

Problem sheet 5

(1) Find the roots of the following polynomials

(a) $x^2 - 3x - 10$.

(b) $16x^4 - 8x^2 + 1$.

(c) $x^4 + 6x^3 + 9x^2$.

(d) $x^5 - x^4 - 16x + 16$.

(e) $x^9 - 4x^7 - x^6 + 4x^4$.

(2) What is the domain of $\frac{x^2-9}{x^3-x}$?

(3) Express $\frac{x^2}{x^2+5x+3}$ as $p(x) + \frac{q(x)}{r(x)}$ where the degree of $q(x)$ is less than the degree of $r(x)$.

(4) Express $\frac{x^4+x^2}{x^3+x^2+1}$ as $p(x) + \frac{q(x)}{r(x)}$ where the degree of $q(x)$ is less than the degree of $r(x)$.

(5) Express the $x^4 + x^2 + 1$ as a product of real quadratic polynomials with no real roots.

(6) Show that $x - 1$ is a factor of a polynomial P if and only if the sum of the coefficients of P is zero.