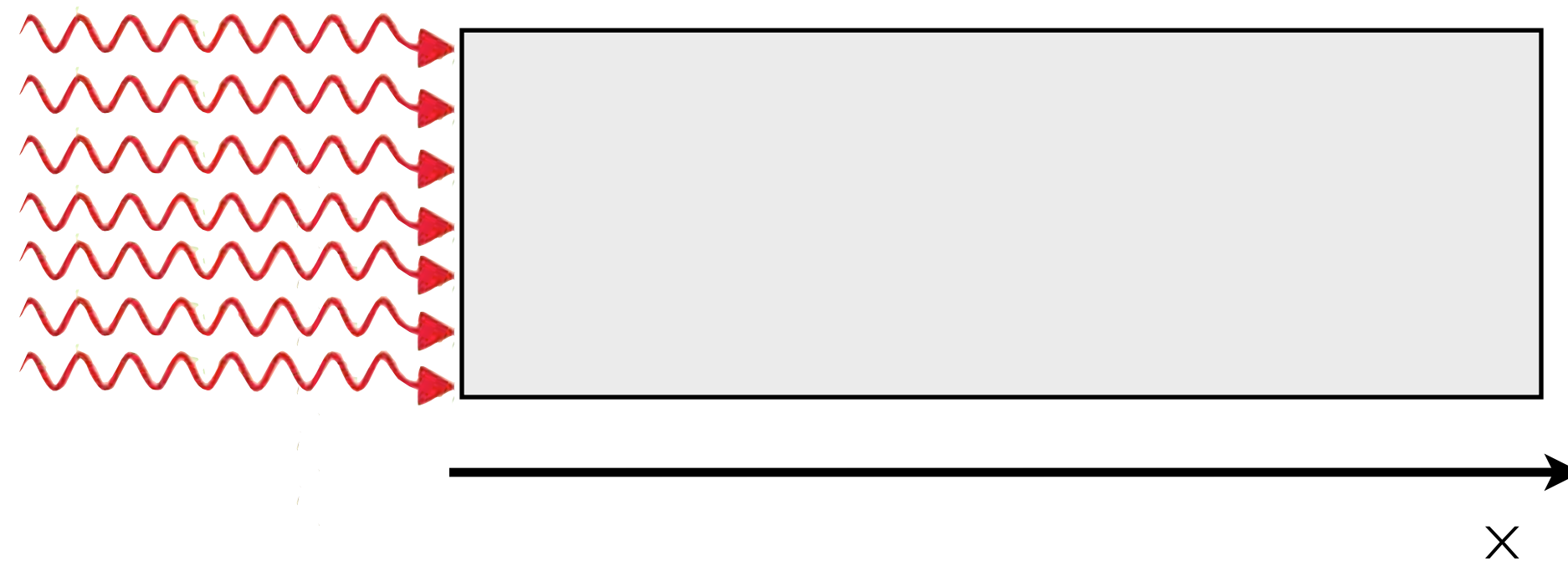


2

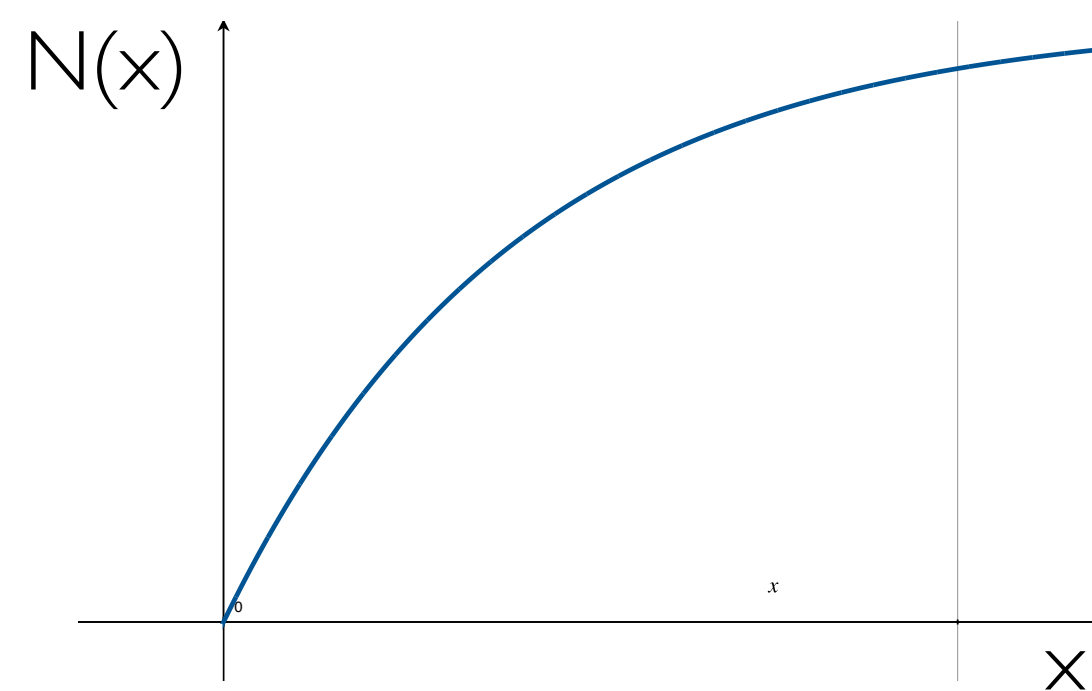


Q:

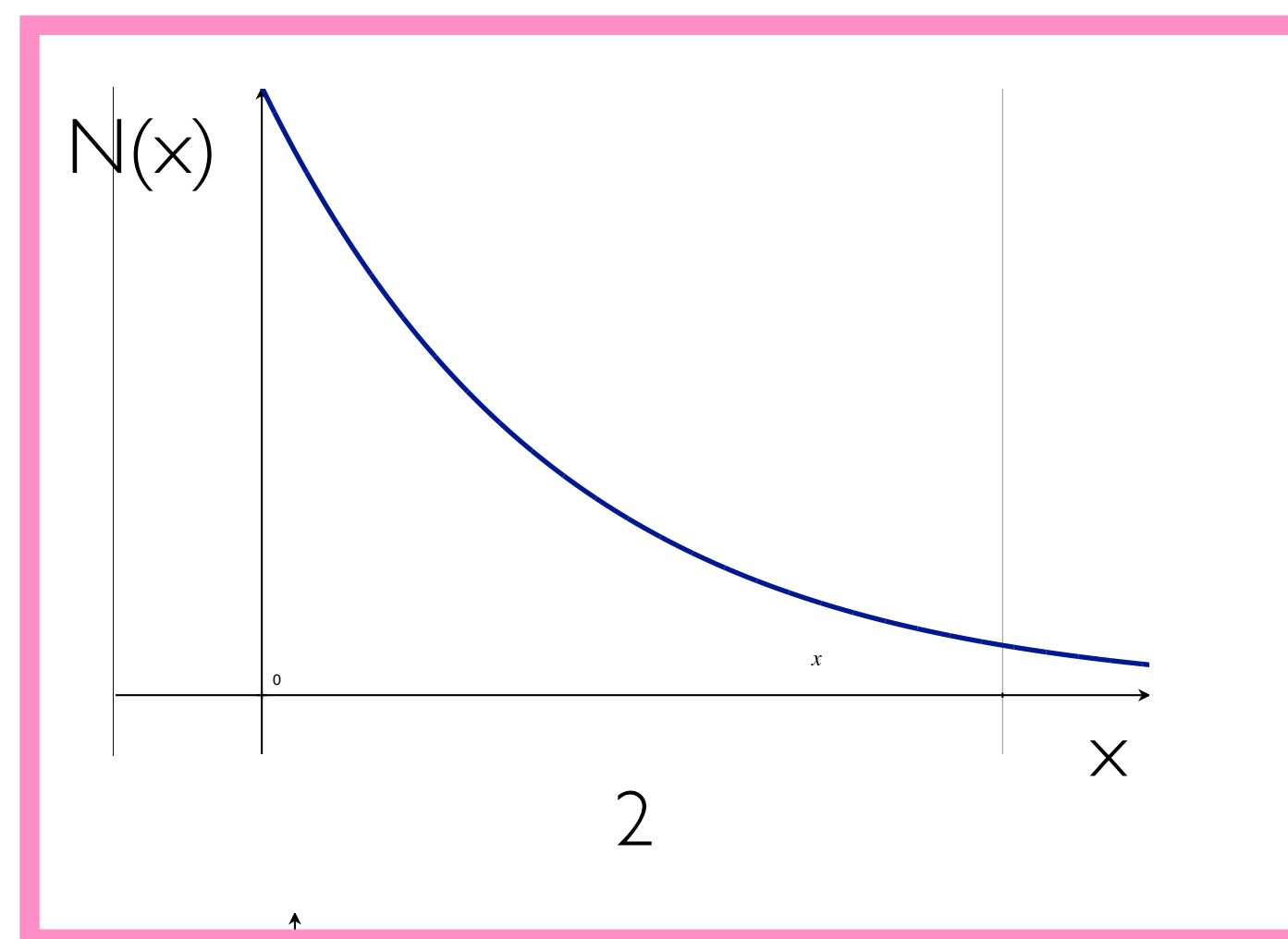
$N(0)$



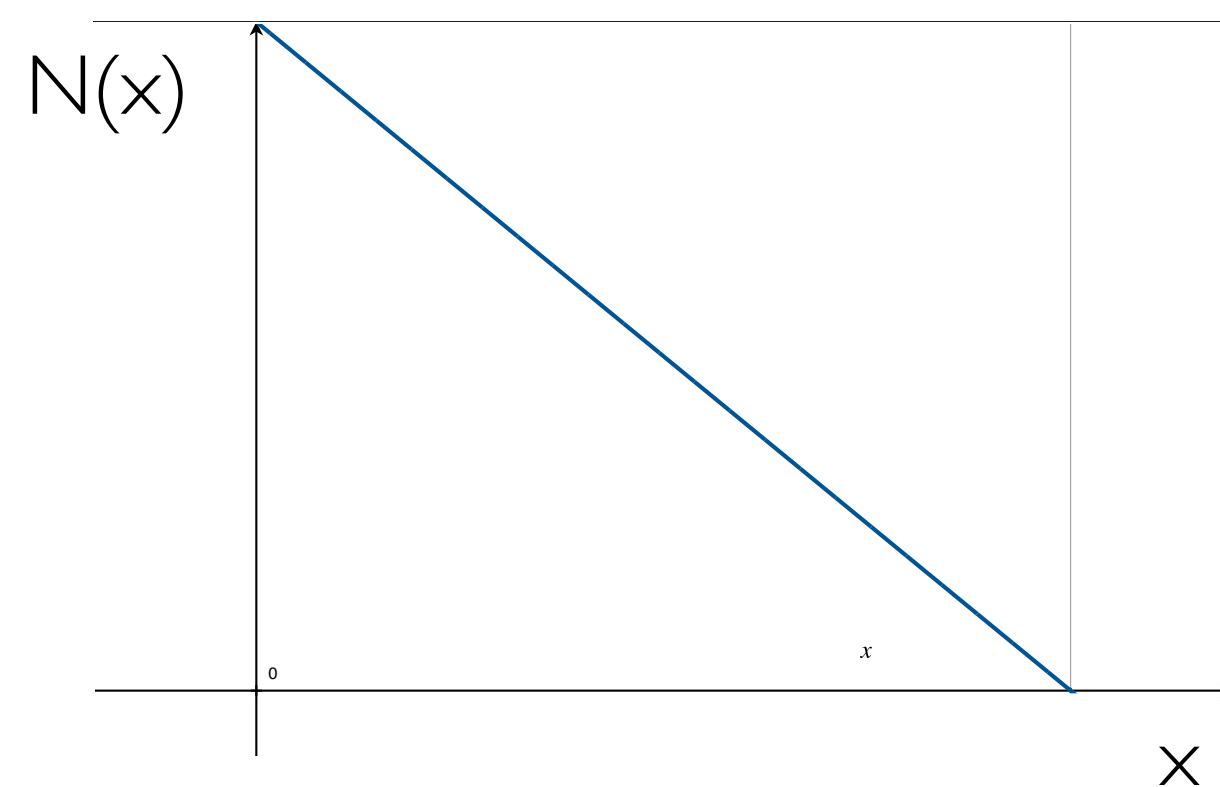
$N(x)$ looks like:



1



2



Q: How many photons are attenuated in a material with linear attenuation coefficient μ and thickness d , if N_0 photons impinge on it?

1. N_0

2. $N_0/2$

3. $N_0 e^{-\mu d}$

4. $N_0 - N_0 e^{-\mu d}$

5. $N_0 e^{-2\mu d}$

6. not sufficient data

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6. not sufficient data

Definitions ahead

The fraction of incoming photons of energy E , going through a target of thickness t and linear attenuation coefficient μ , without interaction (or, equivalently the fraction of primary photons through a target):

$$\frac{N}{N_0} = e^{-\mu(E)t}$$

is called *attenuation factor*

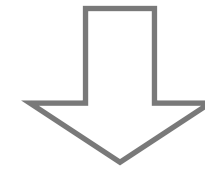
It's confusing, I know! But we're not here to change the world

The fraction of attenuated photons by a target is instead:

$$\frac{N_0 - N}{N_0} = 1 - e^{-\mu(E)t}$$

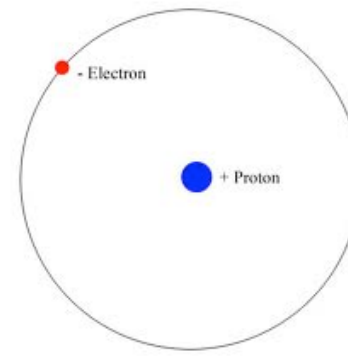
understanding at
macroscopic level
(Physics)

Compton scattering & Photoelectric effect are interactions between photons & electrons

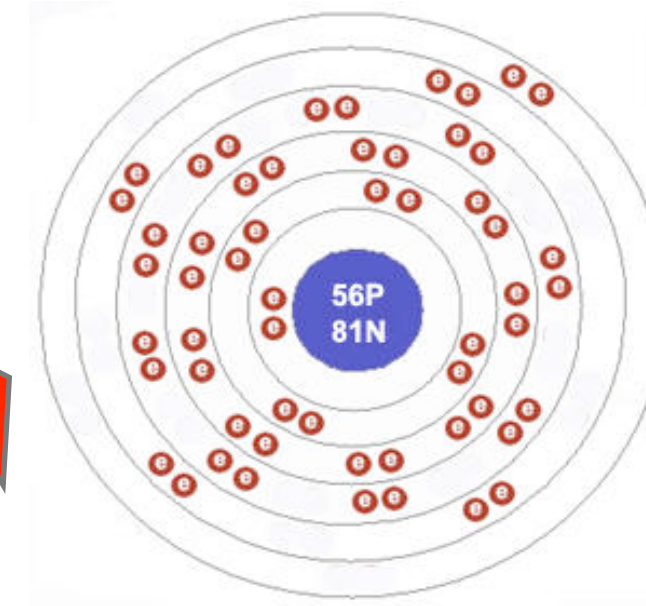
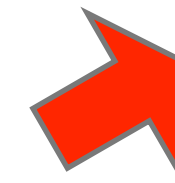


