Advanced Course

Distributed Systems

Basic Abstractions

Paris Carbone

A System's Roadmap

I- Specification

The 'WHAT'

- *Assumptions*
- *Goals*
- *Set of Properties*

II- Solution Design

The 'HOW'

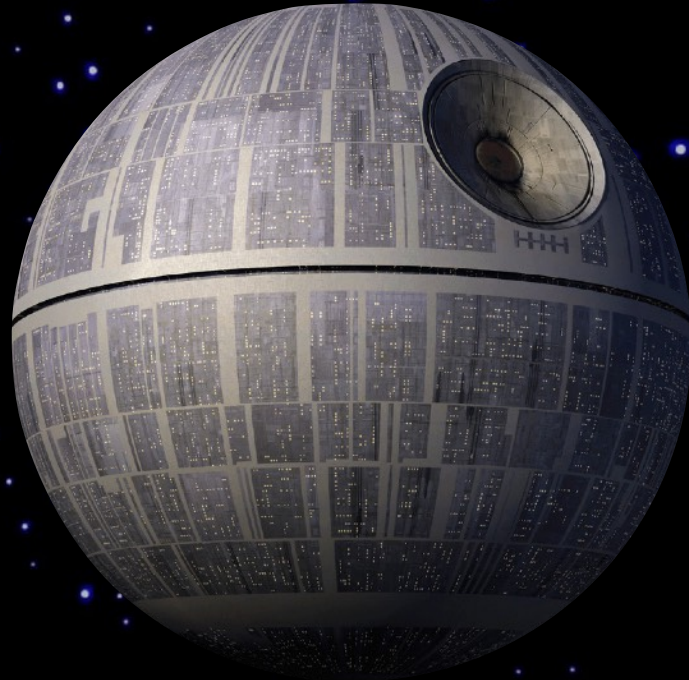- *Satisfies Properties*
- *Abstract yet Accurate Representation*

III- Implementation

- *Execution*
- *Development*

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Let's take a closer look into
…one of the biggest systems of all time

The Death Star

DEATH STAR ROADMAP

I- Specification

II- Solution Design

III- Implementation

- **Gargantuan Scale/Storage**
- **Indestructible**
- **Ultra High-Speed (>light)**
- **Massive Power Projection**
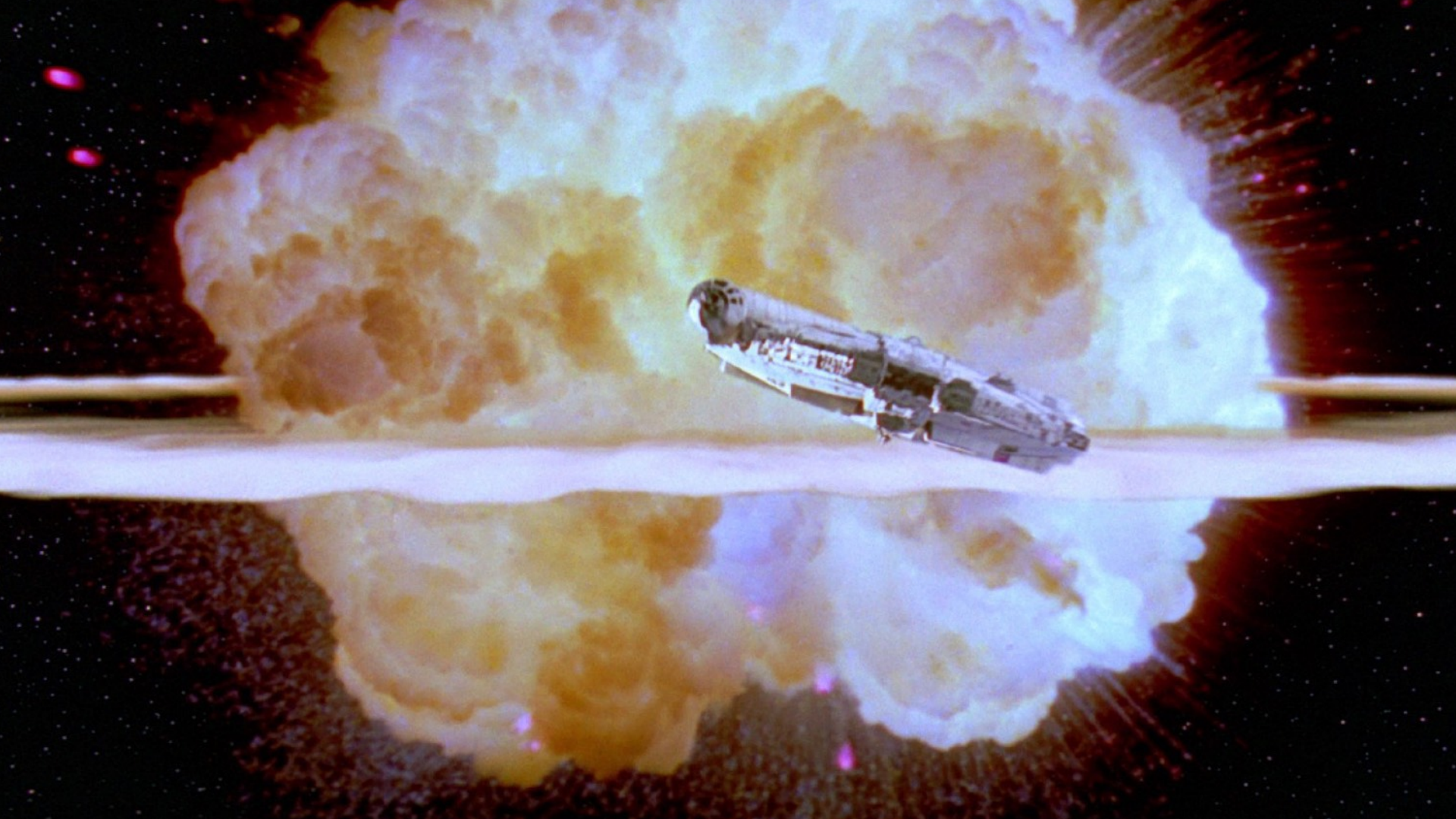
- **Moon-Size Model**
- **Stainless Steel Plates**
- **Hyperdrive, Thermal Reactors**
- **Superlaser Module Design**

THE ISSUE

I- Specification

II- Model (Blueprint)

DEATH STAR

shoot here to detonate

- Gargantuan Scale/Storage
- ~~Indestructible~~
- Ultra High-Speed (>light)
- Massive Power Projection

- Moon-Size Model
- Stainless Steel Plates
- Hyperdrive, **Thermal Reactors**
- Superlaser Module Design

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WE COULD HAVE SAVED DEATH STAR

- As with every type of reliable system

    1. A correct, careful specification of its properties is crucial.

    2. A solution design (algorithm) needs to:

        1. Provably satisfy all properties and

        2. Not violating any property (duh).

    Let's see how this can be done with some core abstractions!

