KTH Teknikvetenskap

SF2719/SF2725 The History of Mathematics
Exam
Thursday, October 20, 2022

Time: 14:00–19:00
Allowed aids: Swedish-English and/or English-Swedish dictionary
Lecturer: Tilman Bauer
Examiner: Tilman Bauer

This exam consists of three parts, each giving up to 12 points. The grade requirements are:

	E	D	C	B	A
minimum points	18	21	24	27	30
of which on part A at least	4	4	4	4	4
and on part B at least	4	4	4	8	10
and on part C at least	4	4	4	8	10

Students achieving at least 4 points on every part, but only 16 or 17 points in total, obtain the grade Fx with the possibility of completion to grade E.

PART A

Answer the following questions briefly. Every question gives up to 2 points. Questions 1 through 4 can be replaced by the in-class quizzes 1 through 4, respectively. If you answer one of these question and at the same time got points on the corresponding quiz, the maximum score will be taken.

1. Briefly describe three fundamentally different systems of notating natural numbers.

2. Solutions to a general quintic cannot be expressed by radicals. Explain what this statement means.

3. Explain briefly what Cavalieri's theory of indivisibles is about. Around what time was this?

4. Who was first to come up with the idea of expectation values of random variables, and around what time did this happen?

5. When did the idea of an ideal in a ring first come up? Name two important contributors.

6. Explain briefly what a perfect number is, when they were first studied, and what important mathematics they led to.

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please turn over

PART B

This part consists of an analysis of an original text (or a translation thereof). Your bonus points from the homework essays 1 and 3 are added to the score achieved on this part. However, the total score cannot exceed 12.

7. Analyze the following translation of a text by Heron of Alexandria (1st century CE) with respect to the questions:

- In modern terms, what is the general algorithm he describes for computing square roots? Is it a good one?
- What differences do you see in the style mathematics is done, compared with earlier Greek works such as Euclid's Elements?

> There is a general method for finding, without drawing a perpendicular, the area of any triangle whose three sides are given. For example, let the sides of the triangle be 7, 8 and 9. Add together 7, 8 and 9; the result is 24. Take half of this, which gives 12. Take away 7; the remainder is 5. Again, from 12 take away 8; the remainder is 4. And again 9; the remainder is 3. Multiply 12 by 5; the result is 60. Multiply this by 4; the result is 240. Multiply this by 3; the result is 720. Take the square root of this and it will be the area of the triangle. Since 720 has not a rational square root, we shall make a close approximation to the root in this manner. Since the square nearest to 720 is 729, having a root 27, divide 27 into 720; the result is $26\frac{2}{3}$; add 27; the result is $53\frac{2}{3}$. Take half of this; the result is $26\frac{1}{2} + \frac{1}{3}(= 26\frac{5}{6})$. Therefore the square root of 720 will be very nearly $26\frac{5}{6}$. For $26\frac{5}{6}$ multiplied by itself gives $720\frac{1}{36}$; so that the difference is $\frac{1}{36}$. If we wish to make the difference less than $\frac{1}{36}$, instead of 729 we shall take the number now found, $720\frac{1}{36}$, and by the same method we shall find an approximation differing by much less than $\frac{1}{36}$.
>
> The geometrical proof of this is as follows: *In a triangle whose sides are given to find the area*. Now it is possible to find the area of the triangle by drawing one perpendicular and calculating its magnitude, but let it be required to calculate the area without the perpendicular.
>
> Let ABC be the given triangle, and let each of AB, BC, CA be given; to find the area. Let the circle DEF be inscribed in the triangle with centre G [Euclid's *Elements* IV. 9], and let AG, BG, CG, DG, EG, FG be joined. Then [Euclid I. 41] $BC \cdot EG = 2 \cdot$ triangle BGC, $CA \cdot FG = 2 \cdot$ triangle AGC, $AB \cdot DG = 2 \cdot$ triangle ABG. Therefore the rectangle contained by the perimeter of the triangle ABC and EG, that is the radius of the circle DEF, is double of the triangle ABC. Let CB be produced and let
>
> ... *etc.*

PART C

This part consists of an essay. It can be replaced by the accumulated points of homework essays 2 and 4. If you write an essay here and at the same time have points from homework essays 2 and/or 4, the maximum score will be taken.

Choose **one** of the following topics and treat it in an essay. The discussion of each topic must be based on or illustrated by concrete and specific examples.

8. Based on lessons learned from the history of mathematics, how can we best foster mathematical talent?

OR

9. Describe and assess the influence of Fermat's work on 17th and 18th century mathematics.

OR

10. Describe the development of rigor in calculus from Newton and Leibniz on.