HE1027 Electrical Principals

Series and Parallel AC Circuits

Inductor in AC Current

Voltage across the inductor is directly related to the inductance of the coil and the rate of change of current through the coil

$$V_L = L\frac{di_L}{dt}$$

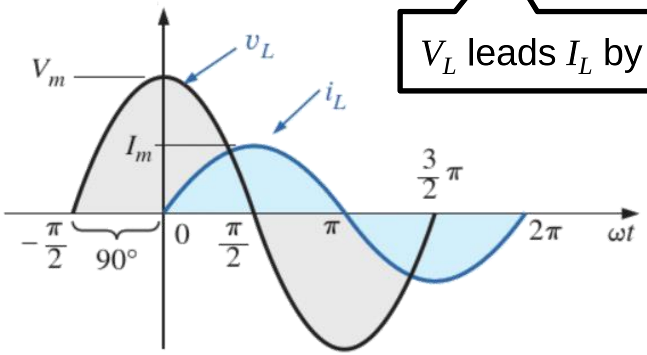
From before, instantaneous value of current $i = I_m \sin\alpha = I_m \sin\omega t$
and $\frac{d}{dt}(sin2x) = 2cos2x$

$$V_L = L\frac{d}{dt}I_m \sin\omega t = LI_m\frac{d}{dt}(\sin\omega t) = LI_m(\omega\cos\omega t) = \omega LI_m \sin(\omega t + 90°)$$

V_L leads I_L by 90°

$\omega L = X_L$ - reactance of an inductor

$$X_L = \frac{V_m}{I_m}$$



Since v_L leads i_L by 90°, impedance of inductive element is $Z_L = X_L\angle 90° = iX_L$

Capacitor in AC Current

The capacitive current is directly related to the rate of the voltage across the capacitor and the rate of change of involved voltage

$$i_C = C\frac{dV_C}{dt}$$

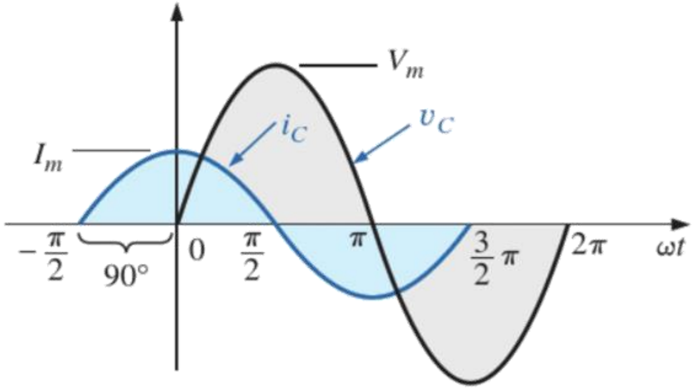
From before, instantaneous value of voltage $v = V_m \sin\alpha = V_m \sin\omega t$

$$i_C = C\frac{d}{dt}V_m \sin\omega t = CV_m(\omega\cos\omega t) = \omega C V_m \sin(\omega t + 90°)$$

i_L leads v_L by 90°

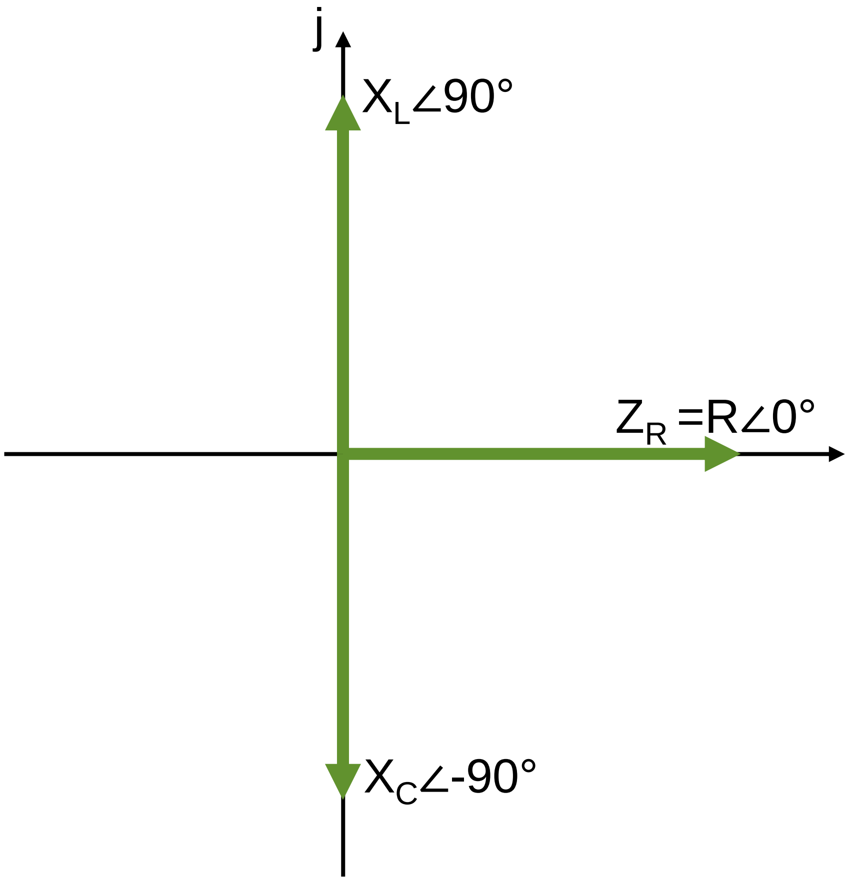
$\frac{1}{\omega C} = X_C$ - reactance of a capacitor

$$X_C = \frac{V_m}{I_m}$$



Since i_C leads v_C by 90°, impedance of capacitive element is $Z_C = X_C \angle -90° = -iX_C$

