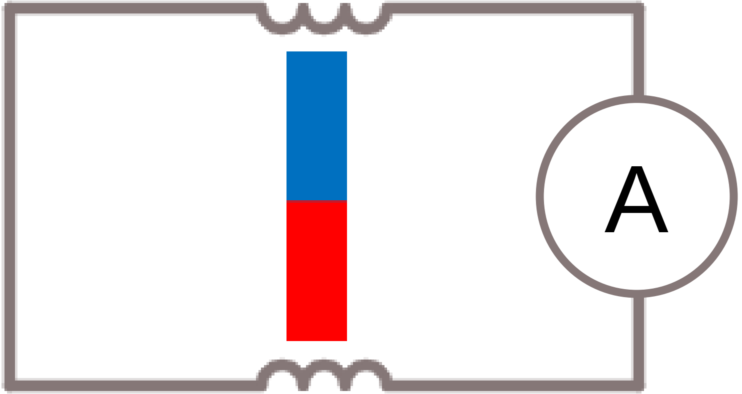
HE1027 Electrical Principals

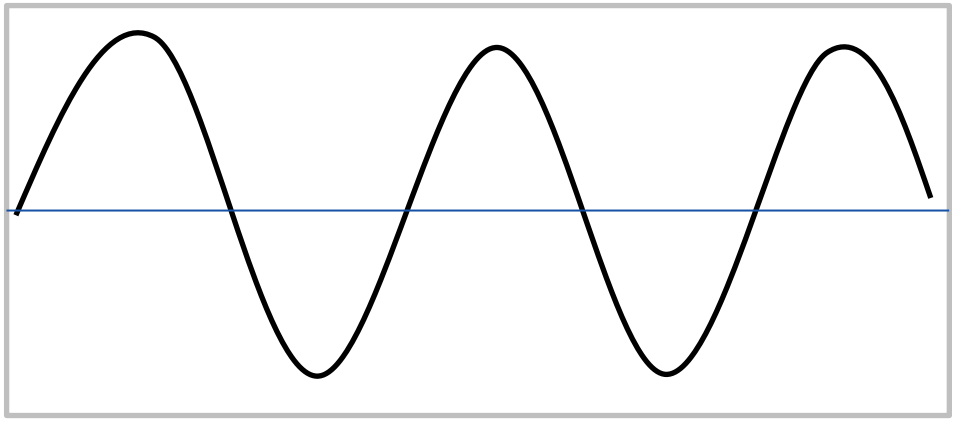
Alternating Current

Why We Need AC Current?

