## Problem sheet 6

In class we defined sine and cosine. The third trigonometric function you will learn is tangent, denoted $\tan x$, and defined by

$$
\begin{equation*}
\tan x=\frac{\sin x}{\cos x} . \tag{1}
\end{equation*}
$$

For these exercises you will need, in addition to the identities from class, the following identities:

$$
\begin{align*}
\sin (x+y) & =\cos x \sin y+\sin x \cos y \\
\cos (x+y) & =\cos x \cos y-\sin x \sin y \tag{2}
\end{align*}
$$

(1) Express each of the following functions in terms of $\sin x$ and $\cos x$
(a) $\cos (\pi+x)$.
(b) $\sin \left(\frac{3 \pi}{2}-x\right)$.
(c) $\cos (2 x)$.
(d) $\sin (2 x)$.
(2) Prove that $\frac{1-\cos x}{1+\cos x}=\left(\tan \frac{x}{2}\right)^{2}$.
(3) Prove that $\frac{\cos x-\sin x}{\cos x+\sin x}=\frac{1}{\cos 2 x}-\tan 2 x$.
(4) Express $\cos 3 x$ in terms of $\sin x$ and $\cos x$.

The cosine and sine function can be extended to hold for all real numbers through the rule $\cos (x+$ $2 \pi)=\cos x$ and $\sin (x+2 \pi)=\sin x$. This is sensible because the circle has $2 \pi$ radians, and so $\sin (2 \pi)=\sin (0)=0$ and $\cos (2 \pi)=\cos (0)=1$.
(5) Sketch $f(x)=\cos \frac{\pi x}{2}$. We say that a function $f$ has period $c$ if $f(x+c)=f(x)$ for all $x \in \mathbb{R}$. What is the period of $f$ ?
(6) Sketch the graph of $y=1+\sin \left(x+\frac{\pi}{4}\right)$.

