

Problem sheet 9

(1) Simplify the following expressions

- (a) $5^3/5^{5/2}$.
- (b) $\log_{10}(100)$.
- (c) $\log_{10}(1000)$.
- (d) $\log_{\sqrt{10}}(100)$.
- (e) $\log_2(16)$.
- (f) $\log_2(48) - \log_2(3)$.
- (g) $\log_a(b) \log_b(a)$.

(2) Solve the following equations

- (a) $2^{x+5} = 3^{2x}$.
- (b) $e^x + e^{-x} = 2$.

(3) Simplify the following expressions

- (a) $\log(x^2 + 10x + 25)$.
 - (b) $\log(\sqrt{x^2 - 1}) - \log(x - 1) - \log(x + 1) + \log\left(\frac{1}{\sqrt{x^2 - 1}}\right)$ for $x > 1$.
 - (c) $2\log(\sin x) - \log(1 - \cos x) - \log(1 + \cos x)$ for $x \in (0, \pi)$.
- (4) Prove that $(ab)^{1/n} = a^{1/n}b^{1/n}$ if $n \in \mathbb{N}$. You may assume that $(ab)^n = a^n b^n$ for $n \in \mathbb{N}$.

(5) Find all x satisfying the following inequality

$$\log(x + 1) - \log(2 - x) > \log(2x), \quad (1) \{?\}$$

such that all of the above quantities are well defined.