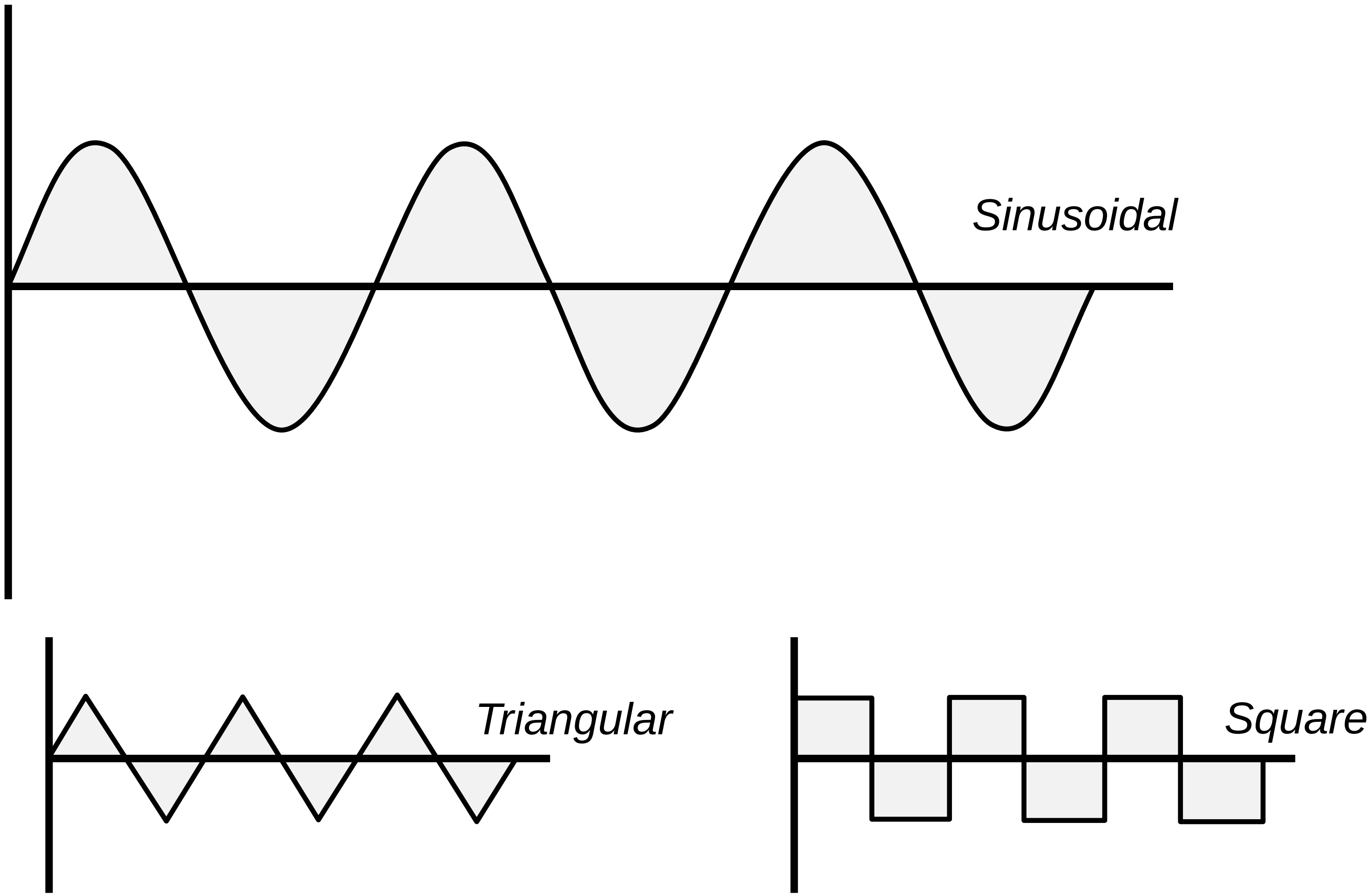
- Easy to transform from one voltage to another
- Energy efficient electrical transmission
- Low maintenance costs of high-speed AC motors
- Easy to interrupt the current (about 1/20th as much DC)
- DC is more lethal than AC
- Electrolytic corrosion is more problematic with DC
- DC produces more heat while operation

How AC Works?