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COURSE TOPICS

▶ Intro to Distributed Systems

▶ Basic Abstractions and Failure Detectors

▶ Reliable and Causal Order Broadcast

▶ Distributed Shared Memory

▶ Consensus (Paxos, Raft, etc.)

▶ Dynamic Reconfiguration

▶ Time Abstractions and Interval Clocks (Spanner etc.)

▶ Consistent Snapshotting (Stream Data Management)

▶ Distributed ACID Transactions (Cloud DBs)

NEED OF DISTRIBUTED ABSTRACTIONS

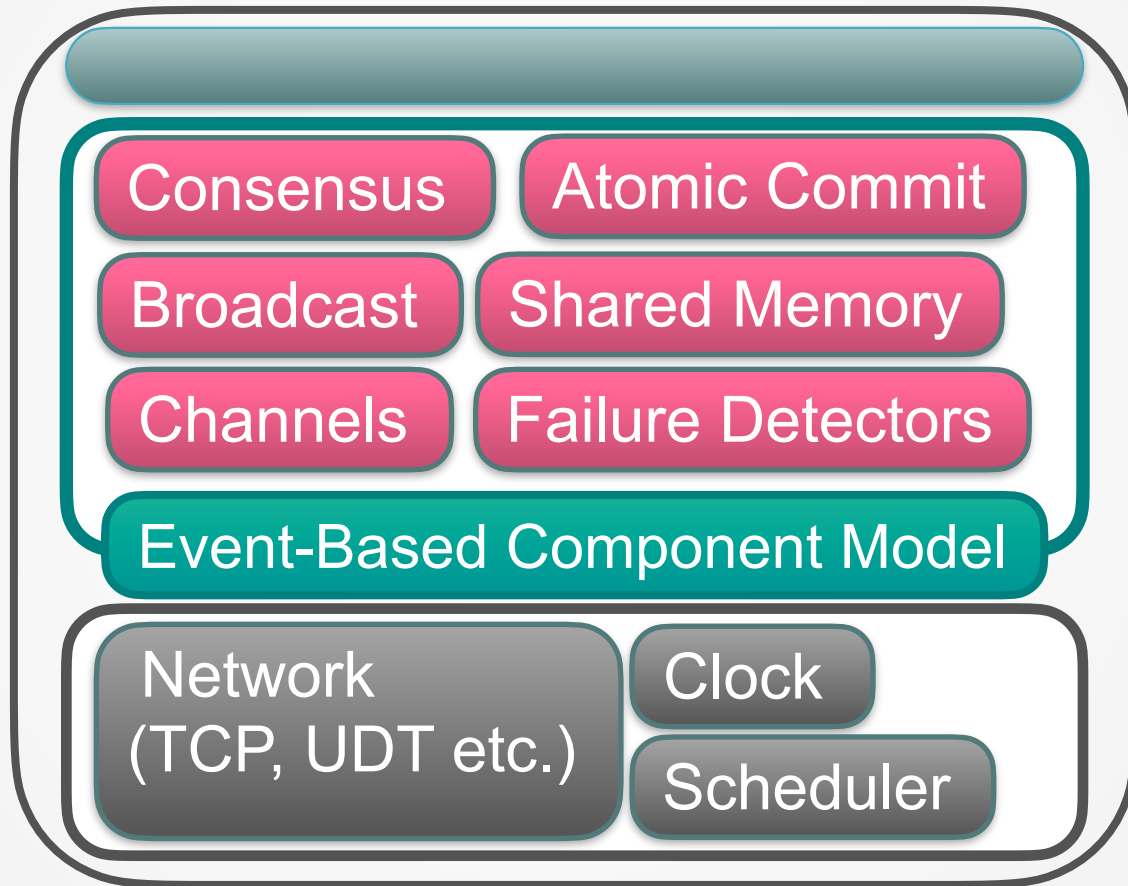*Reliable applications need underlying services stronger than network protocols (e.g. TCP, UDP)*

- The basic building blocks of **any** distributed system is a **set of distributed algorithms**.

- Implemented as a **middleware** between network (OS) and the application.

