KTH Teknik och hälsa

Interaction radiation with matter, quick intro

1 Basic definitions

Suppose to have a flux of particles, $F = \dfrac{N}{A}$, (N number of particles in the beam area A) with energy E hitting a target. Particles could be photons, electrons, protons, α, heavy ions, neutrons ...
One can define the probability of scattering the incoming radiation at a given angle Ω as:

$$\frac{d\sigma(E,\Omega)}{d\Omega} \overset{\text{Def}}{\equiv} \frac{1}{F}\frac{dN}{d\Omega}. \tag{1}$$

This probability is called *differential cross section* and it is a property of the target. The integral over all angles Ω is the cross section $\sigma(E)$.
Simple dimensional analysis gives:

$$\left[\frac{d\sigma(E,\Omega)}{d\Omega}\right] = \frac{\mathbf{L^2}}{\text{angle}}$$

(actually angles are dimensionless)

$$[\sigma(E)] = \mathbf{L^2}$$

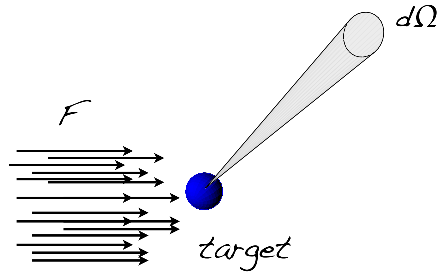


Figure 1: *A schematic view of a scattering experiment. Measuring the number dN of photons in the solid angle $d\Omega$ gives the differential cross section defined in eq.(1).*

suggesting the interpretation of the cross section as the effective area that the target offers to the impinging particles for interaction.
It is easy to convince oneself that the probability of scattering for each particle in a length dx of a given material is $M\sigma(E)dx$, where M is the number of targets per unit volume in the material (here we are making the assumption that targets are not *overlapping*):

1.1 Transport of photons

Considering only primary photons (i. e. photons that haven't interacted) the probability of interaction does not depend on the distance travelled in the target (x in the figure on the right). Note that the restriction to primary photons is due to the fact that σ is a function of the radiation energy, see its definition eq. 1.
Then, Beer's law can be derived:

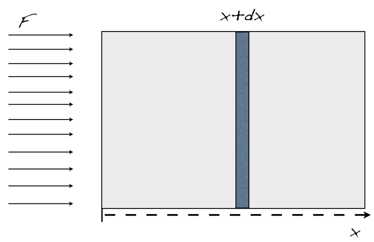


Figure 2:

$$P(x) \equiv \text{ probability of not having interacted at } x$$
$$M\sigma dx \equiv \text{ probability of interaction within } dx$$
$$\Rightarrow P(x)M\sigma dx = \text{ probability of interaction in interval } [x, x+dx]$$

with the above definitions, the probability of not interacting up to $x + dx$ is then:

$$P(x+dx) = \underbrace{\overbrace{P(x)}^{\textit{survived} \text{ up to } x} \cdot \overbrace{(1 - M\sigma dx)}^{\text{does } \textit{not} \text{ interact in } dx}}_{\text{independent events}}$$

solving for $P(x)$ gives a simple expression for the probability of transmission through distance x of photons of energy E:

$$P(x) = e^{-M\sigma(E)x}.$$

Simply note that you can rewrite the eq. as $\frac{dP}{P} = -M\sigma dx$

Beer's law gives the number of photons N transmitted through x of a material with linear absorption coefficient $\mu(E)$ when N_0 photons are shot in, and is a simple consequence of the probability derived above:

$$N = N_0 e^{-\mu(E)x}.$$

1.1.1 Relationship between cross section and linear attenuation and mass attenuation coefficient

In the previous derivation, the identity $M\sigma = \mu$ has been used without any explanation. Here I give a brief account of the relationship between σ and μ and also between σ and μ_ρ.
The cross section for photons interacting with matter at energies relevant to medical imaging regards photon-electron interactions. As a consequence, the cross section, in our case, is the area that a single electron opposes to the propagation of a photon of given energy.
The area of an atom will then be $Z\sigma$, since there are Z electrons in an atom.
The area in a unit volume of a given material will instead be: $N_A \frac{\rho}{m_{\text{mole}}} Z\sigma$, where N_A indicates Avogadro's number, which is the number of atoms contained in a mole of a given material. The area per unit volume has (obviously) the dimensions of the inverse of a length, and it is called the *linear attenuation coefficient*, μ, of the material.
By dividing the linear attenuation coefficient by the density of the material, one obtains the area per unit mass of the material: $N_A \frac{1}{m_{\text{mole}}} Z\sigma$. This quantity is called *mass attenuation coefficient*, it is usually indicated by the symbol μ_ρ, has the units of an area divided by a mass, and is more conveniently tabulated than the linear attenuation coefficient, since it is independent on temperature, pressure, phase or any other factor that can influence the density of a material.
Tables of μ_ρ for all elements and many compounds as a function of energy can be found on the NIST web-site (simply google "NIST mass attenuation coefficient").

1.1.2 Interaction of photons with matter

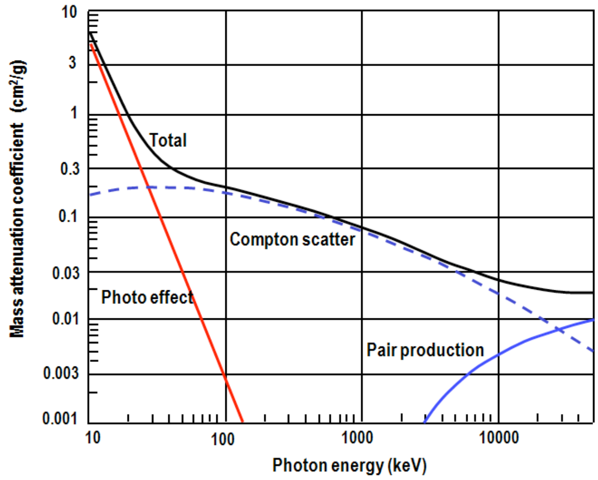
In the energy range of interest for medical imaging the 3 main processes of interaction of photons with matter are:

1. Photoelectric effect
2. Compton scattering
3. Pair production

The total cross section will therefore be the sum of the cross sections for the 3 different processes:

$$\sigma(E) = \sigma_{\text{PE}}(E) + \sigma_{\text{C}}(E) + \sigma_{\text{PP}}(E)|_{E \geq 1022 \text{ keV}}$$

where:
$\sigma_{\text{PE}}(E)$ is the cross section for interaction via photoelectric effect for incoming radiation of energy E,
$\sigma_{\text{C}}(E)$ is the cross section for interaction via Compton scattering for incoming radiation of energy E
and $\sigma_{\text{PP}}(E)|_{E \geq 1022 \text{ keV}}$ is the cross section for interaction via pair production for incoming radiation of energy $E \geq 1022$ keV (threshold energy for production of twice the electron mass).



Actually, Rayleigh scattering is quite relevant for very low energies but we won't consider it here

Figure 3: *Mass attenuation coefficient of photons in muscle ($Z_{eff} = 7$) as a function of photon energy. Contributions from photoelectric effect, Compton scattering and pair production are shown.*

Let me suppose that you are familiar with these 3 processes and let us look at the energy dependence of the probability for the different processes: Photoeffect dominates at low energies while Compton, being more or less independent of energy, becomes relatively more and more dominant as the energy grows. Pair production is not of interest for clinical imaging as it requires E $\geq$1022 keV, it might be of relevance for the dose delivered, though.
An example of the above trends is shown in figure 3 for photons propagating in muscular tissue.
Of special interest for designing medical imaging devices and protocols is the ratio photoelectric/Compton in different materials:

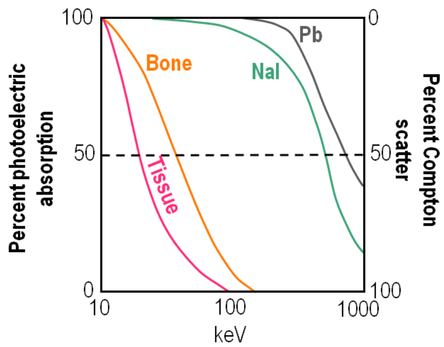


Figure 4: *Ratio PE to Compton in chosen materials as a function of incoming photon energy.*

As you can see in figure 4, the Photo-effect to Compton ratio, in any given material, is strongly dependent on photon energy.