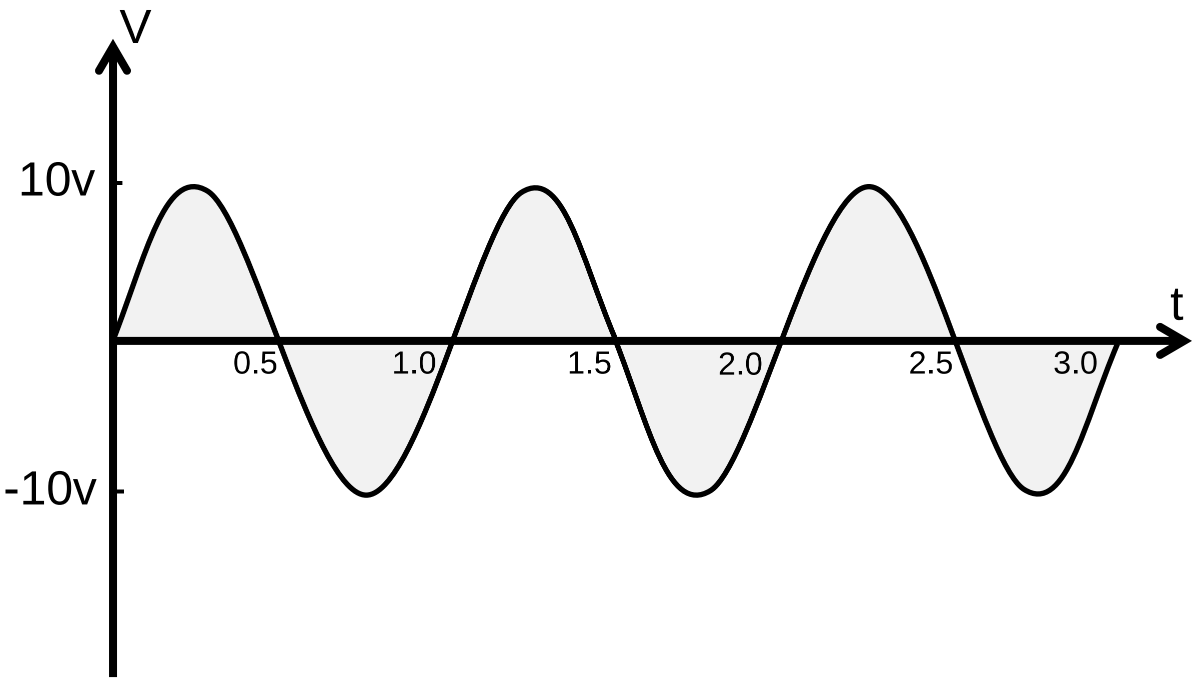




Alternating Waveform

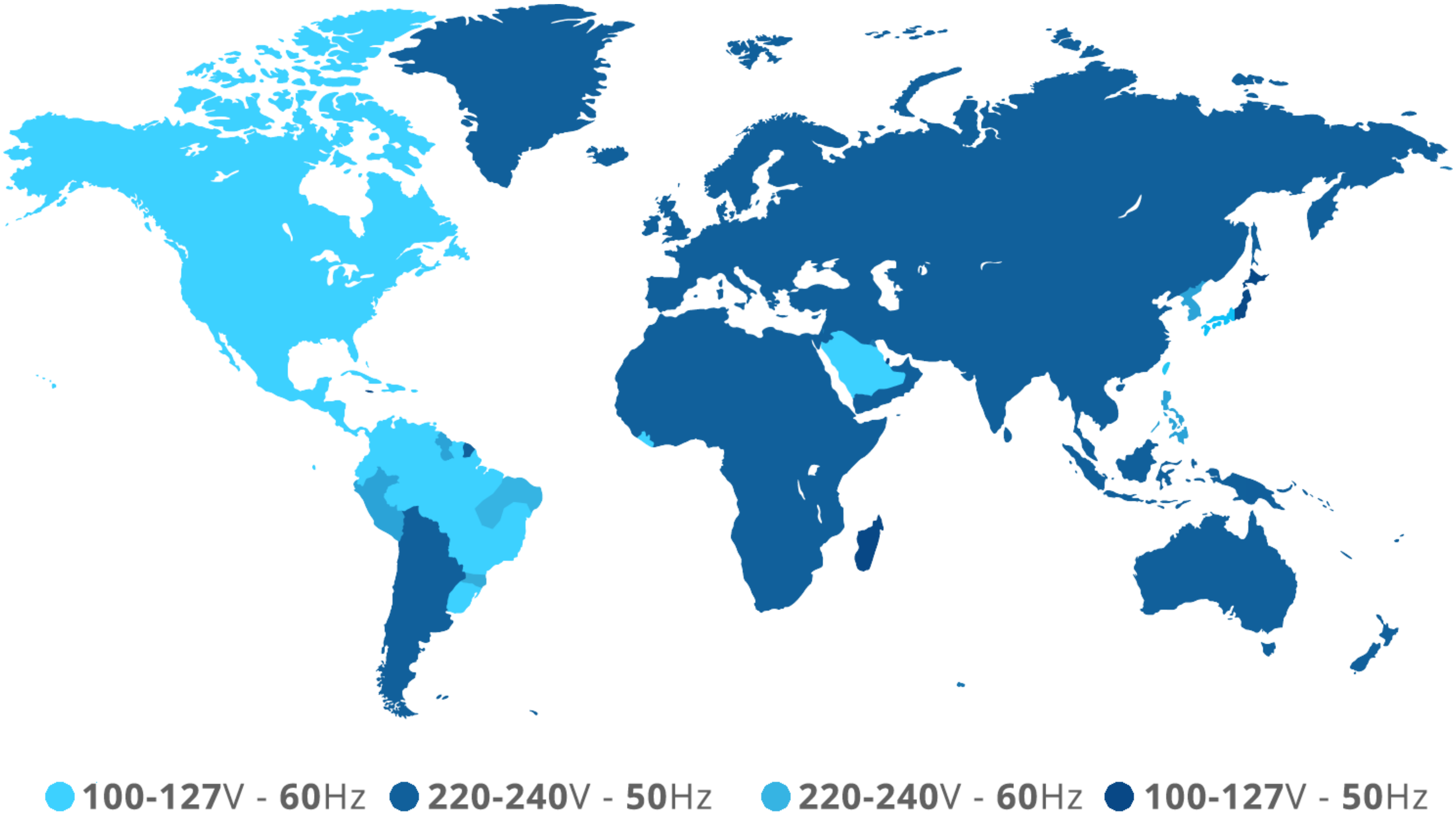


Measurement of AC Signals



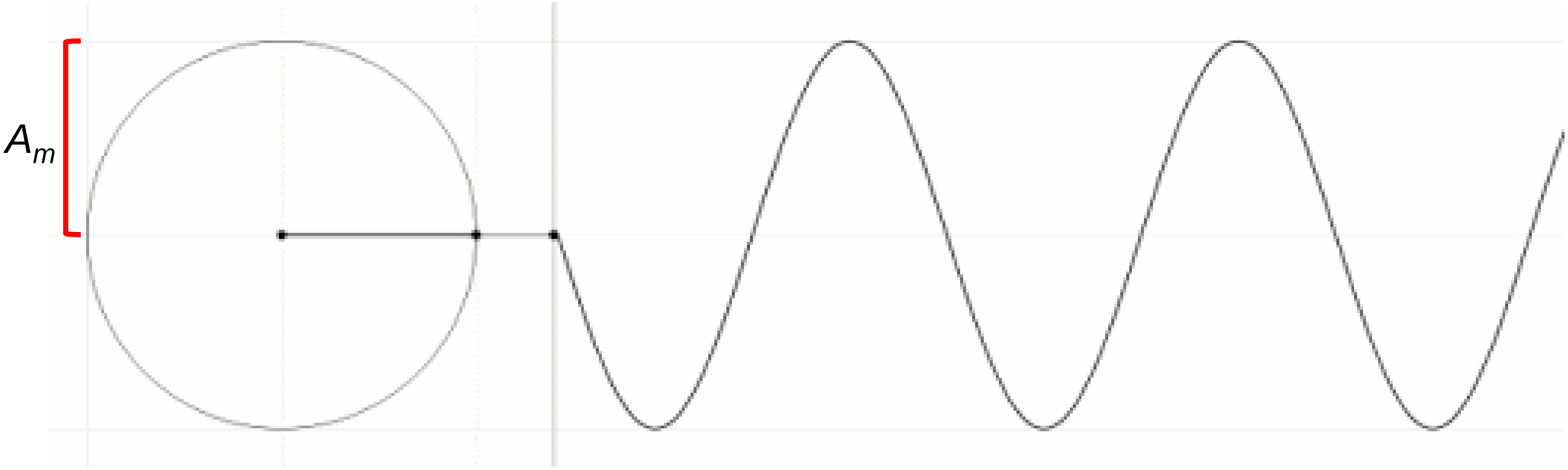
Waveform	sinusoidal
Instantaneous value at t=0.3	7V
Instantaneous value at t=0.75	10V
Peak amplitude	10V
Peak value	10V
Peak-to-peak value	20V
Period	1s
Cycle	3
Frequency	1cps or 1Hz

Power Frequency Worldwide



General Format for the Sinusoidal Voltage

$$y(\alpha)=A_m \sin \alpha$$



General Format for the Sinusoidal Voltage

$y(\alpha)=A_m \sin \alpha$

- A_m – peak value
- α – distance in radians
- $$radians = \frac{\pi}{180°} \times (degrees)$$
- distance in radians=angular velocity (ω) x time

$y(t)=A_m \sin \omega t$

- Instantaneous value of current $i = I_m \sin \alpha = I_m \sin \omega t$
- Instantaneous value of voltage $v = V_m \sin \alpha = V_m \sin \omega t$

Not all waves have to start at 0: $A_m \sin (\omega t+\theta)$

- θ – angle that the waveform has been shifted

Effective value of sinusoidal quantity is $\frac{1}{\sqrt{2}}$ of peak value

- AC current with peak value of 10A will deliver the same power as DC current of 7.07A