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ANATOMY OF A DISTRIBUTED SYSTEM
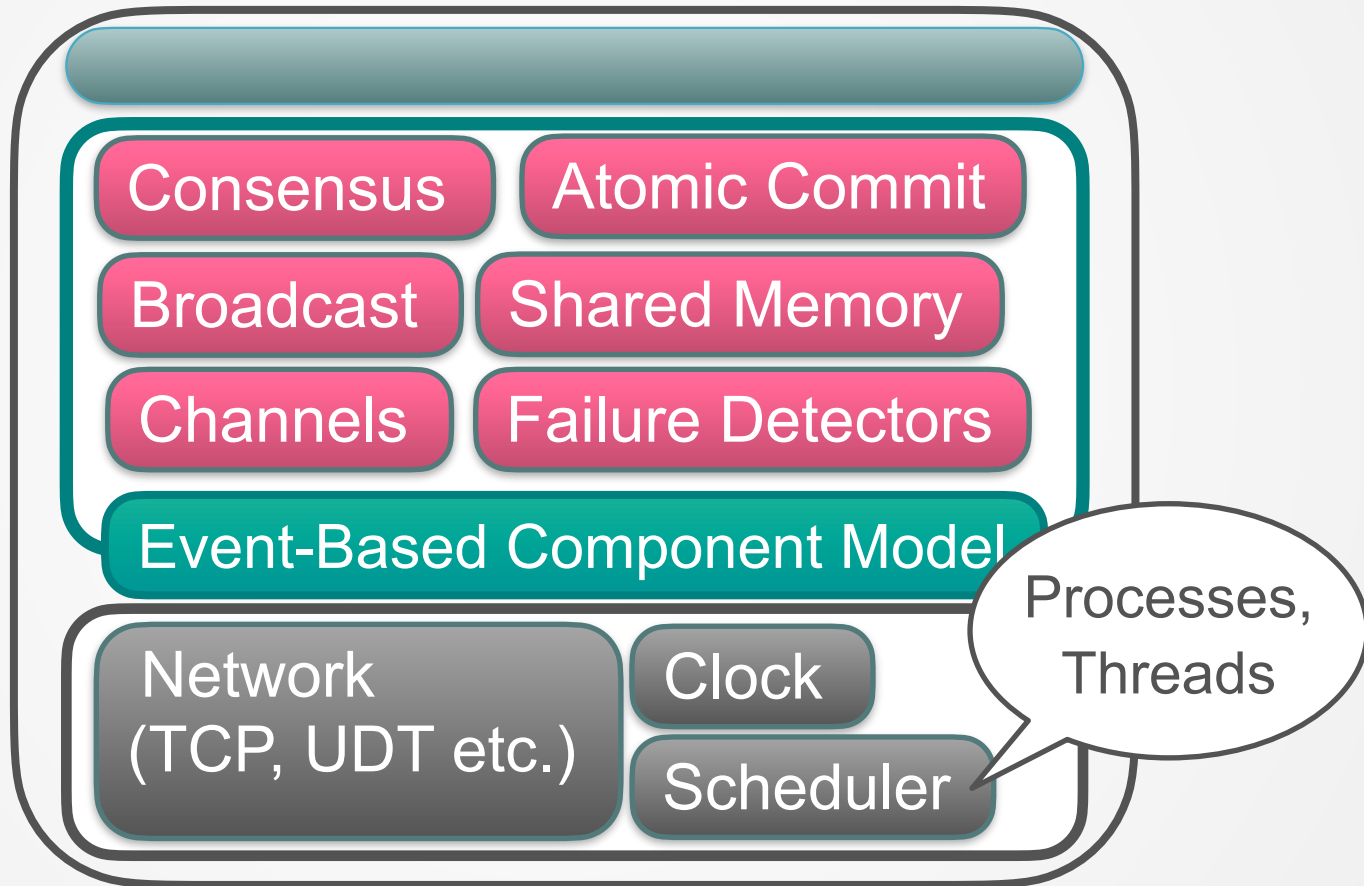
Distributed Applications

Middleware

Consensus	Atomic Commit
Broadcast	Shared Memory
Channels	Failure Detectors

Event-Based Component Model

OS

Network
(TCP, UDT etc.)

Clock

Scheduler

ANATOMY OF A DISTRIBUTED SYSTEM

Distributed Applications

Middleware

- Consensus
- Atomic Commit
- Broadcast
- Shared Memory
- Channels
- Failure Detectors

Event-Based Component Model

OS

- Network (TCP, UDT etc.)
- Clock
- Scheduler

Processes, Threads

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ANATOMY OF A DISTRIBUTED SYSTEM

Distributed Applications

Middleware

OS

Consensus

Atomic Commit

Broadcast

Shared Memory

Channels

Failure Detectors

Event-Based Component Model

Execution Model

Network (TCP, UDT etc.)

Clock

Scheduler

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ANATOMY OF A DISTRIBUTED SYSTEM

Network

Processes

DISTRIBUTED COMPUTING MODEL

Process

- Set of processes and a network (communication links)
- Each process runs a local algorithm (program)
- Each process makes computation steps

- The network makes computation steps
  - to store a message sent by a process
  - to deliver a message to a process

Network   Environment

- Message delivery triggers a computation step at the receiving process

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THE DISTRIBUTED COMPUTING MODEL

- Computation step at a process
  - 1. Receives a message (external, input)
  - 2. Performs local computation
  - 3. Sends one or more messages to some other processes (external, output)

- Communication step:
  - Depends on the network abstraction
  - Receives a message from a process, or
  - Delivers a message to a process

Process

2.

1. **3.**

Network Environment

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- A process consists of a set of components (automata)

- Components are concurrent and access local state.

- Each component receives messages through an input FIFO buffer

- Sends messages to other components

- Events: messages between components in the same process

- Events are handled by procedures (actions) called Event Handlers

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EVENTS VS MESSAGES

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EVENT-BASED PROGRAMMING

- Process executes program
  - Each program consists of a set of modules or **component specifications**

  - At runtime these are deployed as **components**

  - The components in general form a software stack

EVENT-BASED PROGRAMMING

Process executes program

Components interact via events (with attributes):

Handled by Event Handlers

on event $<co_i\ Event_1,\ attr1,\ attr2,...>$ **do**
        // local computation
        trigger $<co_j\ Event_2,\ attr3,\ attr4,...>$

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EVENT-BASED PROGRAMMING

- Events can be almost anything
  - Messages (most of the time)
  - Timers (internal event)
  - Conditions (e.g. x==5 & y<9)

- Two types of events
  - Requests (input)
  - Indications (output)

COMPONENTS IN A PROCESS

Stack of components in a single process

Applications	database_component

request   indication

Algorithms

commit_component

request   indication   request   indication

reliable_bcast_comp   consensus

request   indication

Channels

perfect_link_comp

Local events delivered in FIFO order

CHANNELS AS MODULES

Channels represented by modules (too)

Request event:

Send to destination some message (with data)

trigger *<send | dest*, [data1, data2, …] >

Indication event:

Deliver from source some message (with data)

upon event *<deliver | src,* [data1,data2, …]> **do**

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EXAMPLE

Application uses a Broadcast component

which uses channel component to broadcast

p_1		p_2	p_3
Applications	app	app	app
Algorithms	$\langle$**sendBcast**$|m\rangle$	$\langle$**delBcast**$|p_1,m\rangle$	$\langle$**delBcast**$|p_1,m\rangle$
	bcast	bcast	bcast
	$\langle$**send**$|p_2,m\rangle$ $\langle$**send**$|p_3,m\rangle$	$\langle$**deliver**$|p_1,m\rangle$	$\langle$**deliver**$|p_1,m\rangle$
Channels	channel	channel	channel

