Problem sheet 11

(1) Let $\vec{u} = (1,1)$ and $\vec{v} = (-1,1)$. Draw the following vectors with their initial points at the origin:

    (a) $2\vec{u}$.

    (b) $\vec{u} + \vec{v}$.

    (c) $\vec{u} - \vec{v}$.

    (d) $\vec{u} + 2\vec{v}$.

(2) Given points P and Q in $\mathbb{R}^2$, denote by $\overrightarrow{PQ}$ the vector from Q to P. What are the components of $\overrightarrow{PQ}$ in the following examples?

    (a) $P = (1,5)$ and $Q = (4,1)$.

    (b) $P = (0,0,4)$ and $Q = (2,3,0)$.

(3) Find the components of $\vec{w}$ satisfying

$$2\vec{u} - \vec{v} + \vec{w} = 7\vec{w} + \vec{y},$$

where $\vec{u} = (-3,1,2)$, $\vec{v} = (4,0,-8)$, and $\vec{y} = (6,-1,-4)$.

(4) Let $\vec{u} = (2,-2,3)$ and let $\vec{v} = (1,-3,4)$. Compute the following norms

    (a) $\|\vec{u} + \vec{v}\|$.

    (b) $\|\vec{u} - 3\vec{v}\|$.

    (c) $\|\vec{u}\| - 3\|\vec{v}\|$.

(5) In the following examples, find $\vec{u} \cdot \vec{v}$

    (a) $\vec{u} = (3,1,4)$ and $\vec{v} = (2,2,-4)$.

    (b) $\vec{u} = (1,1,4)$ and $\vec{v} = (2,-2,3)$.

(6) Let $\vec{a}$ and $\vec{b}$ be vectors in $\mathbb{R}^2$, with lengths 9 and 5 respectively. Let b point in the negative y direction and let x make an angle $2\pi/3$ in the counterclockwise direction with the positive x-axis. What is $\vec{a} \cdot \vec{b}$?

(7) Let $\vec{a}$ and $\vec{b}$ be vectors in $\mathbb{R}^3$. Which of the following statements make mathematical sense? Why / why not?

    (a) $(\vec{a} \cdot \vec{b})\vec{b}$.

    (b) $(\vec{a} \cdot \vec{b})\vec{b}$.

    (c) $\|\vec{a} + \vec{b}\|\vec{a}$.

    (d) $(\vec{a} + \vec{b})\vec{a}$.

(8) What is the angle between $\vec{v}$ and $\vec{w}$ if $\vec{v} = (2,3,4)$ and $\vec{w} = (3,4,5)$? What is the angle between $\vec{v}$ and $-\vec{w}$?