Derivative

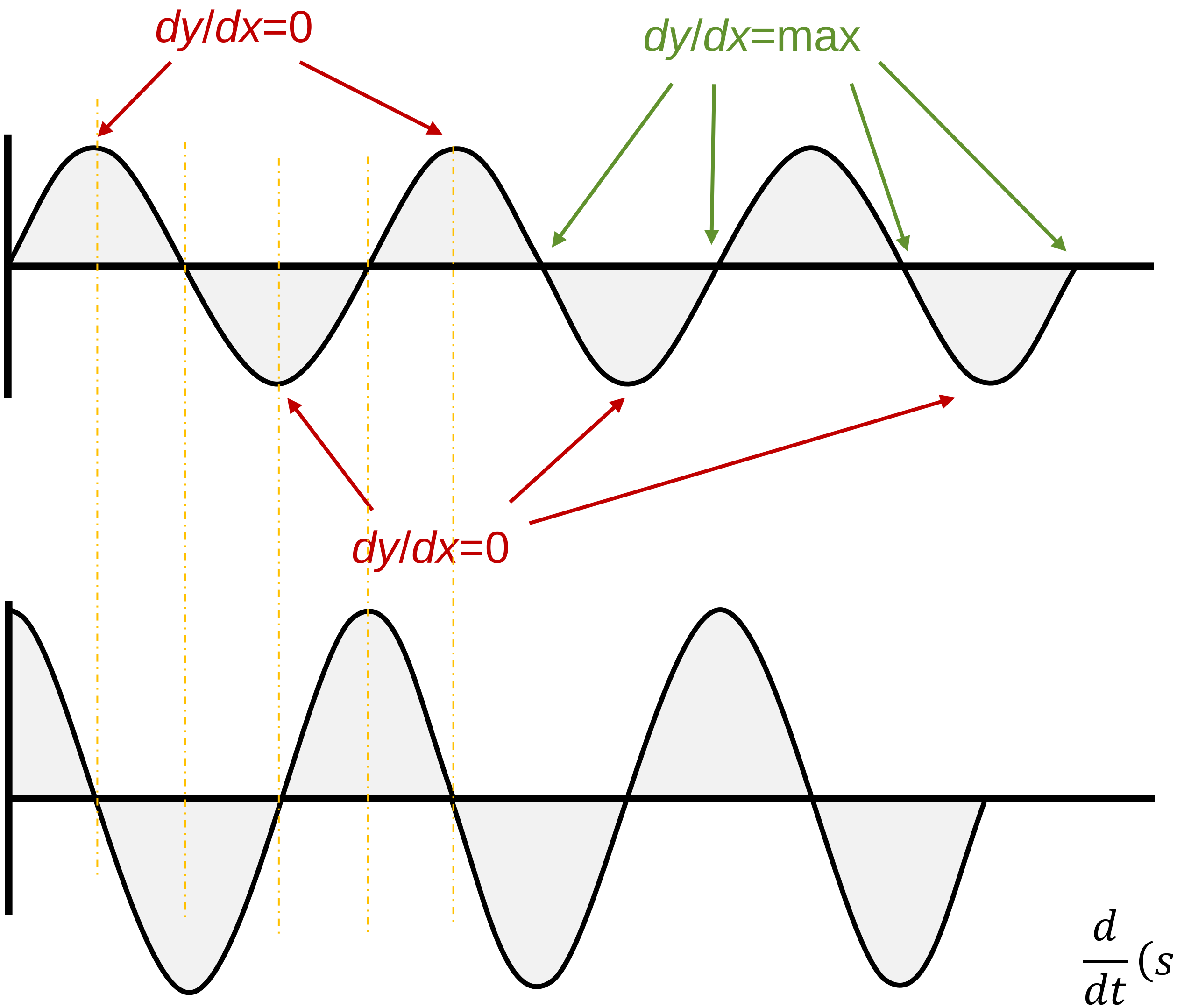
The derivative measures the sensitivity to change of the function value (output value) with respect to a change in its argument (input value)

(derivative of y with respect to x) = $\dfrac{dy}{dx}$

If dy does not change then dy=0 and dy/dx=0

If dy changes quickly then dy=max and dy/dx=max

Derivative of Sine Wave



Derivative of sine was is cosine wave

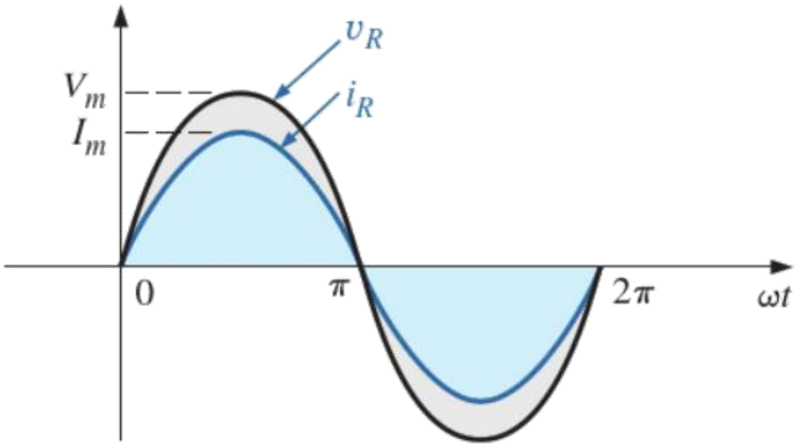
$$\frac{d}{dt}(sin2x) = 2cos2x$$

Resistor in AC Current

Instantaneous value of current $i = I_m \sin\alpha = I_m \sin\omega t$

Instantaneous value of voltage $v = V_m \sin\alpha = V_m \sin\omega t$

- Omh's Law: $R = \dfrac{V}{I}$
- $R = \dfrac{v}{i} = \dfrac{V_m \sin\alpha}{I_m \sin\alpha} = \dfrac{V_m}{I_m}$
- $V_m = R * I_m$
- $I_m = \dfrac{V_m}{R}$