Specifications

SPECIFICATION OF A SERVICE

How to specify a distributed service (abstract)?
    1. Interface (aka Contract, API)
        Requests
        Responses
    2. Correctness Properties
        Safety
        Liveness
    3. Underlying Model
        Assumptions on failures
        Assumptions on timing (amount of synchrony)

declarative
specification
"what"
aka problem

Implementation
        Composed of other services
        Adheres to interface and satisfies correctness
        Has internal events

imperative,
many possible
"how"

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SIMPLE EXAMPLE: JOB HANDLER

Module:
> **Name: JobHandler, instance *jh***

Events:

how to use

> **Request:** *⟨jh, Submit | job⟩ : Requests a job to be processed*
> **Indication:** *⟨jh, Confirm | job⟩ : Confirms that the given job has been (or will be) processed*

Properties:

conditions

> *Guaranteed response: Every submitted job is eventually confirmed*

Synchronous Job Handler

Implements:

JobHandler, **instance *jh***

upon event ⟨*jh, Submit | job*⟩ **do**

process(*job*)

trigger ⟨*jh, Confirm | job*⟩

Implements:

JobHandler, **instance *jh***

upon event ⟨*jh, Init*⟩ **do**
buffer := ∅

upon event ⟨***jh, Submit | job***⟩ **do**
buffer := buffer ∪ {*job*}
trigger ⟨***jh, Confirm | job***⟩

upon *buffer* ≠ ∅ **do**
job := selectjob (buffer)
process(***job***)
buffer := buffer \ {*job*}

⟨*..**Init***⟩ automatically generated upon component creation

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COMPONENT COMPOSITION

⟨th submit …⟩

⟨th Confirm …⟩
⟨th Error⟩

TransformationHandler
(th)

⟨jh submit …⟩

⟨jh Confirm …⟩

JobHandler
(jh)

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Safety and Liveness Properties

SPECIFICATION OF A SERVICE

How to specify a distributed service (abstract)?

> Interface (aka Contract, API)
>> Requests
>> Responses
>
> Correctness Properties
>> Safety
>> Liveness
>
> Model
>> Assumptions on failures
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CORRECTNESS

Always expressed in terms of Safety and Liveness

Safety

Properties that state that nothing bad ever happens

Liveness

Properties that state that something good eventually happens

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CORRECTNESS EXAMPLE

- Correctness of You in ID2203

   Safety

   You should never fail the exam

   (marking exams costs money)

   Liveness

   You should eventually take the exam

   (university gets money when you pass)

• Correctness of traffic lights at intersection

Safety

Only one direction should have a green light

Liveness

Every direction should eventually get a green light

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EXECUTION AND TRACES

An execution fragment of A is sequence of alternating states and events

$s_0, \; \varepsilon_1, s_1, \varepsilon_2, \ldots, s_r, \varepsilon_r, \ldots$

$(s_k, \varepsilon_{k+1}, s_{k+1})$ transition of A for $k \geq 0$

An execution is execution fragment where s_0 is an initial state

A trace of an execution E, trace(E)

The subsequence of E consisting of all external events

$\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_r, \ldots$

SAFETY & LIVENESS ALL THAT MATTERS

A trace property P is a function that takes a trace and returns true/false

    i.e. P is a predicate

Any trace property can be expressed as the conjunction of a safety property and a liveness property"

SAFETY FORMALLY DEFINED

The prefix of a trace T is the first k (for $k \geq 0$) events of T

> I.e. cut off the tail of T

> I.e. finite beginning of T

An extension of a prefix P is any trace that has P as a prefix

SAFETY DEFINED

Informally, property P is a safety property if

Every trace T violating P has a bad event, s.t. every execution starting like T and behaving like T up to the bad event (including), will violate P regardless of what it does afterwards

SAFETY DEFINED

Formally, a property P is a safety property if

Given any execution E such that P(trace(E)) = false,

There exists a prefix of E, s.t. every extension of that prefix gives an execution F s.t. P(trace(F))=false

SAFETY EXAMPLE

Point-to-point message communication

Safety P: "At most once delivery"

A message sent is delivered at most once

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Point-to-point message communication

Safety P: "At most once delivery"

A message sent is delivered at most once

Take an execution where a message is delivered more than once

- Cut-off the tail after the second delivery

- Any continuation (extension) will give an execution which also violates the required property

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LIVENESS FORMALLY DEFINED

- A property P is a liveness property if

  Given any prefix F of an execution E,

  there exists an extension of trace(F) for which P is true

  "As long as there is life there is hope"

Point-to-point message communication

Liveness P: "At least once delivery"

A message sent is delivered at least once

Take the prefix of any execution

- If prefix contains delivery, any extension satisfies P

- If prefix doesn't contain the delivery, extend it so that it contains a delivery, the prefix + extended part will satisfy P

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Safety can only be

      satisfied in infinite time (you're never safe)

      violated in finite time (when the bad happens)

Often involves the word "never", "at most", "cannot",…

Sometimes called "partial correctness"

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Liveness can only be

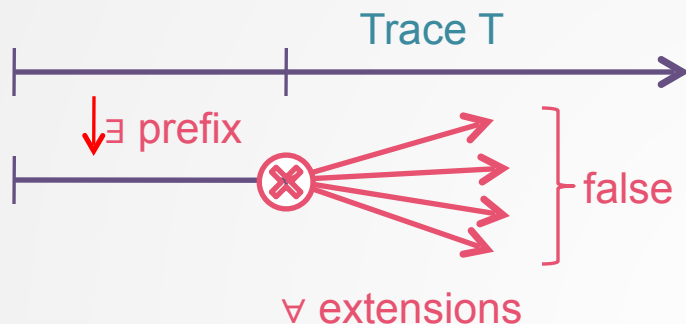   satisfied in finite time (when the good happens)

    violated in infinite time (there's always hope)
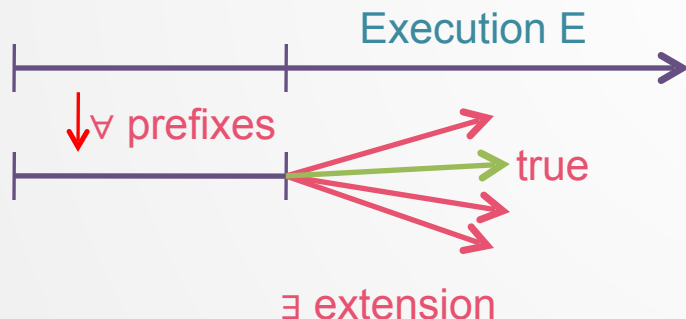
Often involves the words eventually, or must