attenuation probability is proportional to electron density of the material

H
1 proton ($Z = 1$)
1 electron

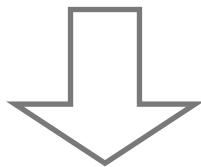


vs

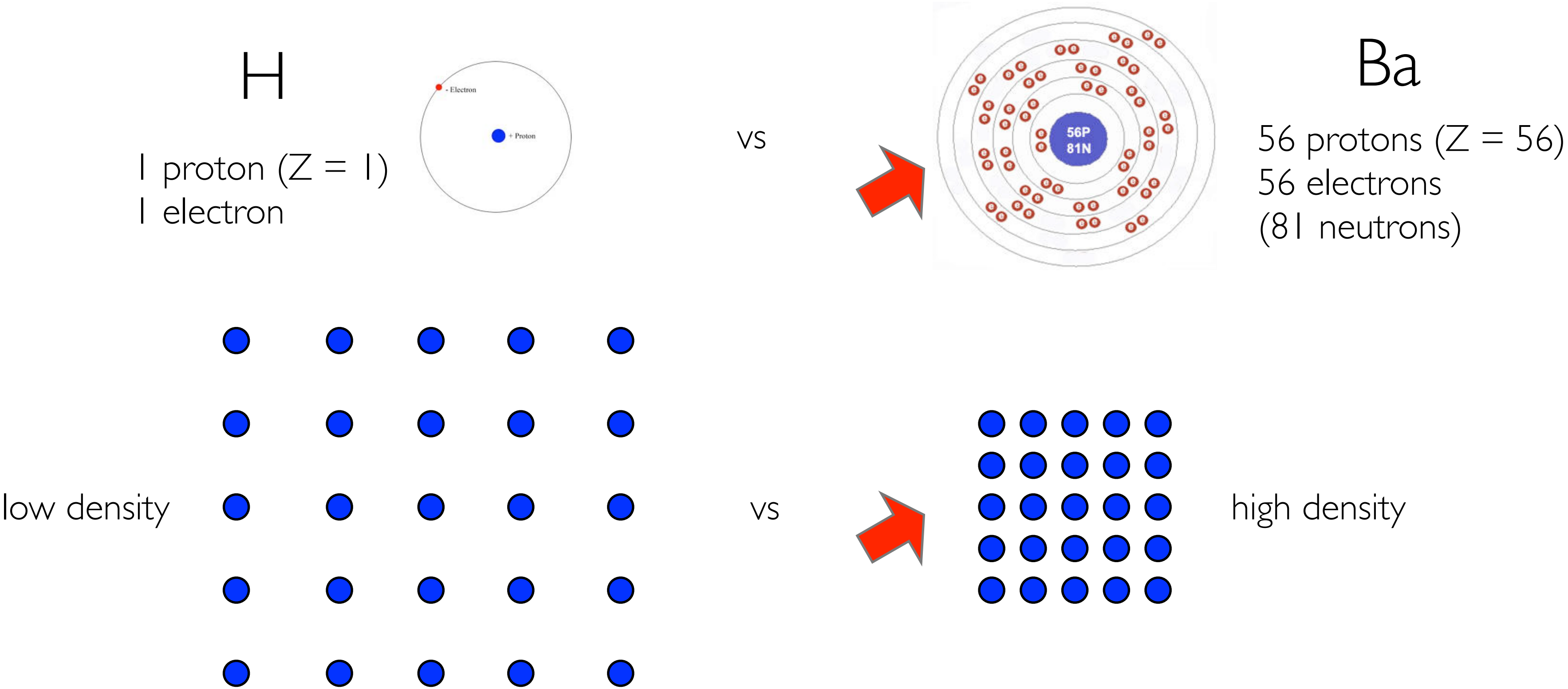


Ba
56 protons ($Z = 56$)
56 electrons
(81 neutrons)

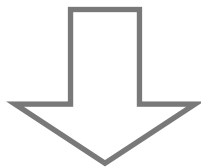
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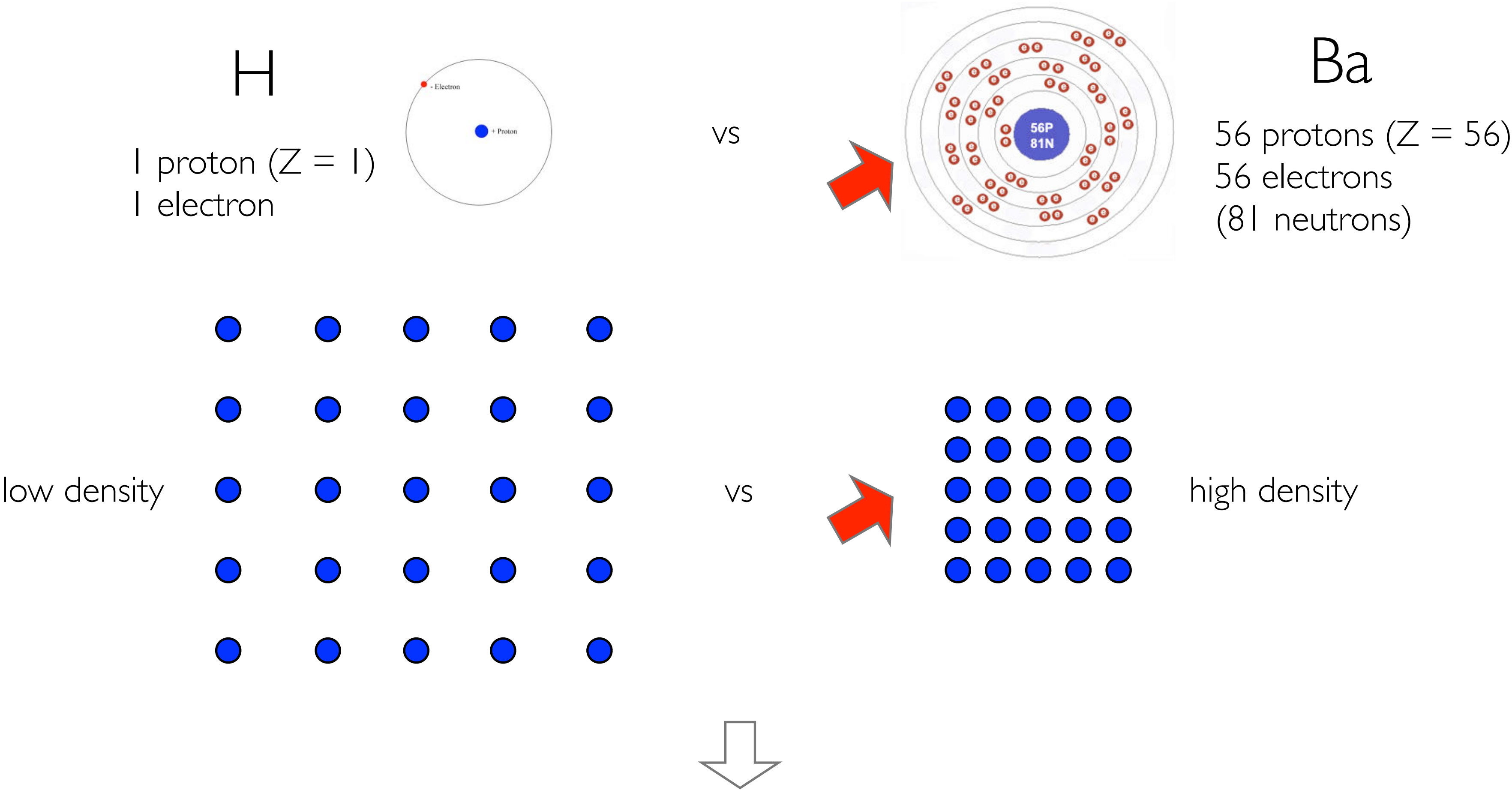
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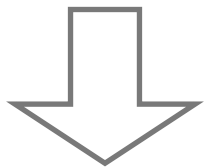
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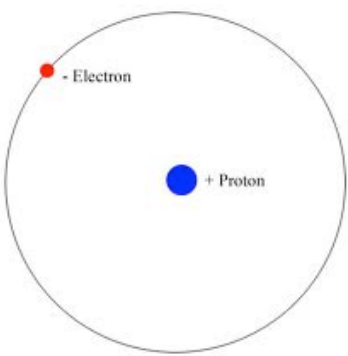


