# HE1027 Electrical Principals

Capacitors and Inductors

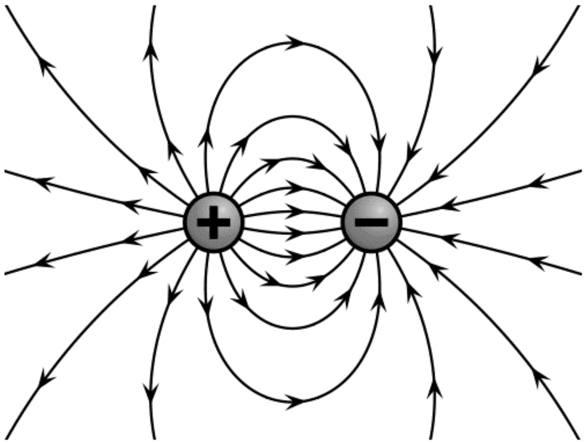
# Electric Field

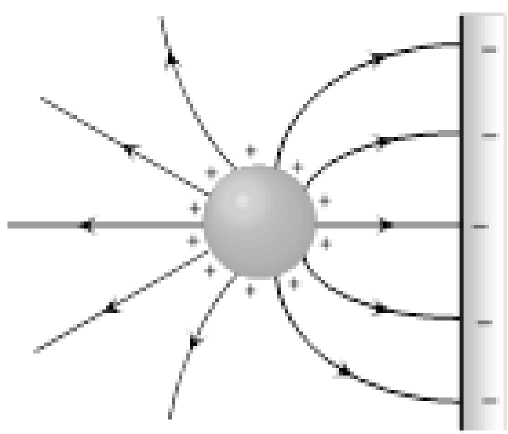
- Around each point in space when charge is present in any form exists an electric field

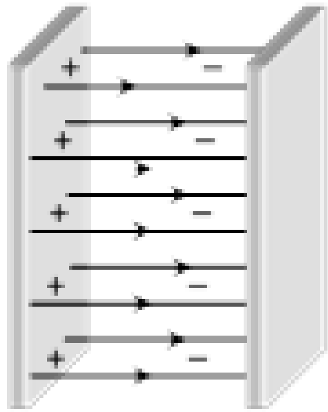
- The strength of electric field is drawn using electric flux lines

- Electric flux lines always go from a positively charged points to negative

- Electric flux lines always go from a perpendicular to charged surfaces

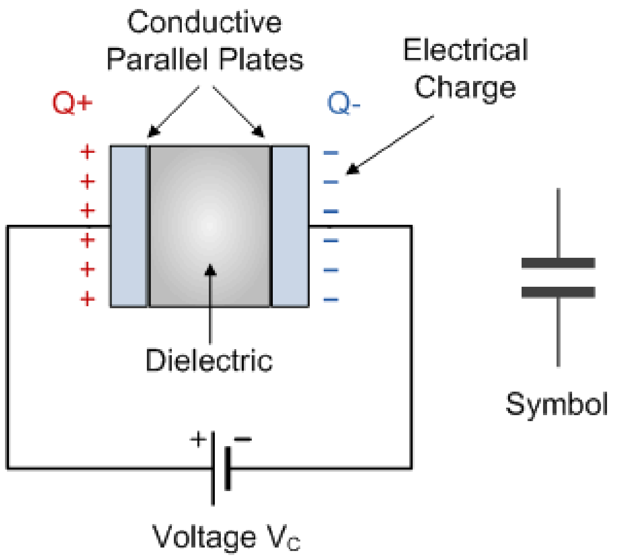






# Capacitor (*kondensator*)

- If two parallel plates are connected to a circuit, these plates will collect a charge
- These plates separated by a gap are known as capacitors
- **Take two electrical conductors and separate them with an insulator and you make a capacitor**
- Capacitors store electrical energy
  - adding electrical energy to a capacitor is called charging
  - releasing the energy from a capacitor is known as discharging
    - a capacitor generally releases its energy much more rapidly (ex. flash camera)