    Eventually means at some (often unknown) point in "future"

Liveness is often just "termination"

FORMAL DEFINITIONS VISUALLY

Trace T

∃ prefix

false

∀ extensions

Execution E

∀ prefixes

true

∃ extension

- Safety can always be violated (false) in finite time

- Safety is violated for an execution E if there exists a prefix such that **all** extensions are false

- Liveness can always be made true in finite time

- Liveness is satisfied (true) for an execution E if for all prefixes there exists an extension that is true

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PONDERING SAFETY AND LIVENESS

Is really every property either liveness or safety?

Every message should be delivered exactly 1 time [d]

Every message is delivered at most once and

Every message is delivered at least once

Process Failure Model

SPECIFICATION OF A SERVICE

How to specify a distributed service (abstract)?

    Interface (aka Contract, API)

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        Liveness

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declarative specification "what" aka problem

    Implementation

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imperative, many possible "how"

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MODEL/ASSUMPTIONS

Specification needs to specify the distributed computing model

- Assumptions needed for the algorithm to be correct

Model includes assumptions on

- Failure behavior of processes & channels
- Timing behavior of processes & channel

PROCESS FAILURES

Processes may fail in four ways:

- Crash-stop

- Omissions

- Crash-recovery

- Byzantine/Arbitrary

- Processes that don't fail in an execution are correct

CRASH-STOP FAILURES

- Crash-stop failure

  - Process stops taking steps

    - Not sending messages

    - Nor receiving messages

- Default failure model is crash-stop

  - Hence, do not recover

  - But processes are not allowed to recover? [d]

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OMISSION FAILURES

- Process omits sending or receiving messages
  - Some differentiate between
    - Send omission
      - Not sending messages the process has to send according to its algorithm
    - Receive omission
      - Not receiving messages that have been sent to the process
  - For us, omission failure covers both types

CRASH-RECOVERY FAILURES

The process might crash

  It stops taking steps, not receiving and sending messages

It may recover after crashing

  Special \<Recovery\> event automatically generated

  Restarting in some initial recovery state

Has access to stable storage

  May read/write (expensive) to permanent storage device

  Storage survives crashes

  E.g., save state to storage, crash, recover, read saved state

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CRASH-RECOVERY FAILURES

- Failure is different in crash-recovery model
  - A process is faulty in an execution if
    - It crashes and never recovers, or
    - It crashes and recovers infinitely often (unstable)
  - Hence, a correct process may crash and recover
    - As long as it is a finite number of times

BYZANTINE FAILURES

- Byzantine/Arbitrary failures
  - A process may behave arbitrarily
    - Sending messages not specified by its algorithm
    - Updating its state as not specified by its algorithm

  - May behave maliciously, attacking the system
    - Several malicious processes might collude

Fault-tolerance Hierarchy

FAULT-TOLERANCE HIERARCHY

- Is there a hierarchy among the failure types
  - Which one is a special case of which? [d]
  - An algorithm that works correctly under a general form of failure, works correctly under a special form of failure

- Crash special case of Omission
  - Omission restricted to omitting everything after a certain event

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FAULT-TOLERANCE HIERARCHY

- In Crash-recovery

  - Under assumption that processes use stable storage as their main memory

- Crash-recovery is identical to omission

  - Crashing, recovering, and reading last state from storage

  - Just same as omitting send/receiving while being crashed

FAULT-TOLERANCE HIERARCHY

- In crash-recovery it is possible to use volatile memory
  - Then recovered nodes might not be able to restore all of state
  - Thus crash-recovery extends omission with amnesia

- Omission is special case of Crash-recovery
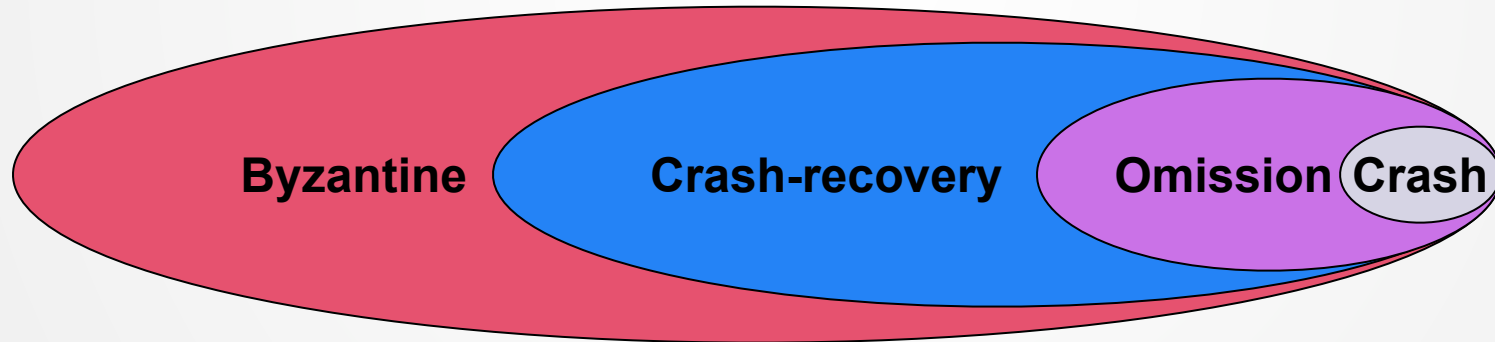 - Crash-recovery , not allowing for amnesia

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FAULT-TOLERANCE HIERARCHY

Crash-recovery special case of Byzantine

      Since Byzantine allows anything

Byzantine tolerance → crash-recovery tolerance

      Crash-recovery → omission, omission → crash-stop

Channel Behavior (failures)