Impendence Diagram

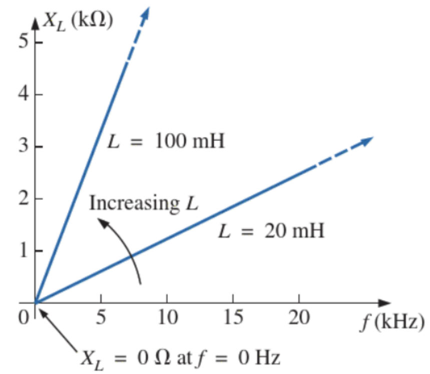


- Combination of different elements will have total impedances that extend from -90° to +90°

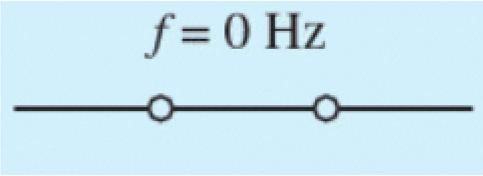
- If the total impedance is close to 0°, it is resistive in nature
- If it is closer to 90°, it is inductive in nature
- If it is closer to -90°, it is capacitive in nature

Frequency and Inductor

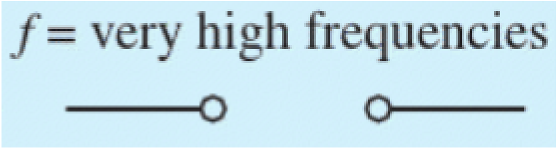
- $\omega = 2 * \pi * f$
- $X_L = \omega L = 2 * \pi * f * L$



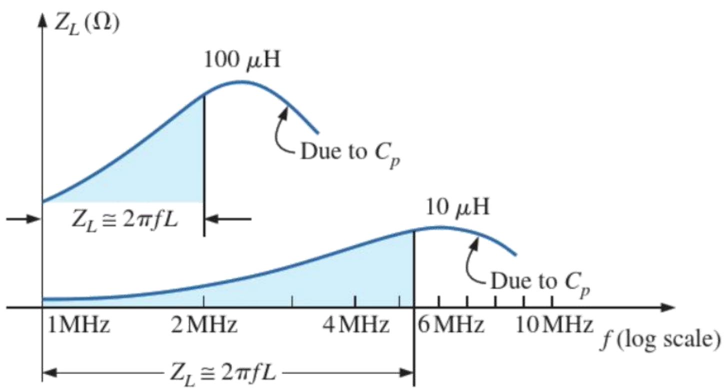
If f=0Hz -> X_L=0Ω



If f=∞Hz -> X_L=∞Ω

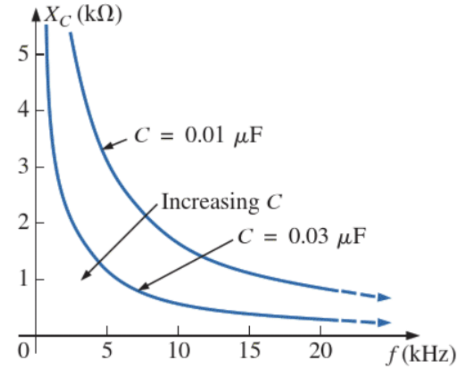


Reality:

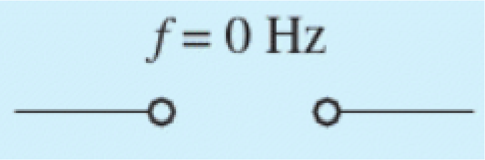


Frequency and Capacitor

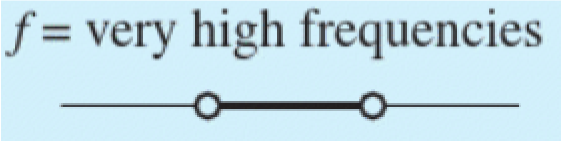
- $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2*\pi*f*C}$



