Problem sheet 15

(1) Find a vector orthogonal to both $(2, 1, 0)$ and $(4, 3, -1)$.

(2) What is the area of the triangle in $\mathbb{R}^3$ with vertices $(2, 1, 4)$, $(2, 1, 1)$ and $(0, 4, 1)$?

(3) Prove that $u \cdot (u \times v) = 0$.

(4) Find all pairs $\vec{u}$ and $\vec{v}$ satisfying $\vec{u} \times (\vec{u} \times \vec{v}) = \vec{v}$.

(5) Find all pairs $\vec{u}$ and $\vec{v}$ satisfying $\vec{u} \times (\vec{v} \times \vec{u}) = \vec{v}$.

(6) Find the equation of the plane passing through the points $(3, 4, 1)$, $(6, 1, 1)$ and $(2, 3, 1)$.

(7) Let S be the plane given by

$$t\vec{u} + s\vec{v} + \vec{P}, \qquad t, s \in \mathbb{R}, \tag{1}$$ {?}

where $\vec{u}$ and $\vec{v}$ are vectors and $\vec{P}$ is a point in the plane. Why is an alternative description for the plane S, the points (x, y, z) satisfying

$$(\vec{u} \times \vec{v}) \cdot \left((x, y, z) - \vec{P}\right) = 0? \tag{2}$$ {?}