Department of Mathematics

SF1624
Algebra and Geometry
Year 2017/2018

Problems for Seminar 6

Check the canvas page of the course for information on how seminars are run and what you are expected to do before and during the seminars.

Start by doing the simpler recommended exercises from the book:

7.1: A1, A2
7.2: A2, A3
7.3: A1
8.1: A2
8.2: A2

The seminar starts with a test. The problem will be about finding an approximation to a solution of a linear system of equations using the least squares method.

In the seminar, the following problems will be discussed.

Problem 1. Three vectors in $\mathbf{R}^5$ are given by

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 2 \\ 0 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 2 \\ 1 \\ -3 \\ 5 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{w} = \begin{bmatrix} 1 \\ 4 \\ -4 \\ 0 \\ -4 \end{bmatrix}.$$

(a) Construct an orthonormal base $\mathcal{B}$ of the vector space W spanned by $\vec{u}$, $\vec{v}$ and $\vec{w}$.
(b) Construct an orthonormal base $\mathcal{B}'$ of the orthogonal complement $W^{\perp}$ of W.
(c) Let $p\colon \mathbf{R}^5 \to \mathbf{R}^5$ be the projection onto W. Compute the projection $p(\vec{x})$, where $\vec{x} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}^T$.
(d) Determine the matrix of p with respect to the basis $\mathcal{B} \cup \mathcal{B}'$ of $\mathbf{R}^5$, i. e. the basis consisting of the elements of $\mathcal{B}$ together with the elements of $\mathcal{B}'$.

Problem 2. A line $y = kx + m$ is to fit the points $(-2, 1)$, $(1, 2)$, $(4, 2)$, and $(7, 6)$.

(a) Determine the values of the constants k and m giving the bet fit in the sense of least squares.
(b) Sketch the line together with the points in a coordinate system and illustrate what it is that is minimized for these values of the constants.

Problem 3. The quadratic form Q on $\mathbb{R}^2$ is given by

$$Q(\vec{x}) = x_1^2 + x_1 x_2 + x_2^2.$$

(a) Determine the symmetric matrix A which satisfies $Q(\vec{x}) = \vec{x}^T A\vec{x}$.
(b) Determine whether Q is positive definite, negative definite, positive semidefinite, negative semidefinite, or indefinite.

MISCELLANEOUS

Here are some other topics that are important and interesting to discuss.

- What is an orthogonal matrix?
- What is the relationship between symmetric and orthogonal matrices?
- Why are symmetric matrices diagonalizable?
- What is a quadratic form and how does one classify them?
- What is the purpose of the least squares method?
- What is the connection between the least squares method and (orthogonal) projections?