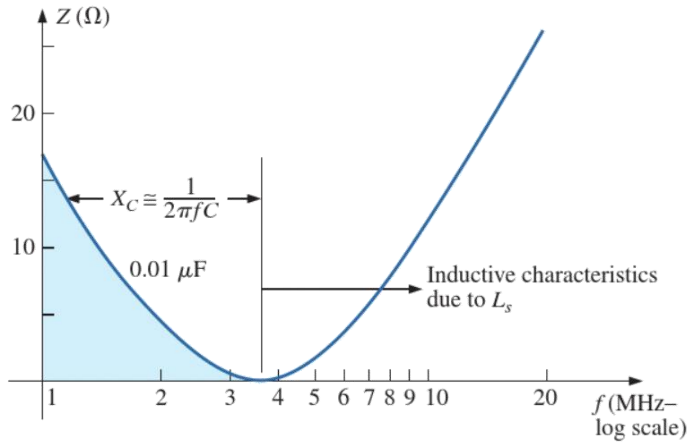
If f=0Hz -> X_C= ∞ Ω



If f=∞Hz -> X_C=0Ω



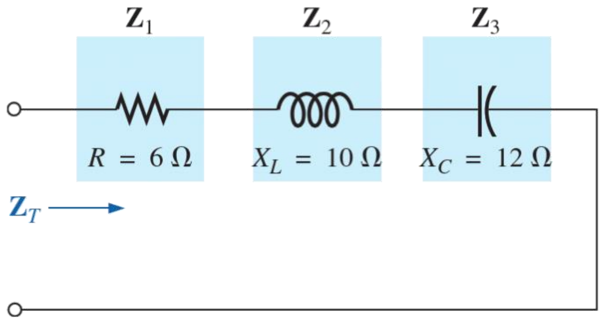
Reality:



Series Configuration

- Total impendence is a sum of all individual impendences

$Z_T = Z_1 + Z_2 + Z_3 + Z_4 + \ldots + Z_n$

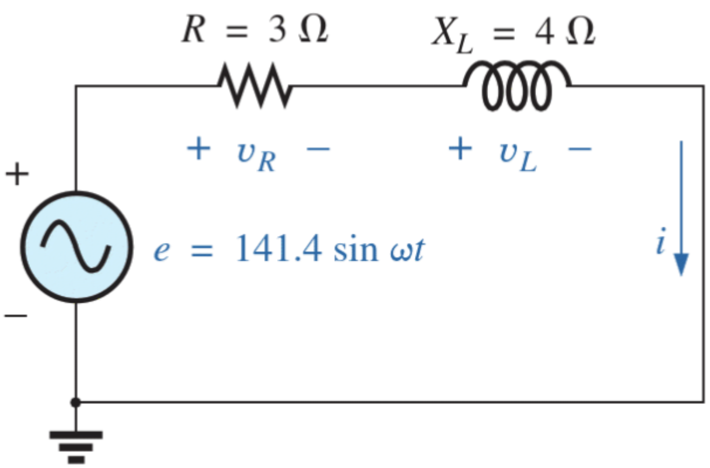


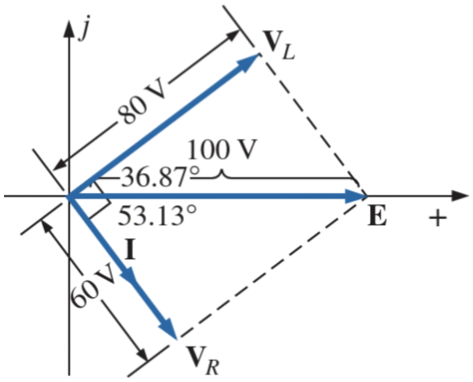
- $Z_T = Z_1 + Z_2 + Z_3 = R + iX_L - iX_C = 6 + i10 - i12 = 6\Omega - i2\Omega$ $\quad Z_T = 6.32\Omega \angle -18.43°$

- $I_T = I_1 = I_2 = I_3 = \ldots = I_n$
- $E = V_1 + V_2 + V_3 + V_4 + \ldots + V_n$
- $P = EI * \cos|\theta_E - \theta_I|$

Example

Find total impendence, current, V_R, V_L and P_T.
Draw Impendence Diagram





1. Convert e into phasor notation

   $e=141.4\sin\omega t=100\angle 0°$

2. Find total impendence

   $Z_T=Z_R+Z_L=3\Omega+i4\Omega=5\Omega\angle 53.13°$

3. Find current

   $I=E/Z_T=(100\angle 0°)/(5\angle 53.13°)=100/5\angle(0-53.13)=20A\angle -53.13°$

4. Find V_R

   $V_R=IZ_R=20A\angle -53.13°*3\Omega\angle 0°=60V\angle -53.13°$

5. Find V_L

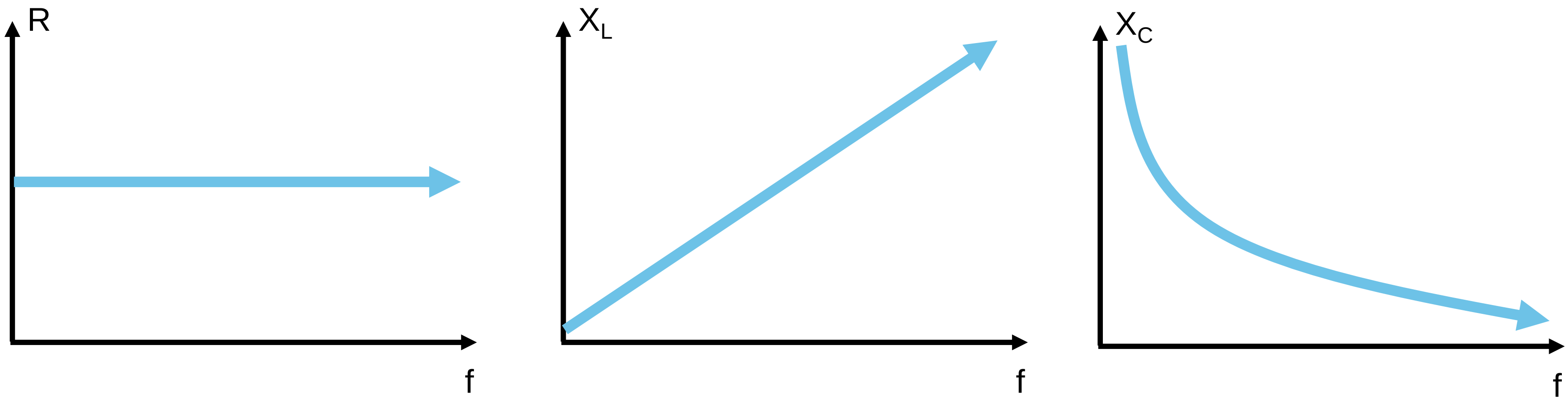
   $V_L=IZ_L=20A\angle -53.13°*\Omega\angle 90°=80V\angle 36.87°$

