# FORMELBLAD

**Kurser: HF1008, Hf1006. TEN2 (Analys)** **Program: Elektroteknik, Datateknik**

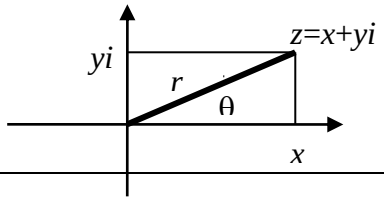
## TRIGONOMETRISKA FORMLER

| | | | |
|---|---|---|---|
| 1. | $\sin 30° = \sin\frac{\pi}{6} = \frac{1}{2}$ | 16. | $\sin^2\alpha + \cos^2\alpha = 1$ |
| 2. | $\sin 45° = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ | 17. | $\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}, \quad \cot(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)}$ |
| 3. | $\sin 60° = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ | 18. | $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ |
| 4. | $\cos 30° = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ | 19. | $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ |
| 5. | $\cos 45° = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ | 20. | $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ |
| 6. | $\cos 60° = \cos\frac{\pi}{3} = \frac{1}{2}$ | 21. | $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ |
| 7. | $\tan 30° = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ | 22. | $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta}$ |
| 8. | $\tan 45° = \tan\frac{\pi}{4} = 1$ | 23. | $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\cdot\tan\beta}$ |
| 9. | $\tan 60° = \tan\frac{\pi}{3} = \sqrt{3}$ | 24. | $\sin 2\theta = 2\sin\theta\cdot\cos\theta$ |
| 10. | $\cos(-\theta) = \cos\theta$ | 25. | $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$ |
| 11. | $\sin(-\theta) = -\sin\theta$ | 26. | $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ |
| 12. | $\tan(-\theta) = -\tan\theta$ | 27. | $\cos\theta = x \Leftrightarrow \theta = \pm\arccos x + n\cdot 2\pi$ |
| 13. | $\cot(-\theta) = -\cot\theta$ | 28. | $\sin\theta = x \Leftrightarrow \theta_1 = \arcsin x + n\cdot 2\pi;\quad \theta_2 = \pi - \arcsin x + n\cdot 2\pi$ |
| 14. | $\cos(\frac{\pi}{2} - \theta) = \sin\theta$ | 29. | $\tan\theta = x \Leftrightarrow \theta = \arctan x + n\cdot\pi$ |
| 15. | $\sin(\frac{\pi}{2} - \theta) = \cos\theta$ | 30 | $\cot\theta = x \Leftrightarrow \theta = arc\cot x + n\cdot\pi$ |

## KOMPLEXA TAL

Låt $z = x + yi$ vara ett komplext tal .

$z = x + yi = r(\cos\theta + i\sin\theta) = re^{i\theta}$




| | | | |
|---|---|---|---|
| 31. | Absolutbelopp: $\lvert z\rvert = r = \sqrt{x^2+y^2}$ | 39. | $\arg(z) = \theta = \begin{cases} \arctan\frac{y}{x} & \text{då } x>0 \\ \pi + \arctan\frac{y}{x} & \text{då } x<0 \\ \frac{\pi}{2} & \text{då } x=0,\ y>0 \\ -\frac{\pi}{2} & \text{då } x=0,\ y<0 \\ \text{ej definierat då både } x=0 \text{ och } y=0 \end{cases}$ |
| 32. | $\lvert z_1\cdot z_2\rvert = \lvert z_1\rvert\cdot\lvert z_2\rvert$ | | |
| 33. | $\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$ | | |
| 34. | $\lvert z^n\rvert = \lvert z\rvert^n$ | | |
| 35. | $\lvert z_1 \pm z_2\rvert \le \lvert z_1\rvert + \lvert z_2\rvert$ | | |
| 36. | $\arg(z_1\cdot z_2) = \arg(z_1) + \arg(z_2)$ | | |
| 37. | $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$ | | |
| 38. | $\arg(z^n) = n\cdot\arg(z)$ | | |

## DERIVERINGSREGLER

$(af+bg)' = af'+bg'$, a, b konstanter $f = f(x)$, $g = g(x)$

$(fg)' = f'g + fg'$ $\qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

KEDJEREGELN. Om $f$ och $g$ är deriverbara så är också $f[g(x)]$ deriverbar och

$\frac{d}{dx} f[g(x)] = f'[g(x)] \cdot g'(x)$ dvs $\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$ då $y = f(z)$ och $z = g(x)$.

## REGLER FÖR INTEGRATION

PARTIELL INTEGRATION

$$\int f(x)\,g'(x)dx = f(x)g(x) - \int f'(x)\,g(x)dx$$

$$\int_a^b f(x)\,g'(x)dx = \left[f(x)g(x)\right]_a^b - \int_a^b f'(x)\,g(x)dx$$

INTEGRATION GENOM SUBSTITUTION

$$\int f[g(x)]\cdot g'(x)dx = \left\{\begin{matrix} t = g(x) \\ dt = g'(x)dx \end{matrix}\right\} = \int f(t)dt$$

$$\int_a^b f[g(x)]\cdot g'(x)dx = \left\{\begin{matrix} t = g(x) \\ dt = g'(x)dx \end{matrix}\right| \begin{matrix} x = a \Rightarrow t = g(a) \\ x = b \Rightarrow t = g(b) \end{matrix} \Bigg\} = \int_{g(a)}^{g(b)} f(t)dt$$

NÅGRA OBESTÄMDA INTEGRALER

| | | | |
|---|---|---|---|
| 40. | $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ | 49. | $\int \frac{dx}{\sin ax} = \frac{1}{a}\ln\lvert\tan\frac{ax}{2}\rvert + C$ |
| 41. | $\int e^{ax+b}dx = \frac{e^{ax+b}}{a} + C$ | 50. | $\int \frac{dx}{\cos ax} = \frac{1}{a}\ln\lvert\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)\rvert + C$ |
| 42. | $\int a^x dx = \frac{a^x}{\ln a} + C \quad (0 < a \neq 1)$ | 51. | $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} + C$ |
| 43. | $\int \cos(ax+b)dx = \frac{\sin(ax+b)}{a} + C$ | 52. | $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\lvert\frac{x-a}{x+a}\rvert + C$ |
| 44. | $\int \sin(ax+b)dx = \frac{-\cos(ax+b)}{a} + C$ | 53. | $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\lvert\frac{a+x}{a-x}\rvert + C$ |
| 45. | $\int \frac{1}{ax+b}dx = \frac{\ln\lvert ax+b\rvert}{a} + C$ | 54. | $\int \frac{dx}{\sqrt{x^2+a}} = \ln\lvert x + \sqrt{x^2+a}\rvert + C$ |
| 46. | $\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + C$ | 55. | $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$ |
| 47. | $\int \frac{dx}{\sin^2 x} = -\cot x + C$ | 56. | $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$ |
| 48. | $\int \frac{dx}{\cos^2 x} = \tan x + C$ | 57. | $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + C$ |