SPECIFICATION OF A SERVICE

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    Implementation
        Composed of other services
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CHANNEL FAILURE MODES

- Fair-Loss Links
  - Channels delivers any message sent with non-zero probability (no network partitions)
- Stubborn Links
  - Channels delivers any message sent infinitely many times
- Perfect Links
  - Channels that delivers any message sent exactly once

CHANNEL FAILURE MODES

- Logged Perfect Links
  - Channels delivers any message into a receiver's persistent store (message log)

- Authenticated Perfect Links
  - Channels delivers any message m sent from process p to process q, that guarantees the m is actually sent from p to q
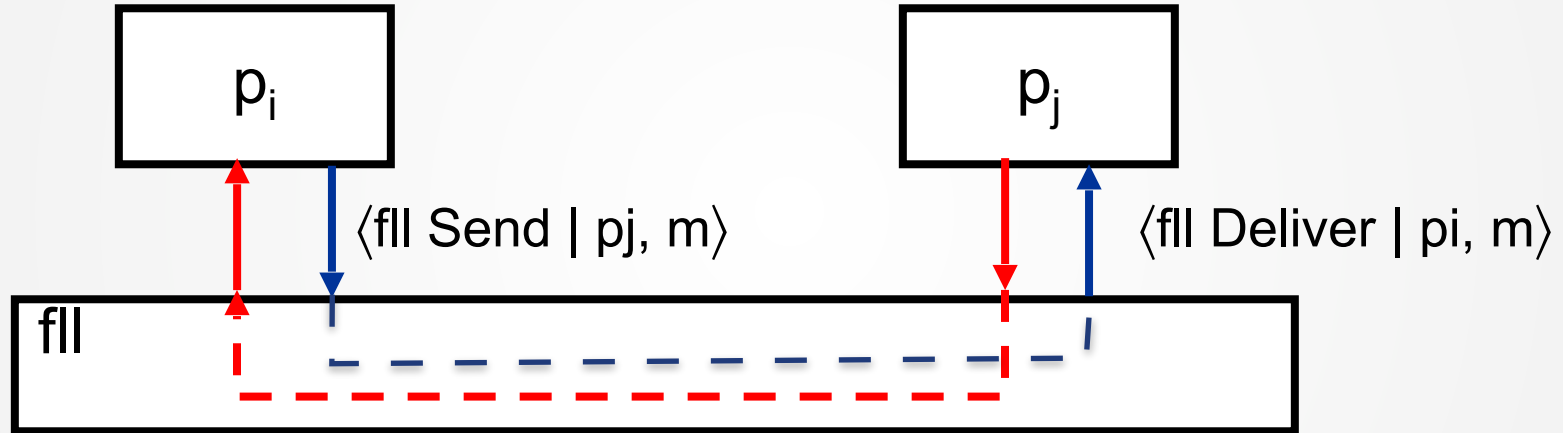
Fair Loss Links

Fair-Loss Links

Channels delivers any message sent with non-zero probability (no network partitions)

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⟨fll Send | pj, m⟩

⟨fll Deliver | pi, m⟩

p_i

p_j

fll

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Module:

Name: FairLossPointToPointLink **instance** fll

Events:

Request: ⟨fll, Send | dest, m⟩

Request transmission of message m to process dest

Indication:⟨fll, Deliver | src, m⟩

Deliver message m sent by process src

Properties:

FL1, FL2, FL3.

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FAIR-LOSS LINKS

Properties

> **FL1. Fair-loss**: If m is sent infinitely often by p_i to p_j, and neither crash, then m is delivered infinitely often by p_j
>
> **FL2. Finite duplication**: If a m is sent a finite number of times by p_i to p_j, then it is delivered at most a finite number of times by p_j
>
>> I.e. a message cannot be duplicated infinitely many times
>
> **FL3. No creation**: No message is delivered unless it was sent

Stubborn Links

Stubborn Links

Channels delivers any message sent infinitely many times

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Module:

Name: StubbornPointToPointLink **instance** sl

Events:

Request: ⟨sl, Send | dest, m⟩

Request the transmission of message m to process dest

Indication:⟨sl, Deliver src, m⟩

deliver message m sent by process src

Properties:

SL1, SL2

- Properties
  - **SL1. Stubborn delivery**: if a correct process p_i sends a message m to a correct process p_j, then p_j delivers m an infinite number of times
  - **SL2. No creation**: if a message m is delivered by some process p_j, then m was previously sent by some process p_i

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- Implementation
  - Use the Lossy (fair-loss) link
  - Sender stores every message it sends in **sent**
  - It periodically resends all messages in **sent**

Implements: StubbornLinks **instance** sl

Uses: FairLossLinks, **instance** fll

- **upon event** ⟨sl, Init⟩ **do**

    sent := ∅

    startTimer(TimeDelay)

- **upon event** ⟨Timeout⟩ **do**

    forall (dest, m) ∈ sent **do**

        trigger ⟨fl, Send | dest, m⟩

    startTimer(TimeDelay)

- **upon event** ⟨sl, Send | dest, m⟩ **do**

    trigger ⟨fll, Send | src, m⟩

    sent := sent ∪ { (dest, m) }

- **upon event** ⟨fll, Deliver | src, m⟩ **do**

    trigger ⟨sl Deliver | src, m⟩

- Implementation
  - Use the Lossy link
  - Sender stores every message it sends in **sent**
  - It periodically resends all messages in **sent**

- Correctness
  - **SL1. Stubborn delivery**
    - If process doesn't crash, it will send every message infinitely many times. Messages will be delivered infinitely many times. Lossy link may only drop a (large) fraction.
  - **SL2. No creation**
    - Guaranteed by the Lossy link

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Perfect Links

- Perfect Links
  - Channels that delivers any message sent exactly once

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- **Module:**
  - Name: PerfectPointToPointLink, **instance** pl
- **Events:**
  - **Request**: ⟨pl, Send | dest, m⟩
    - Request the transmission of message m to node dest
  - **Indication**: ⟨pl, Deliver | src, m⟩
    - deliver message m sent by node src
- **Properties:**
  - *PL1, PL2, PL3*