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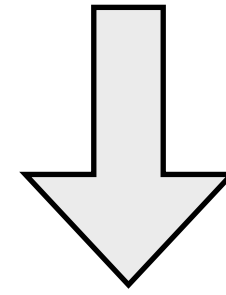


vs

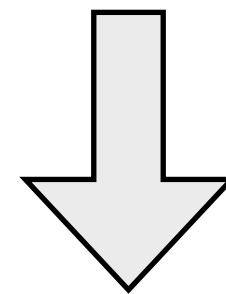
</

mass attenuation coefficient

linear attenuation coefficient, μ , depends on Z & ρ



same material in different phases or pressure will have different μ



more convenient to tabulate $\mu_\rho = \mu/\rho$

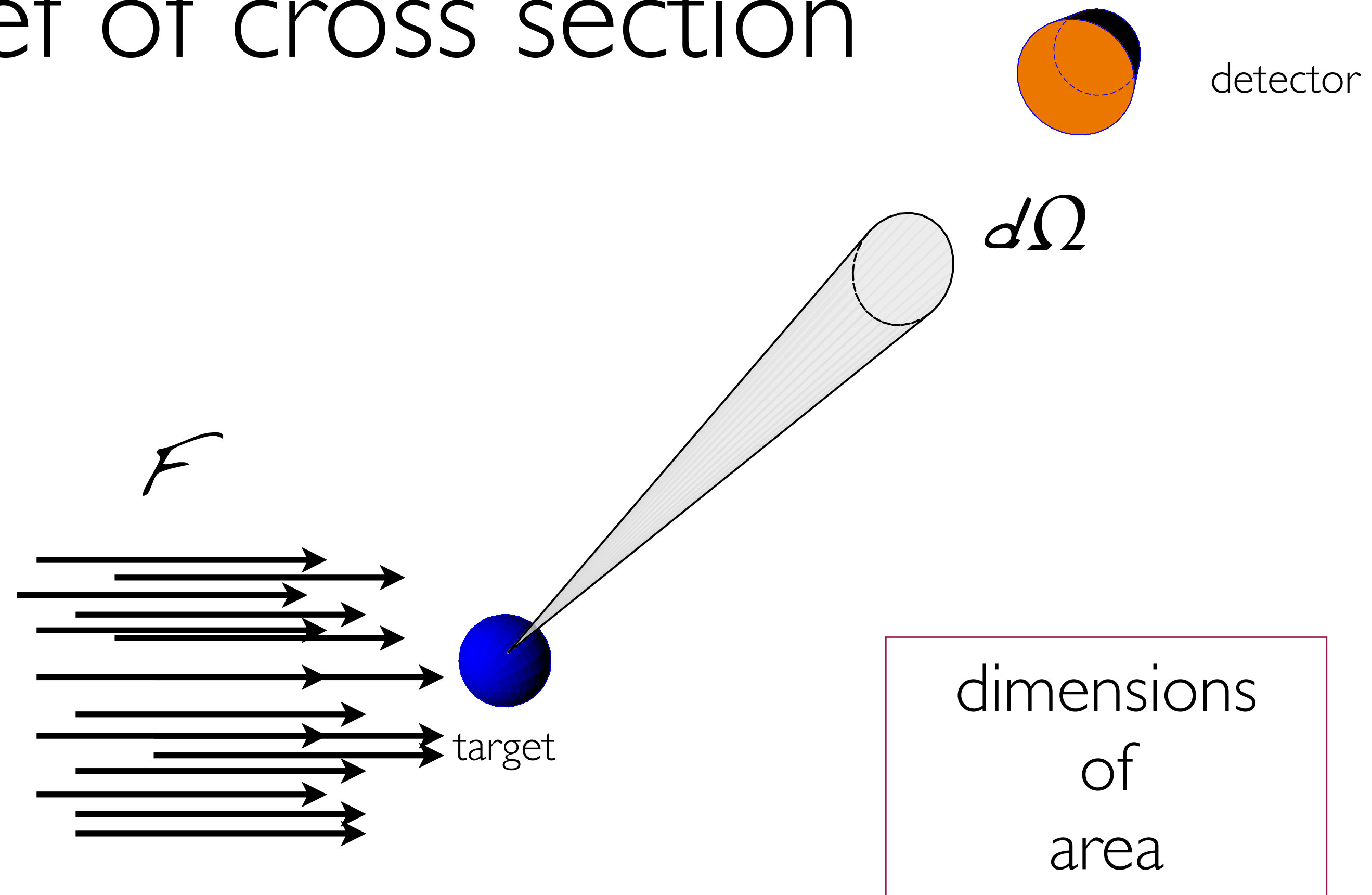
mass attenuation coefficient

<http://www.nist.gov/pml/data/xraycoef/>

(google: mass attenuation coefficient NIST)

Physics:
Let's go microscopic!

