

Problem sheet 4

- (1) Let $f(x) = \sqrt{1-x}$ and $g(x) = \sqrt{1+x}$. Find the domain of $f+g$, $f-g$, fg , f/g , and g/f .
- (2) If $f(x) = x+5$ and $g(x) = x^2-3$, sketch the graphs of $f(g(x))$ and $g \circ f(x)$.
- (3) Let $f(x) = (x+1)/(x-1)$ and $g(x) = \operatorname{sgn}(x)$. Find the domain and range of, and provide a sketch of the graph of, the following composite functions:
- (a) $f \circ f(x)$.
 - (b) $f \circ g(x)$.
 - (c) $g \circ f(x)$.
 - (d) $g \circ g(x)$.
- (4) Sketch the graph of $g(x) = \begin{cases} \sqrt{x} & \text{if } 0 \leq x \leq 1, \\ 2-x & \text{if } 1 < x \leq 2. \end{cases}$
- (5) Find the real values of the constants A and B for which the function $F(x) = Ax + B$ satisfies
- (a) $F \circ F(x) = F(x)$ for all x .
 - (b) $F \circ F(x) = x$ for all x .
- (6) For what values of x is $\lfloor x \rfloor = 0$? For what values of x is $\lceil x \rceil = 0$?
- (7) What real numbers satisfy the equation $\lfloor x \rfloor = \lceil x \rceil$?
- (8) True or false: $\lceil -x \rceil = -\lfloor x \rfloor$?
- (9) Sketch the graph of $y = x - \lfloor x \rfloor$.
- (10) Sketch the graph of the function

$$f(x) = \begin{cases} \lfloor x \rfloor & \text{if } x \geq 0, \\ \lceil x \rceil & \text{if } x < 0. \end{cases} \quad (1) \{?\}$$

Why is $f(x)$ called the integer part of x ?

- (11) Assume that f is even and g is odd, and the domains of f and g are $\mathbb{R}$.