Properties

- ***PL1. Reliable Delivery***: If p_i and p_j are correct, then every message sent by p_i to p_j is eventually delivered by p_j

- ***PL2. No duplication***: Every message is delivered at most once

- ***PL3. No creation***: No message is delivered unless it was sent

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PERFECT LINKS (RELIABLE LINKS)

Which one is safety/liveness/neither

(liveness) **PL1. Reliable Delivery**: If neither p_i nor p_j crashes, then every message sent by p_i to p_j is eventually delivered by p_j

(safety) **PL2. No duplication:** Every message is delivered at most once

(safety) **PL3. No creation:** No message is delivered unless it was sent

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PERFECT LINK IMPLEMENTATION

- Implementation
  - Use Stubborn links
  - Receiver keeps a log of all received messages in **Delivered**
    - Only deliver (perfect link Deliver) messages that weren't delivered before
- Correctness
  - *PL1. Reliable Delivery*
    - Guaranteed by Stubborn link. In fact the Stubborn link will deliver it infinite number of times
  - *PL2. No duplication*
    - Guaranteed by our log mechanism
  - *PL3. No creation*
    - Guaranteed by Stubborn link (and its lossy link? [D])

FIFO PERFECT LINKS (RELIABLE LINKS)

Properties

PL1. Reliable Delivery:

PL2. No duplication:

PL3. No creation: No message is delivered unless it was sent

FFPL. Ordered Delivery: if m_1 is sent before m_2 by p_i to p_j and m_2 is delivered by p_j then m_1 is delivered by p_j before m_2

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INTERNET TCP VS. FIFO PERFECT LINKS

- TCP provides reliable delivery of packets

- TCP reliability is so called "session based"

- Uses sequence numbers

  - ACK: "I have received everything up to byte X"

- Implementing Perfect Link abstraction on TCP requires reconciling messages between the sender and receiver when reestablishing connection after a session break

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DEFAULT ASSUMPTIONS IN COURSE

- We assume perfect links (aka reliable) most of time in the course (unless specified otherwise)

- Roughly, reliable links ensure messages exchanged between correct processes are delivered exactly once

- Messages are uniquely identified and

  - the message identifier includes the sender's identifier

  - i.e. if "same" message sent twice, it's considered as two different messages

- Many algorithm for crash-recovery process model assume either a Stubborn link, or Logged perfect link

Timing Assumptions

SPECIFICATION OF A SERVICE

How to specify a distributed service (abstract)?

      Interface (aka Contract, API)

            Requests

            Responses

      Correctness Properties

            Safety

            Liveness

      Model

            Assumptions on failures

            Assumptions on timing (amount of synchrony)

declarative specification "what" aka problem

      Implementation

            Composed of other services

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imperative, many possible "how"

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TIMING ASSUMPTIONS

- Timing assumptions
  - Processes
    - bounds on time to make a computation step
  - Network
    - Bounds on time to transmit a message between a sender and a receiver
  - Clocks:
    - Lower and upper bounds on clock rate-drift and clock skew w.r.t. real time

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Recap - Models

- Synchronous (systems build on solid timed operations + clocks)

- Partially Synchronous (eventually every execution will exhibit

  period of synchrony - to make progress - satisfy liveness)
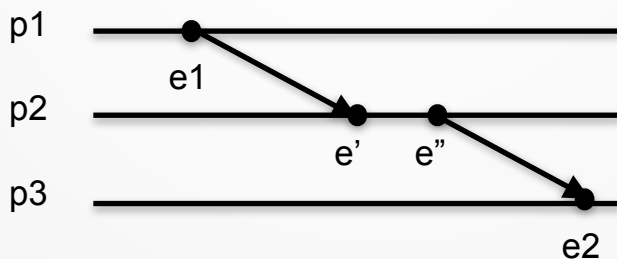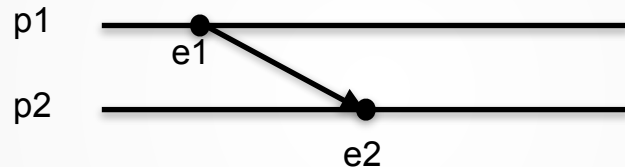
- Asynchronous (?)

Asynchronous Model and Causality

ASYNCHRONOUS SYSTEMS

- No timing assumption on processes and channels
  - Processing time varies arbitrarily
  - No bound on transmission time
  - Clocks of different processes are not synchronized
- Reasoning in this model is based on which events may cause other events
  - Causality

- Total order of event not observable locally, no access to global clocks

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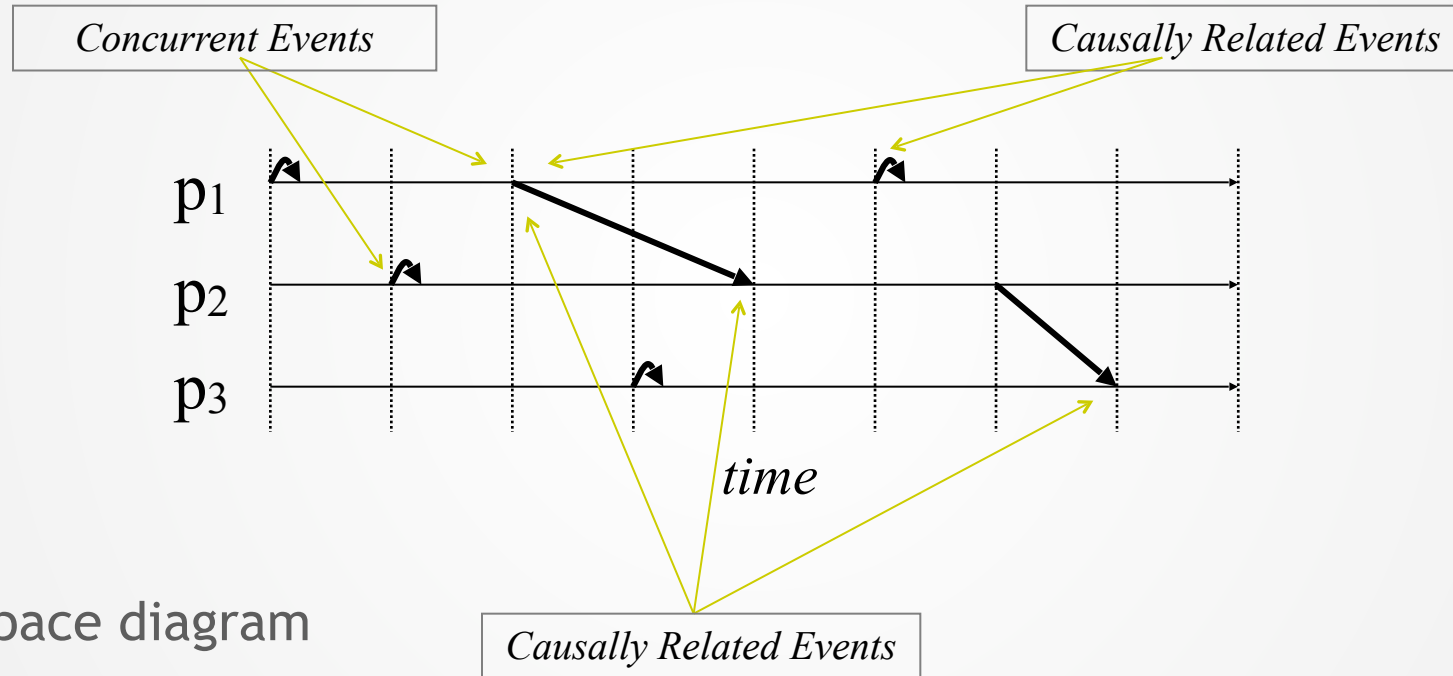KTH-2022