Some Math

- For DC we used often sum of different voltages or currents.
- For AC we can do it for each time point (NO) or use complex numbers

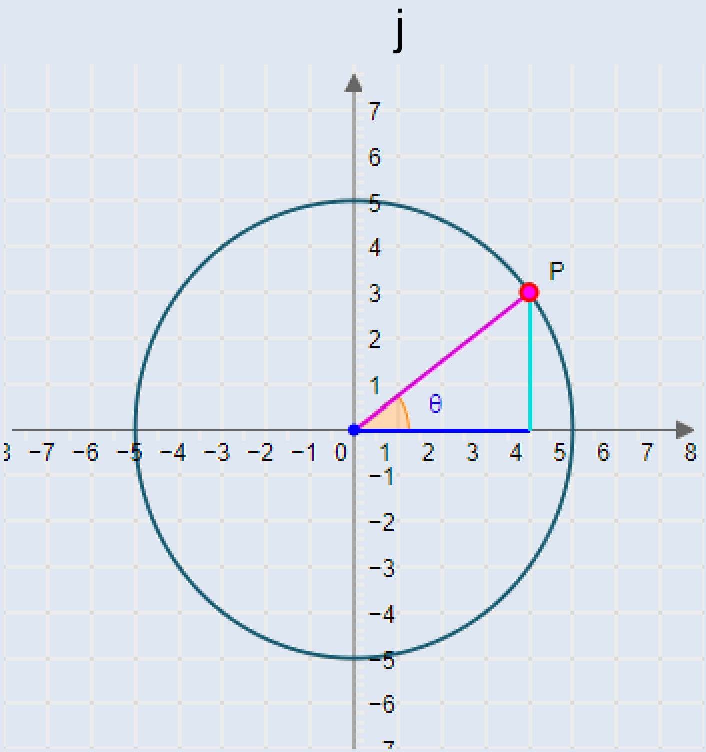
- Complex numbers are two-dimensional: real and imaginary

- What is $\sqrt{-4}$?

- Numbers can be recorded in *rectangular form* or *polar form*

- Rectangular form: C=X+iY
- Polar form: $C = Z\angle\theta$

Plotting Complex Numbers



- Rectangular form (C=X+iY):
  - Real part X on horizontal axis (4)
  - Imaginary part Y on vertical axis (3)
- Polar form:
  - Magnitude Z is a radius (5)
  - Angle θ counterclockwise from positive real axis
- To convert from rectangular to polar:

$$Z = \sqrt{X^2 + Y^2}$$

$$\theta = \tan^{-1}\frac{Y}{X}$$

- To convert from polar to rectangular:

$$X = Z\cos\theta$$
$$Y = Z\sin\theta$$

Mathematical Operations with Complex Numbers

Addition	Subtraction
$C_1+C_2=(X_1+X_2)+i(Y_1+Y_2)$ $(2+i4)+(3+i1)=5+i5$ Addition can be in polar form only if the same angle or difference is 180° $2\angle 37°+3\angle 37°=5\angle 37°$	$C_1-C_2=(X_1-X_2)+i(Y_1-Y_2)$ $(2-i3)-(-5+i4)=7-i7$ Subtraction in polar form only if the same angle or difference is 180° $6\angle 45°-2\angle 225°=6\angle 45°-(-2\angle 45°)=8\angle 45°$
Multiplication	**Division**
$C_1*C_2=(X_1X_2-Y_1Y_2)+i(X_1Y_2+X_2Y_1)$ or just remember that $j^2=-1$ $C_1*C_2=Z_1*Z_2\angle(\theta_1+\theta_2)$	$\frac{C_1}{C_2}=\frac{X_1X_2+Y_1Y_2}{X_2^2+Y_2^2}+i\frac{X_1Y_2+X_2Y_1}{X_2^2+Y_2^2}$ or just remember multiply all by conjugate of the denominator $\frac{C_1}{C_2}=\frac{Z_1}{Z_2}\angle(\theta_1-\theta_2)$