Def of cross section



Prob. of scattering particle
of energy E to angle Ω :

$$\frac{d\sigma(E, \Omega)}{d\Omega} \stackrel{\text{Def}}{=} \frac{1}{F} \frac{dN}{d\Omega}$$

Integrate over all angles
(measure at all Ω)
 $\Rightarrow$ prob. of scattering particle of
energy E :

$$\sigma(E, \Omega)$$

Mer om sambandet mellan σ , μ_ρ , μ

I g av materia av isotopen ${}^A_Z\text{X}$ innehåller $\frac{N_{\text{Avogadro}}}{A}$ atomer
(till exempel ${}^{26}_{13}\text{Al}$, ${}^{19}_{13}\text{Al}$)

och därför $\frac{N_{\text{Avogadro}}}{A}Z$ elektroner

Note on mass and linear attenuation coefficient

$$\sigma = \text{area / electron} \quad [L^2]$$

$$N_A \frac{m}{m_{\text{molar}} V} Z \sigma = \text{area / atom} \quad [L^2]$$

$$N_A \frac{\rho}{m_{\text{molar}}} Z \sigma = \text{area / volume} \quad [L^{-1}]$$

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linear attenuation
coefficient
 μ

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linear attenuation
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 μ

$$N_A \frac{\rho}{m_{molar}} Z\sigma = \text{area} / \text{volume} \quad [L^{-1}]$$

mass attenuation
coefficient
 μ_ρ

$$N_A \frac{1}{m_{molar}} Z\sigma = \text{area} / \text{mass} \quad \left[\frac{L^2}{M}\right]$$

Tabulated!

Google: NIST mass attenuation coefficient
(till government shutdown är last titta på pdf I CANVAS)

Let's see if we understand
how to use this for clinical
applications!

Q: In order to better image veins a water solution of Ba is injected to patients before an x-ray examination. What is the reason for this?

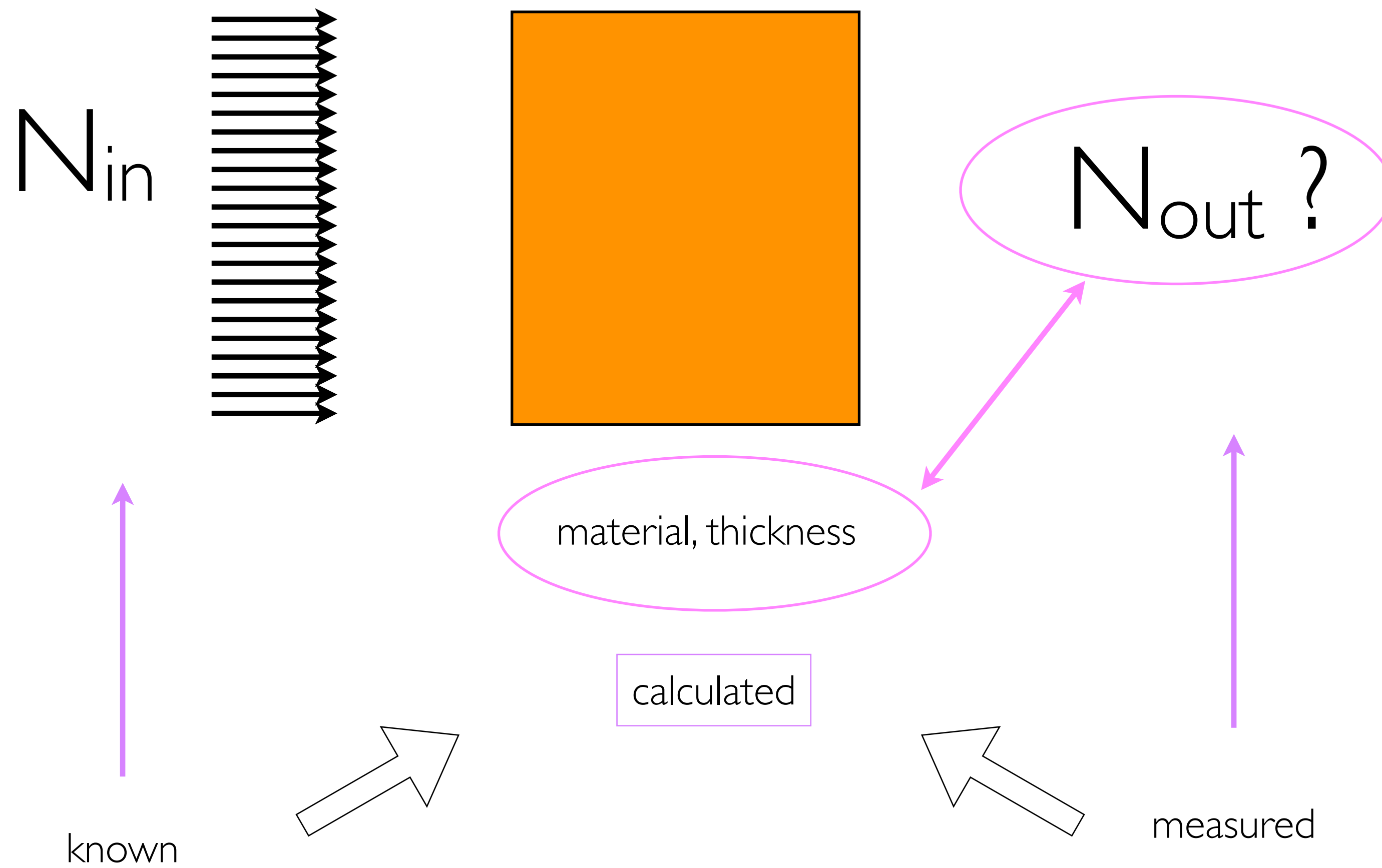
1. Ba has high Z , this will result in a much higher x-ray absorption than in surrounding tissue
2. Ba in standard condition has around 3,5 times the density of blood => the higher density will produce much higher absorption
3. both 1 and 2

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Physics:
back to a macroscopic

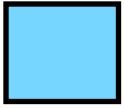
Transmission basic idea



toy model



in order to hope to get an image:

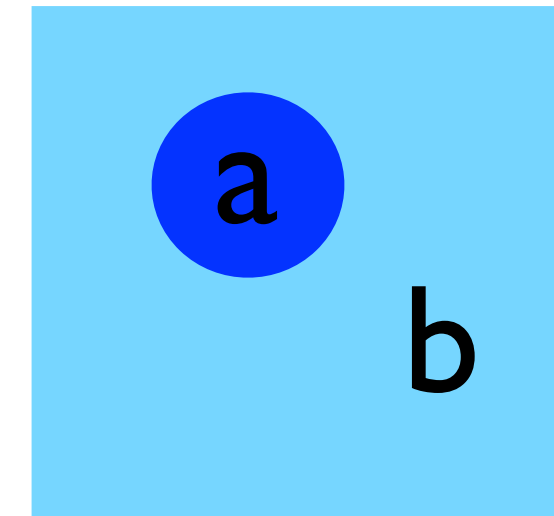
1.  and  must attenuate x-ray differently
2. D must translate x-rays in something visible or measurable
3. a lot of other things you'll know by the end of this week

toy model

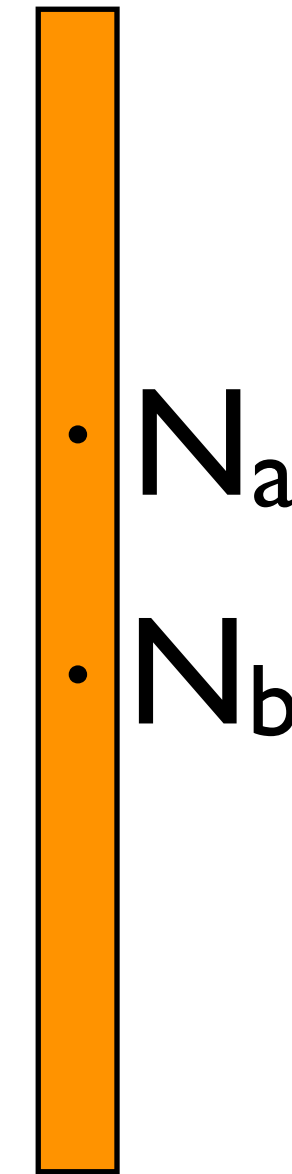
S



O



D



object contrast defined as:

$$\frac{N_b - N_a}{N_b}$$

toy model



object contrast depends on:

1. energy of incoming photons
2. thickness of b
3. thickness of a
4. difference in linear attenuation coefficient between a and b

recall: $\mu \equiv N_A \frac{\rho}{m_{$

toy model



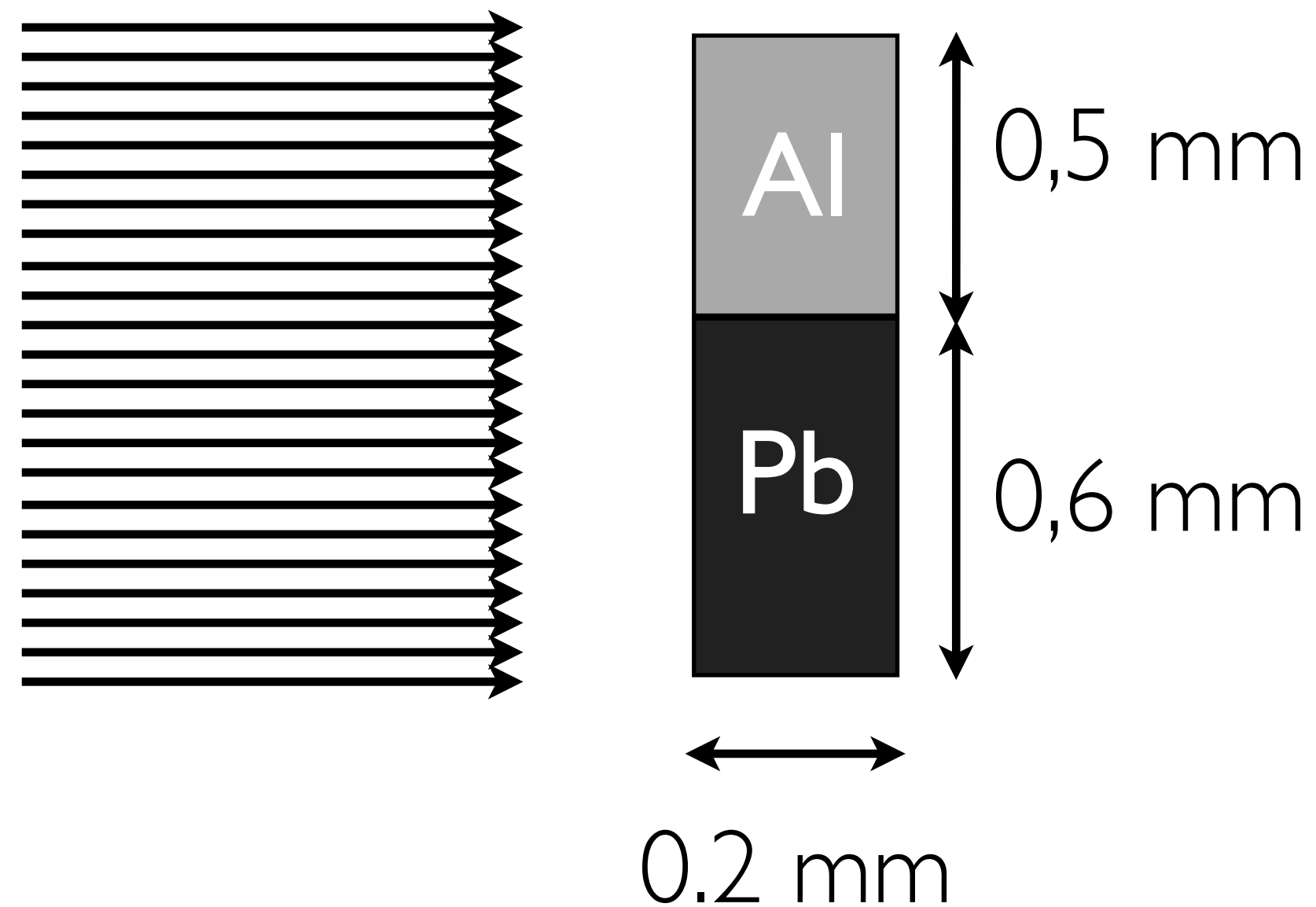
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- ~~2. thickness of b~~
3. thickness of a
4. difference in linear attenuation coefficient between a and b