6. Find P_T

   $P_T=EI*\cos|\theta_E-\theta_I|=100*20*\cos|0°-53.13°|=2000*\cos(53.13°)=1200W$

Frequency Response for Series AC Circuits

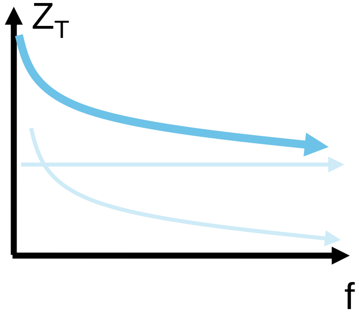
- For ideal resistor frequency has no effect
- $X_L = 2\pi f L$
- $X_C = \frac{1}{2\pi f C}$



In series connection, element with largest impedance has the greatest impact

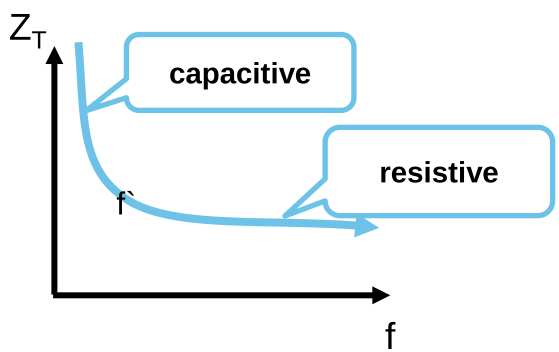
Series R-C (Resistor-Capacitor) AC Circuit

- $Z_T = Z_1 + Z_2$

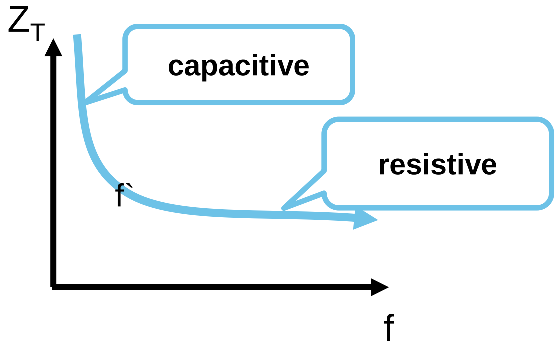


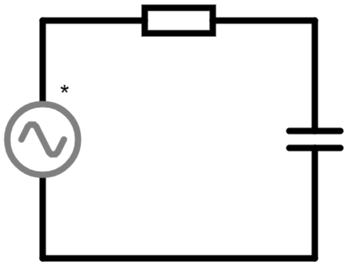
- At low frequency impendence of capacitor has a larger impact
- At high frequency impendence of resistor has a larger impact

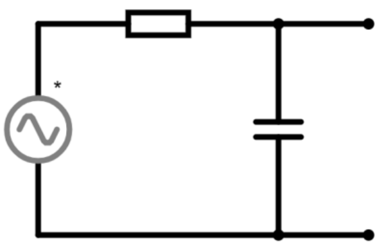
- Breaking point is at $X_c = R$
- Since $X_C = \frac{1}{2\pi f C}$, then $f^{`} = \frac{1}{2\pi R C}$

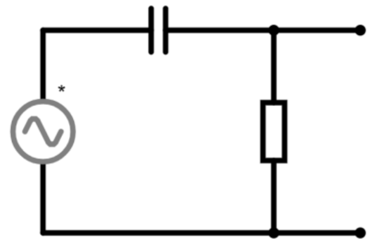


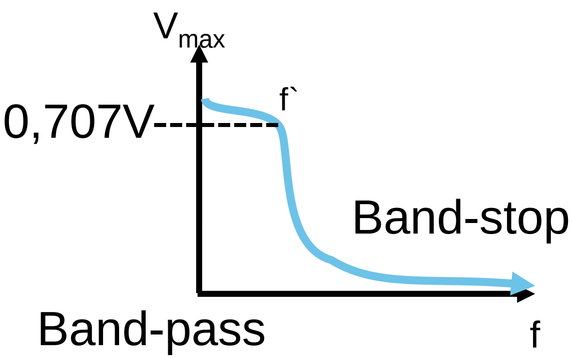
R-C low pass filter and high pass filter