Phasors

- Math operations with sinusoidal functions are hard
- Easier to work with phasors
- Phasor is a complex number representing a sinusoidal function

$$v = V_m \sin(\omega t \pm \theta) \rightarrow V_m \angle \pm \theta$$

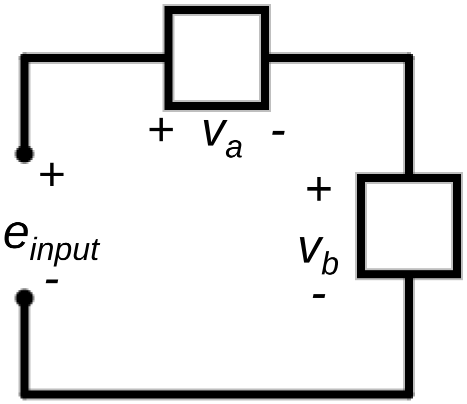
- After all math, it can be converted back

- Effective value ($0.707A_m$), rather than the peak, values are used almost exclusively in the analysis of AC circuits

- Phasor algebra for sinusoidal quantities is applicable **only for waveforms having the same frequency**

Example

Find the input voltage of the circuit if v_a=50sin(377t+30°) and v_b=30sin(377t+60°)



$e = v_a + v_b$

$V_a = 50*0.707\angle 30° = 35.35\angle 30°$ and $V_b = 30*0.707\angle 60° = 21.21\angle 60°$

$V_a = 35.35*\cos 30° + i*35*\sin 30° = 35.35*0.866 + i*35.35*0.5 =$
$= 30.613 + i17.675$

$V_b = 21.21*\cos 60° + i*21.21*\sin 60° = 10.605 + i18.368$

$e = V_a + V_b = 41.22v + i36.05v$

$$Z = \sqrt{X^2 + Y^2} \text{ and } \theta = \tan^{-1}\frac{Y}{X}$$

$$Z = \sqrt{41.22^2 + 36.05^2} = 54.760$$

$$\theta = \tan^{-1}\frac{36.05}{41.22} = 41.17$$

$$Z = \sqrt{2} * Z = 77.43$$

E=77.43sin(337t+41.17°)

Resistance Elements

- $I_m = \frac{V_m}{R}$
- In phasor form $v = V_m \sin \omega t$ -> **V**=V ∠0°, where V=0.707V_m

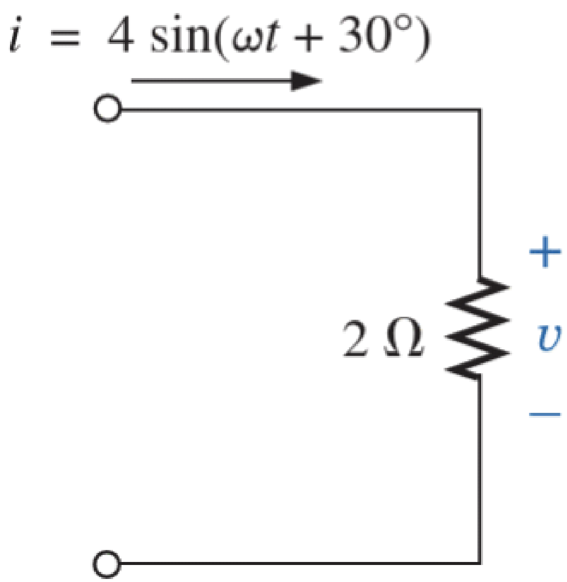
- $I = \frac{V\angle 0°}{R\angle \theta°} = \frac{V\angle 0°}{R\angle 0°} = \frac{V}{R}\angle(0° - 0°) = \frac{V}{R}\angle 0°$

R∠0°=Z_R

- $i = \sqrt{2}\left(\frac{V}{R}\right)\sin \omega t$

Example

Find voltage v



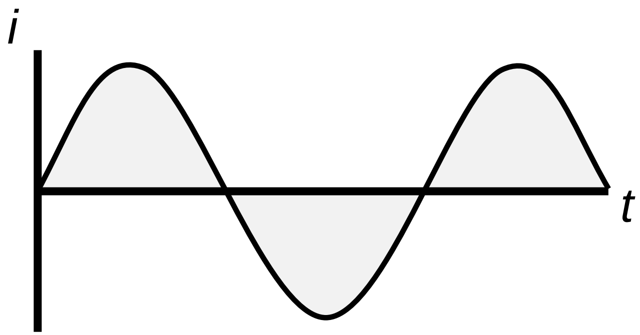
$i = 4\sin(\omega t + 30°)$ -> $I = 4 * 0.707\text{A} \angle 30° = 2.828\text{A} \angle 30°$