recall: $\mu \equiv N_A \frac{\rho}{m_{$

object contrast: numerisk uppgift

$E = 100 \text{ keV}$

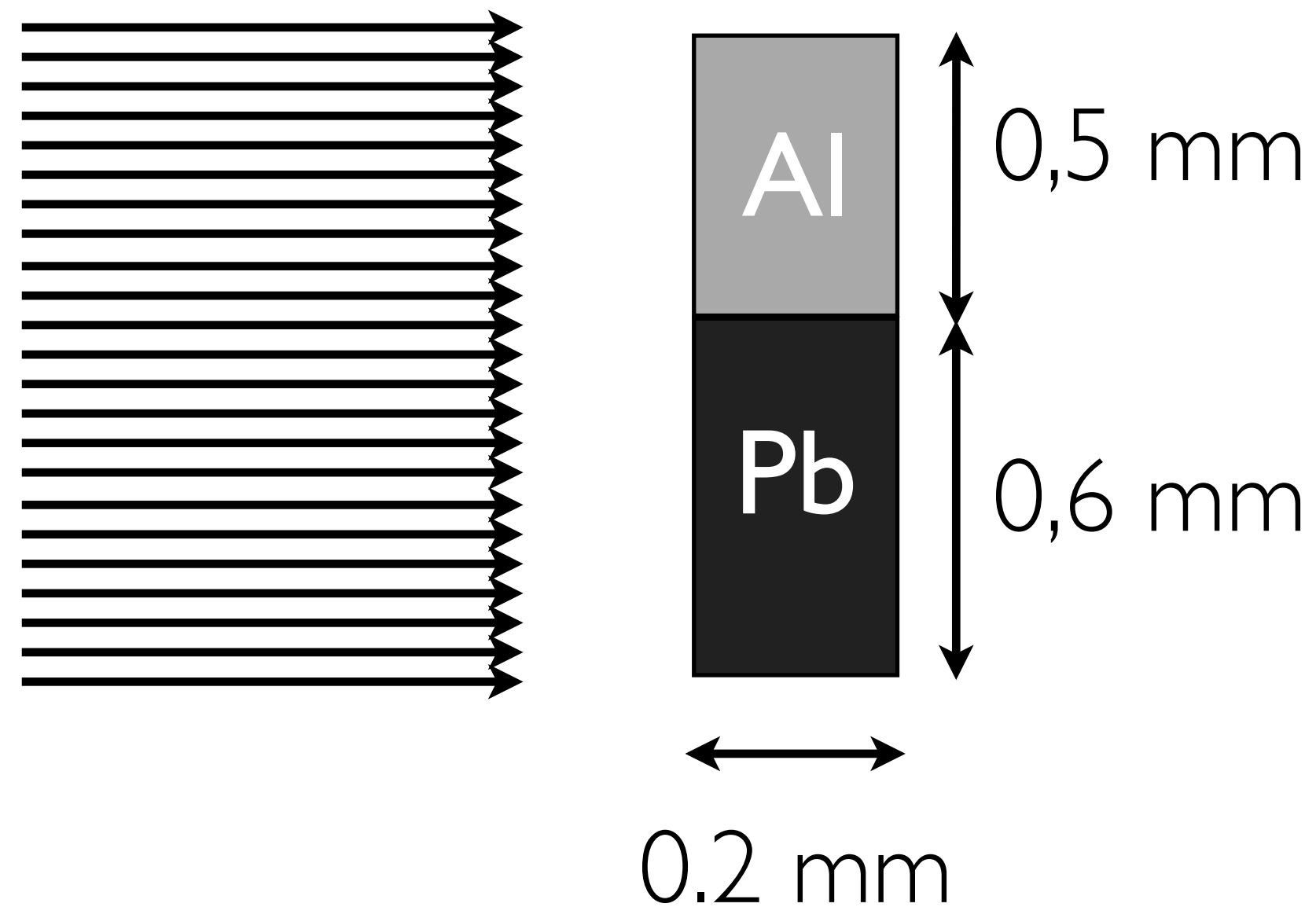


Contrast?

$$= \frac{N_0 e^{-\mu_{Al}(100 \text{ keV$$

object contrast: numerisk uppgift

$E = 100 \text{ keV}$



$$\text{Object contrast} = \frac{N \text{ through Al} - N \text{ through Pb}}{N \text{ through Al}} = \frac{N_0 e^{-\mu_{Al}(100 \text{ keV})t_{Al}}$$

tables given by Hubbell in the International Journal of Applied Radiation and Isotopes **33**, 1269 (1982).

Note on NIST X-ray Attenuation Databases

edges are indicated by the shell designation.

Z	Element	Z	Element	Z
---	---------	---	---------	---

Al

Al

tables given by Hubbell in the International Journal of Applied Radiation and Isotopes **33**, 1269 (1982).

Z	Element	Z/A	I (eV)	Density (g/cm ³)
1 H	Hydrogen	0.99212	19.2	8.375E-05

Z	Element	Z/A	I (eV)	Density (g/cm ³)
1 H	Hydrogen	0.99212	19.2	8.375E-05

Pb

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

