SF1811 Optimization
Theory questions 2022/2023

One of the twelve theory questions below will be given at the final exam as an alternative question.
Note that you have to motivate your conclusions carefully at the final exam. It must be clear that you understand what you write. A correct argumentation without logical or formal deficiencies is necessary for full score.

1. Formulate and prove the fundamental theorem of linear programming.

2. Formulate and prove weak duality for linear programming.

3. Formulate and prove strong duality for linear programming. The proof may use the existence of optimal solutions given by the simplex method.

4. Formulate and prove the complementary theorem for linear programming. The proof may use strong duality.

5. Assume that $\hat{x}$ is a local minimizer of $f : \mathbb{R}^n \to \mathbb{R}$, which is differentiable. Prove that $\nabla f(\hat{x}) = 0$.

6. Assume that $\hat{x}$ is a local minimizer to a convex optimization problem. Prove that $\hat{x}$ is also a global minimizer.

7. Assume that $\hat{x}$ is a local minimizer of $f : \mathbb{R}^n \to \mathbb{R}$ and f is two times continuously differentiable. Prove that $\nabla f(\hat{x}) = 0$ and $\nabla^2(\hat{x})$ is positive semidefinite, where $\nabla^2(\hat{x})$ denotes the Hessian of f at $\widehat{x}$.

8. Prove that if $f : \mathbb{R}^n \to \mathbb{R}$ is convex and continuously differentiable, then $\hat{x}$ is a global minimizer if and only if $\nabla f(\widehat{x}) = 0$.

9. State and motivate the Karush-Kuhn-Tucker conditions for a local minimizer that is also a regular point.

10. Assume that $f : \mathbb{R}^n \to \mathbb{R}$ and $g_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, m$, are all convex and continuously differentiable. Prove that if $\hat{x}$ satisfies the KKT-conditions then $\hat{x}$ is a global minimizer to $\min f(x)$ subject to $g_i(x) \leq 0$, $i = 1, \ldots, m$.

11. Formulate and prove weak duality for a relaxed Lagrange problem.

12. Formulate and prove a theorem in which the global optimality conditions for a relaxed Lagrange problem give a global minimizer to the original problem.