Causal Order (happen before)

- The relation $\rightarrow_\beta$ on the events of an execution (or trace β), called also causal order, is defined as follows
  - If a occurs before b on the same process, then $a \rightarrow_\beta b$
  - If a is a send(m) and b deliver(m), then $a \rightarrow_\beta b$
  - $a \rightarrow_\beta b$ is transitive
    - i.e. If $a \rightarrow_\beta b$ and $b \rightarrow_\beta c$ then $a \rightarrow_\beta c$

- Two events, a and b, are concurrent if not $a \rightarrow_\beta b$ and not $b \rightarrow_\beta a$
- a||b

Causal Order (happen before)

Time-space diagram

SIMILARITY OF EXECUTIONS

- The view of p_i in E, denoted $E|p_i$, is
  - the subsequence of execution E restricted to events and state of p_i
- Two executions E and F are similar w.r.t p_i if
  - $E|p_i = F|p_i$
- Two executions E and F are similar if
  - E and F are similar w.r.t every process

EQUIVALENCE OF EXECUTIONS

- **Computation Theorem:**
  - Let E be an execution $(c_0, e_1, c_1, e_2, c_2, \ldots)$, and V the trace of events $(e_1, e_2, e_3, \ldots)$
  - Let P be a permutation of V, preserving causal order
    - $P = (f_1, f_2, f_3 \ldots)$ preserves the causal order of V when for every pair of events $f_i \rightarrow_V f_j$ implies f_i is before f_j in P
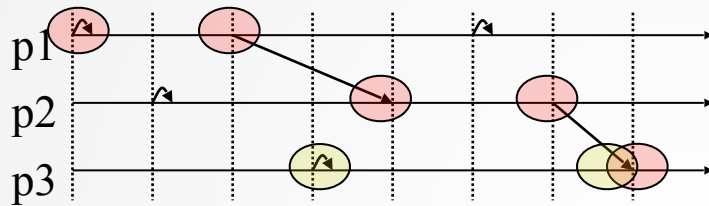 - Then E is similar to the execution starting in c_0 with trace P

EQUIVALENCE OF EXECUTIONS

- If two executions F and E have the same collection of events, and their causal order is preserved, F and E are said to be similar executions, written $F{\sim}E$
  - F and E could have different permutation of events as long as causality is preserved!
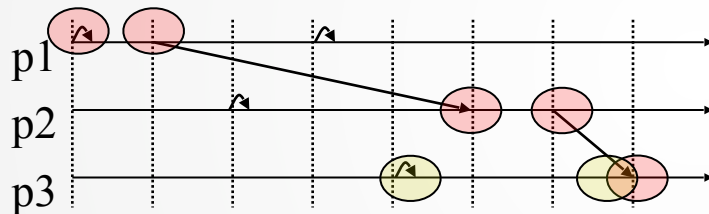
COMPUTATIONS

- Similar executions form equivalence classes where every execution in a class is similar to the other executions in the same class

- I.e. the following always holds for executions:
    - ~ is reflexive
        - I.e. a~ a for any execution
    - ~ is symmetric
        - I.e. If a~b then b~a for any executions a and b
    - ~ is transitive
        - If a~b and b~c, then a~c, for any executions a, b, c

- Equivalence classes are called computations of executions

EXAMPLE OF SIMILAR EXECUTIONS

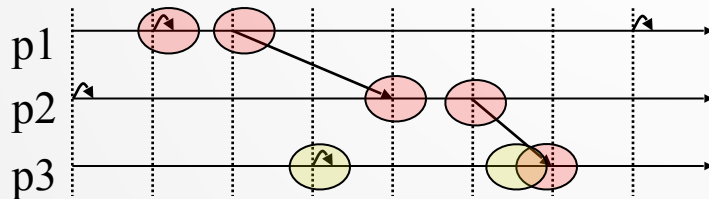*Same color ~ Causally related*

All three executions are part of the same computation, as causality is preserved

Computation theorem gives two important results

Result 1: There is no algorithm in the asynchronous system model that can observe the order of the sequence of events (that can "see" the time-space diagram, or the trace) for all executions

TWO IMPORTANT RESULTS (1)

Proof:

- Assume such an algorithm exists. Assume p knows the order in the final (repeated) configuration

- Take two distinct similar executions of algorithm preserving causality

- Computation theorem says their final repeated configurations are the **same**, then the algorithm <u>cannot</u> have observed the actual order of events as **they differ**

Two important results (2)

Result 2: The computation theorem does not hold if the model is extended such that each process can read a local hardware clock

Proof:

- Similarly, assume a distributed algorithm in which each process reads the local clock each time a local event occurs
- The final (repeated) configuration of different causality preserving executions will have different clock values, which would contradict the computation theorem

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