# Capacitance

- Capacitance is a measure of a capacitor's ability to store charge on it (to store capacity)
- Capacitance is measured in units called farads: 1-farad capacitor can store one coulomb of charge at 1 volt:

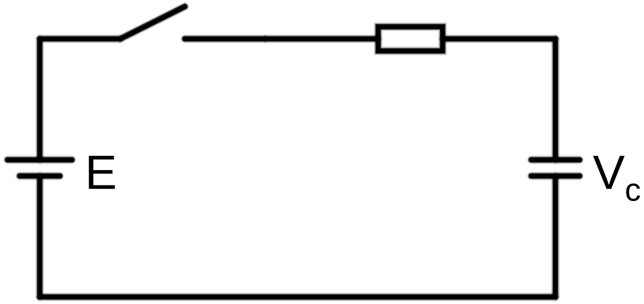
$$C(\text{capacitance}) = \frac{Q\ (\text{charge})}{V\ (\text{voltage})}$$

*A 1-farad capacitor would typically be pretty big. It might be as big as a can of tuna or a 1-liter soda bottle, depending on the voltage it can handle. For this reason, capacitors are typically measured in microfarads*

- Capacitance value and depends upon three main factors:
  - the type of material which separates the two plates ($\epsilon$)
  - surface area of conductive plates (A)
  - distance between the two plates (d)