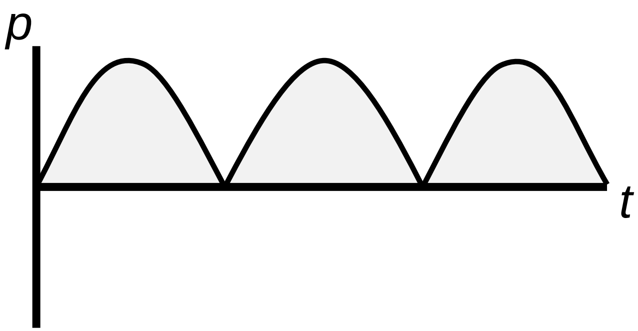
$V = IZ_R = (2.828\text{A} \angle 30°)(2\Omega \angle 0°)$

$V = 2.828 * 2 \angle (30° + 0°) = 5.656\text{V} \angle 30°$

$v = \sqrt{2}(5.656)\sin(\omega t + 30°) = 8\sin(\omega t + 30°)$

Power





- $P=I^2R$

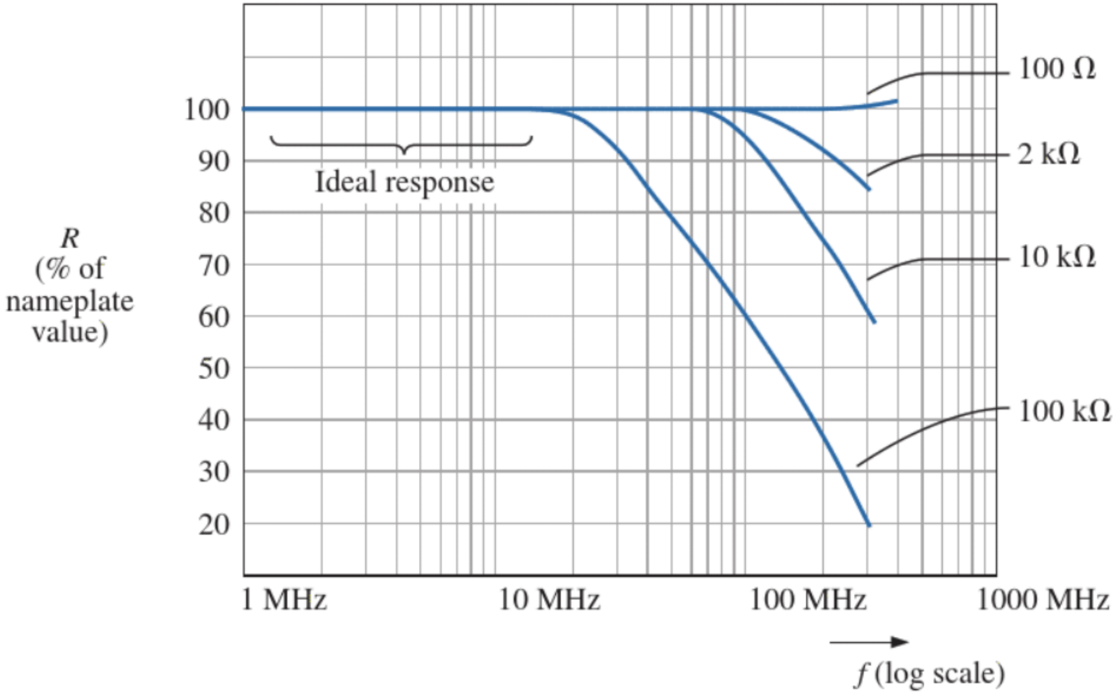
$i=0A$	$P=0w$
$i=I_m=5A$	$P=25R$
$i=0A$	$P=0w$
$i=-5A$	$P=25R$

- Power curve is always above horizontal axis

Frequency and Resistors

- For ideal resistor frequency has no effect

- In reality, resistor has some capacitance and some inductance
- When frequency is beyond megahertz, there are some changes:



Suggested reading

Introductory Circuit Analysis

– Kap 13: 13.1 - 13.4, **13.5 - 13.8**
– Kap 14: 14:5-14:9, **14:11**

*suggestions in bold are more important

Suggested exercises

- AC (kapital 13): 1, 11, 49, 50
- Complex math (kapital 14): 37, 39, 41, 43, 49
- Phasors (kapital 14): 53, 55, 57