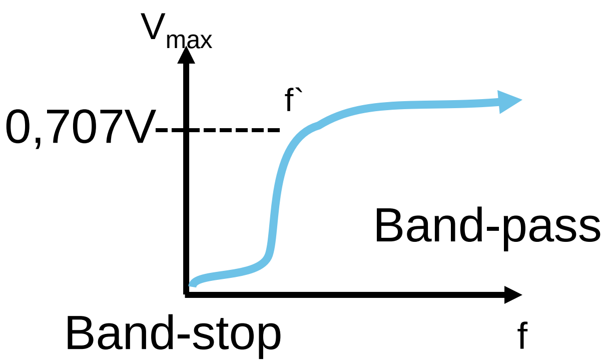








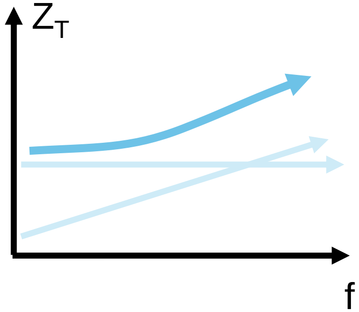
RC low pass filter



RC high pass filter

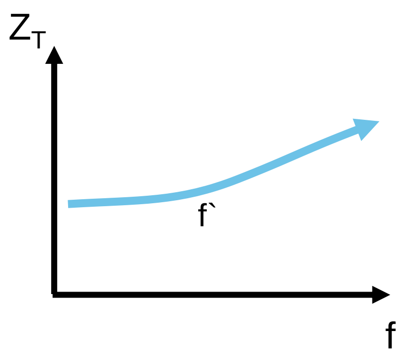
Series R-L (Resistor-Inductor) AC Circuit

- $Z_T = Z_1 + Z_2$

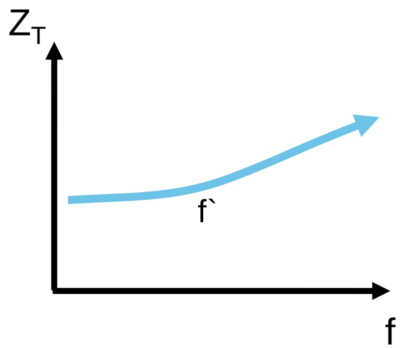


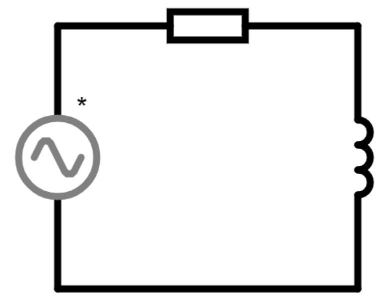
- At low frequency impendence of resistor has a larger impact
- At high frequency impendence of inductor has a larger impact

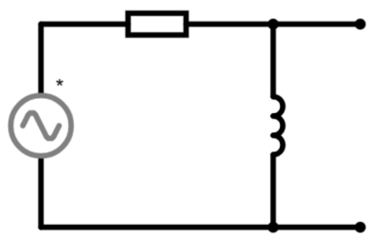
- Breaking point is at $X_L = R$
- Since $X_L = 2\pi f L$, then $f^` = \frac{R}{2\pi C}$

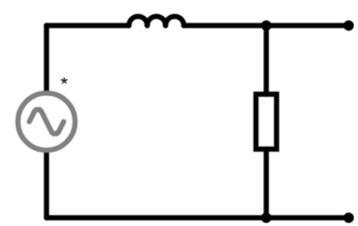


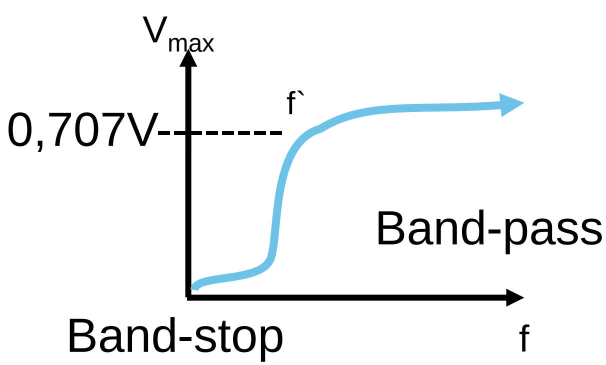
R-L low pass filter and high pass filter