$$C = \epsilon \frac{A}{d}$$

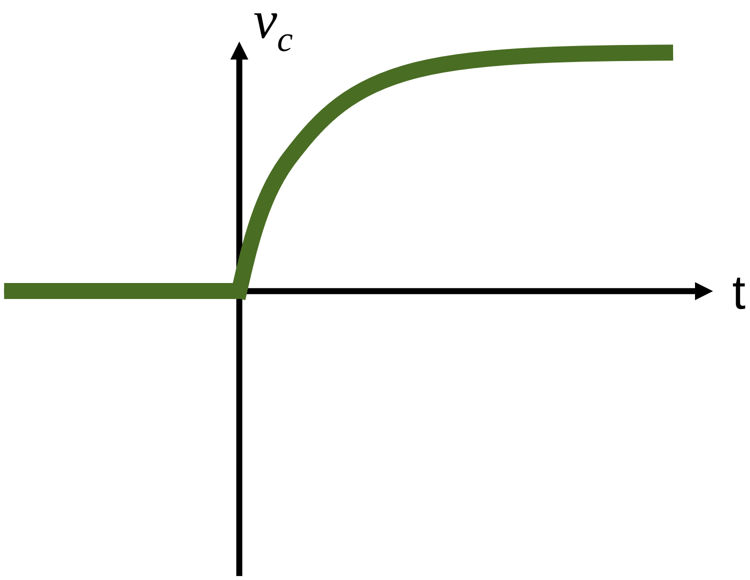
# Charging Phase in Capacitive Network

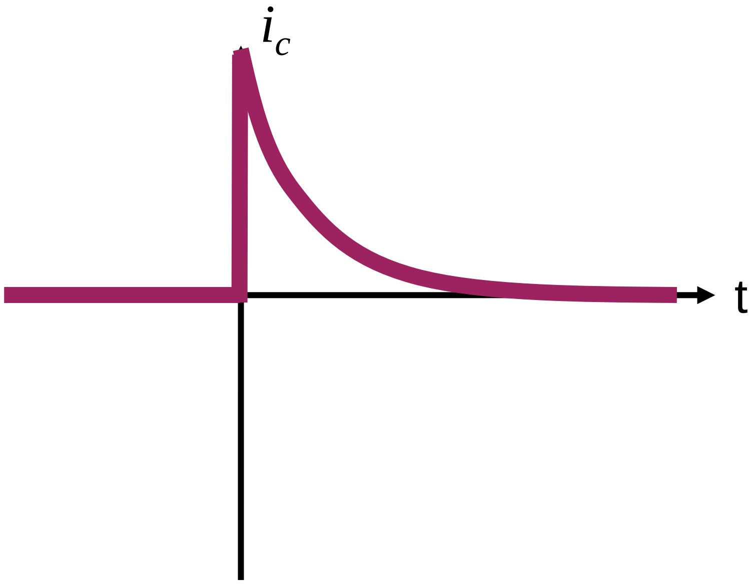




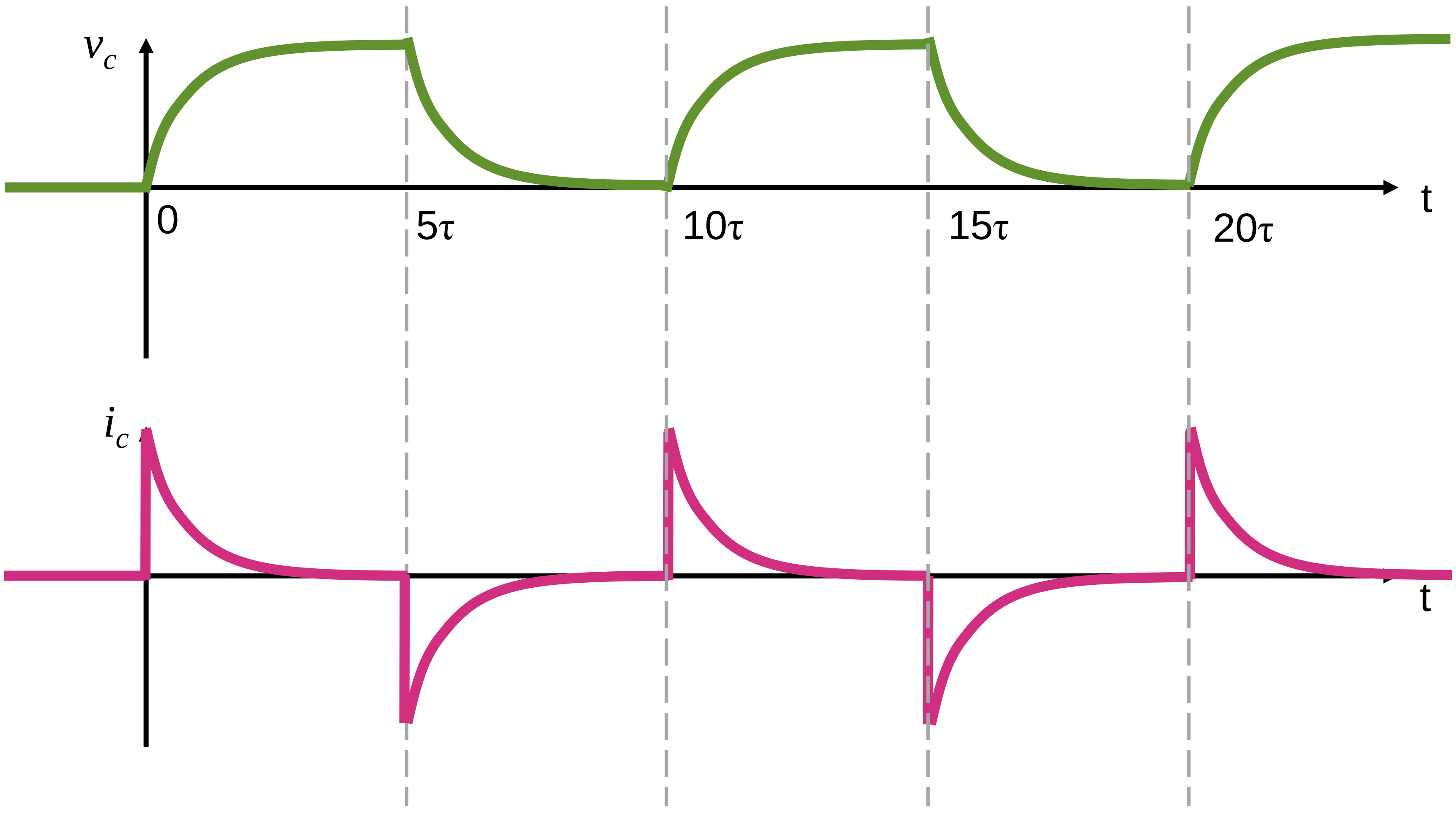

| Time | E | $V_c$ | $E\text{-}V_c$ | Current |
|---|---|---|---|---|
| 0 | 100 | 0 | 100 | Really fast |
| 1 | 100 | *50* | 50 | Fast |
| 2 | 100 | *80* | 20 | Medium |
| 3 | 100 | *90* | 10 | Slow |
| 4 | 100 | *95* | 5 | Very slow |
| 5 | 100 | *96* | 4 | Very slow |
| 6 | 100 | *97* | 3 | Very slow |
| 7 | 100 | *98* | 2 | Very slow |
| 8 | 100 | *99* | 1 | Very slow |
| 9 | 100 | *100* | 0 | Stopped |

# Concept of $t_{0-}$ and $t_{0+}$



# Switching between Contacts



# R-C Circuit

RC circuit is a circuit with both a resistor (R) and a capacitor (C)

| | |
|---|---|
| Time constant $\tau$ (tau) | $\tau = RC$ |
| Voltage of charging capacitor over time with an initial value $V_{initial}$ | $v_c(t) = V_{final} + (V_{initial} - V_{final})e^{-\frac{t}{\tau}}$ |
| Voltage of charging capacitor over time with no initial value ($V_{initial}$=0 and $V_{fianl}$=E) | $v_c(t) = E + (0 - E)e^{-\frac{t}{\tau}} = = E\left(1 - e^{-\frac{t}{\tau}}\right)$ |
| Voltage of discharging capacitor over time | $v_c(t) = E - E\left(1 - e^{-\frac{t}{\tau}}\right) = Ee^{-\frac{t}{\tau}}$ |