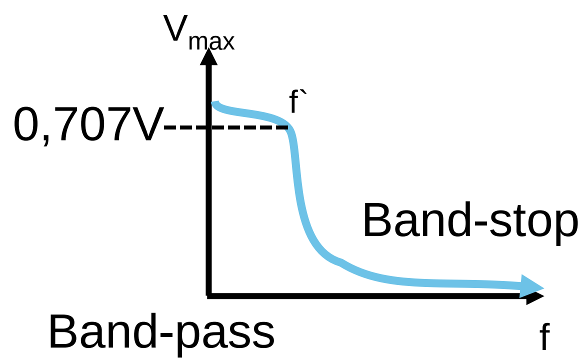






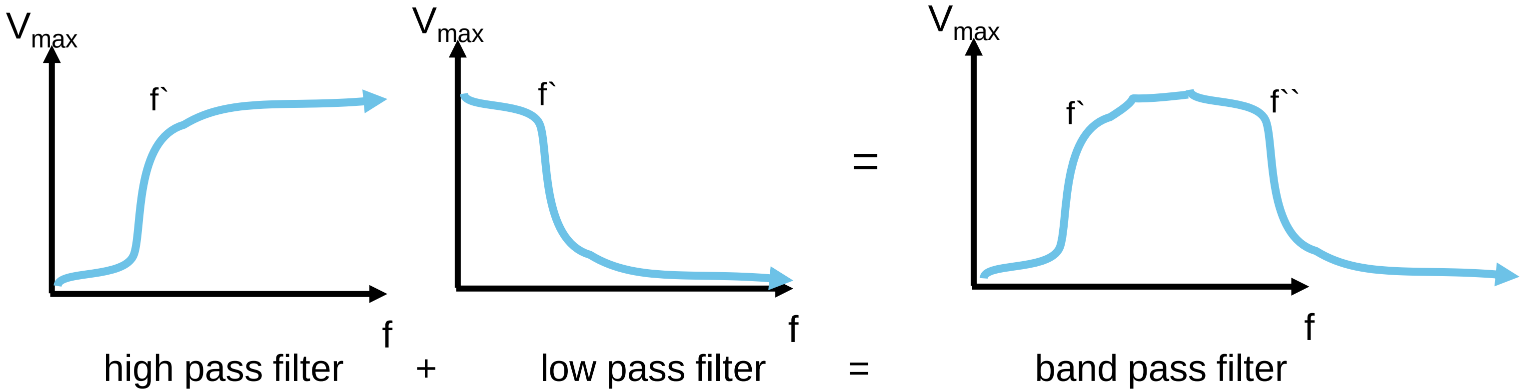


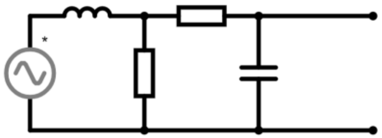
RL high pass filter

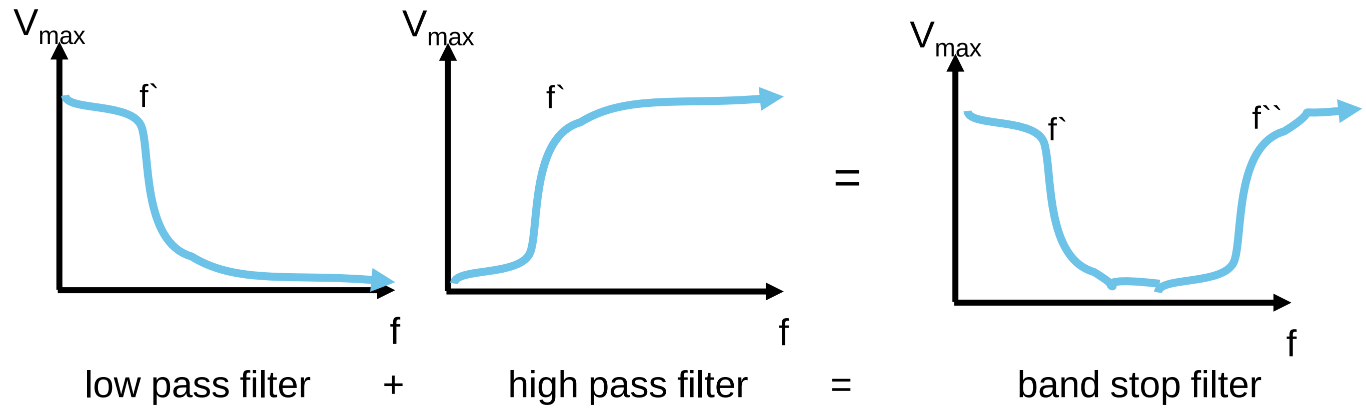


RL low pass filter

Band-pass filter and band-stop filter

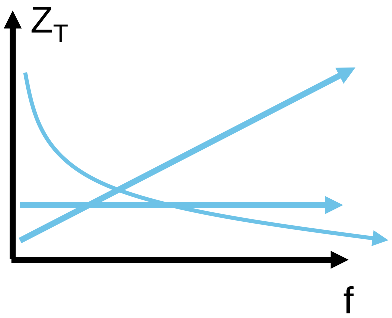






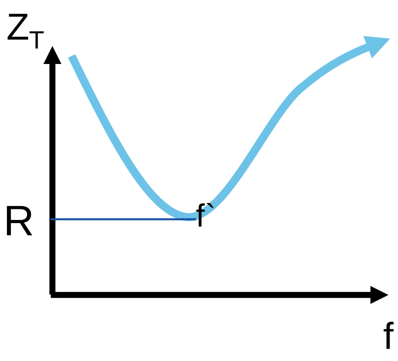
Series R-L-C (Resistor-Inductor-Capacitor) AC Circuit

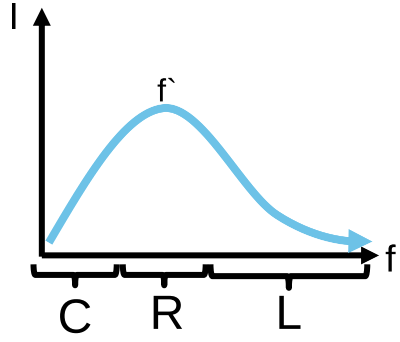
- $Z_T = Z_1 + Z_2 + Z_3$



- Since resistor don’t change over time it basically just ignored
- At low frequency impendence of capacitor has a larger impact
- At hight frequency impendence of inductor has a larger impact

- Breaking point is at $X_L = X_c$ -> $2\pi f L = \frac{1}{2\pi f C}$ -> $f^{`} = \frac{1}{2\pi\sqrt{LC}}$





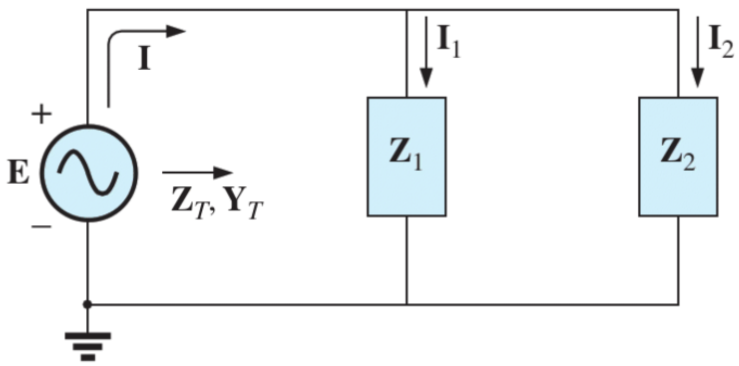
Parallel Configuration

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_n}$$

$E=V_1=V_2=V_3=\ldots=V_n$

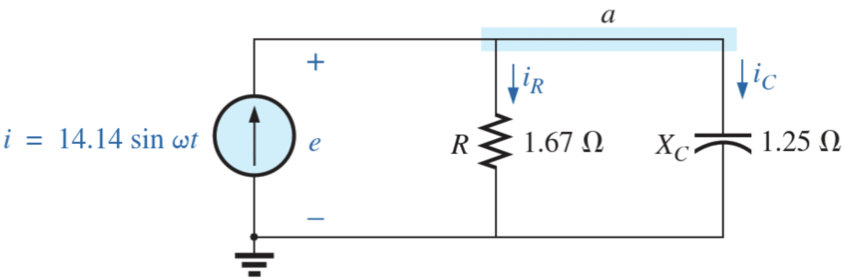
$I_T=I_1+I_2+I_3+\ldots+I_n$

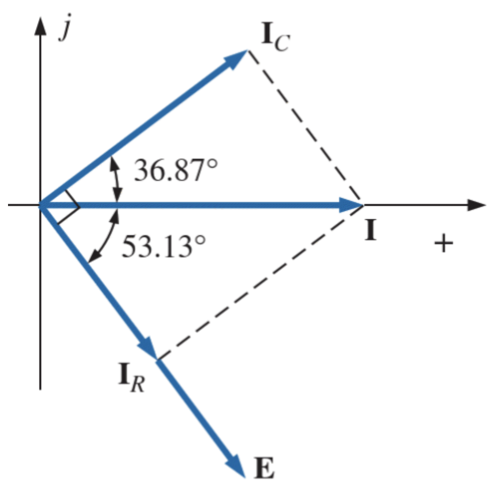
$P=EI^*\cos|\theta_E- \theta_I|$



Example

Find total impendence, current, V_R, V_L and P_T.
Draw Impendence Diagram





1. Convert *i* into phasor notation

   i=14.14sinωt=10∠0°

2. Find total impendence

   $1/Z_T=1/Z_R+1/Z_C=1/1.67\Omega+1/i1.25\Omega=0.599\Omega+i0.8\Omega$

   $Z_T= 1\ \Omega\ \angle -53.13°$

3. Find voltage

   $E=I*Z_T=(10\angle 0°)*(1\angle -53.13°)=100*1\ \angle(0+(-53.13))=10V\ \angle -53.13°$

4. Find I_R

   $I_R=E/Z_R=(10V\angle -53.13°)/(1.67\Omega\angle 0°)=6A\angle -53.13°$

5. Find I_C

   $I_C=E/Z_C=(10V\angle -53.13°)/(1.25\Omega\angle -90°)=8A\angle 36.87°$

6. Find P_T

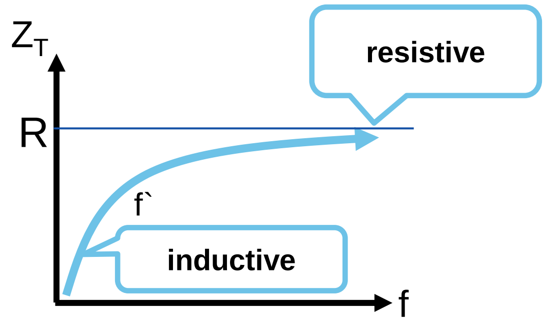
   $P_T=EI*\cos|\theta_E- \theta_I|=10*10*\cos|-53.13°-0°|=100*\cos(53.13°)=60W$

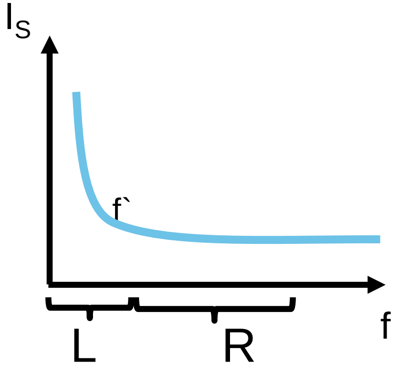
Parallel R-L (Resistor-Inductor) AC Circuit

- $1/Z_T = 1/Z_1 + 1/Z_2$

- In parallel connection, element with smallest impedance has the greatest impact
- At low frequency impendence of inductor has a larger impact
- At high frequency impendence of resistor has a larger impact

- Breaking point is at $X_L = R$
- Since $X_L = 2\pi f L$, then $f^{`} = \frac{R}{2\pi L}$



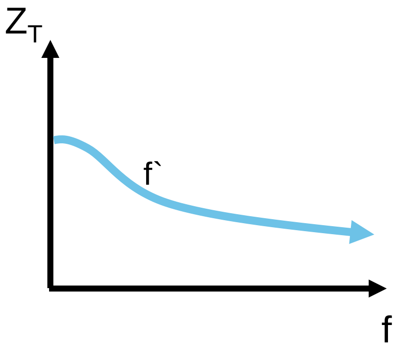


Parallel R-C (Resistor-Capacitor) AC Circuit

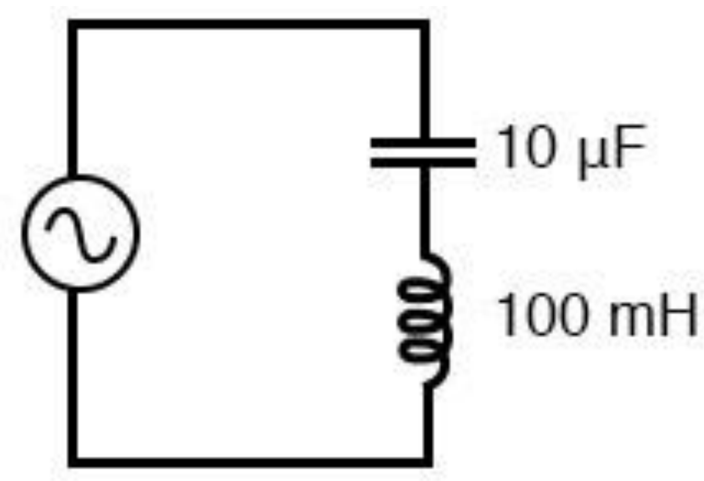
- $Z_T = Z_1 + Z_2$

- At low frequency impendence of resistor has a larger impact
- At high frequency impendence of capacitor is larger than of capacitor

- Breaking point is at $X_c = R$
- Since $X_C = \frac{1}{2\pi f C}$, then $f^{`} = \frac{1}{2\pi RC}$



Resonance



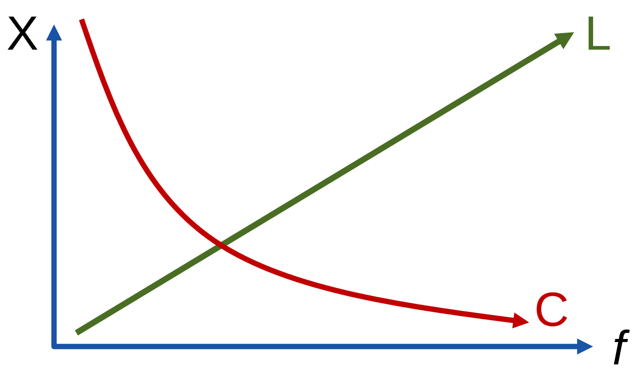
- $Z_L = 2\pi f L$
- $Z_C = \frac{1}{2\pi f C}$

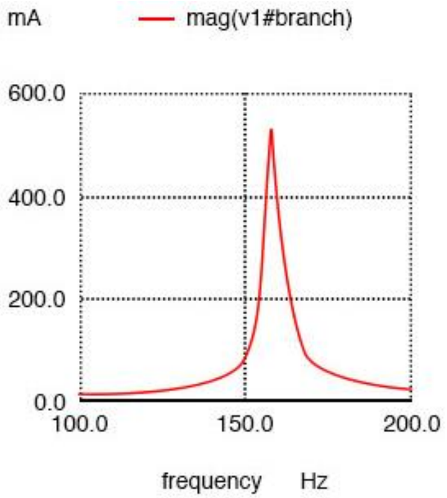
At 159.155Hz:

$Z_L = 2\pi * 159.155 * 0.1 = 100\angle 90°$

$Z_C = \frac{1}{2\pi * 159.155 * 0.0001} = 100\angle -90°$

$Z_T = 100\angle 90° + 100\angle -90° = 0\Omega$





Suggested reading

Introductory Circuit Analysis

- Kap 14: **14.2 - 14.9**
- Kap 22: 22:1-22.8, 22.11

*suggestions in bold are more important

Suggested exercises

- Kap 14: 5, 17, 35, 37, 39, 41, 43, 49, 53, 55
- Kap 22: 19, 21, 23, 25