| Current of capacitor over time | $i_c(t) = \frac{E}{R} = \frac{E}{R}e^{-\frac{t}{\tau}}$ |
| Voltage of a resistor over time | $v_r(t) = i_c R = \left(\frac{E}{R}e^{-\frac{t}{\tau}}\right)R = Ee^{-\frac{t}{\tau}}$ |
| Math contestant $e$ | $e \approx 2.718$ |

# R-C Circuit

| | $v_c = E(1-e^{-t/\tau})$ | $i_c = \dfrac{E}{R} e^{-t/\tau}$ |
|---|---|---|
| t=0 | $v_c = E(1-e^0) = E(1-1) = 0V$ | $i_c = (E/R)*e^0 = (E/R)*1 = E/R$ |
| t=τ | $v_c = E(1-e^{-\tau/\tau}) = E(1-e^{-1}) =$<br>$= E(1-0.368) = 0.632E$ | $i_c = (E/R)*e^{-\tau/\tau} = 0.368*(E/R)$ |
| t=2τ | $v_c = E(1-e^{-2\tau/\tau}) = E(1-e^{-2}) =$<br>$= E(1-0.135) = 0.865E$ | $i_c = (E/R)*e^{-2\tau/\tau} = 0.135*(E/R)$ |
| t=5τ | $v_c = E(1-e^{-5\tau/\tau}) = E(1-e^{-5}) =$<br>$= E(1-0.007) = 0.993E \approx E$ | $i_c = (E/R)*e^{-5\tau/\tau} = 0.007*(E/R) \approx 0$ |

# R-C Circuit

$$i_c = \frac{E}{R} e^{-t/\tau}$$

A capacitor is a short-circuit at the moment when a switch is just closed

| | | |
|---|---|---|
| t=0 | $v_c=E(1-e^{0})=E(1-1)=0V$ | $i_c=(E/R)*e^{0}=(E/R)*1= E/R$ |
| $t=\tau$ | $v_c=E(1-e$ … <br> $=E(1-0.3$ … | … $^{-\tau/\tau}=0.368*(E/R)$ |
| $t=2\tau$ | $v_c=E(1-e^{-2\tau/\tau})=E(1-e^{-2})=$ | $i_c=(E/R)*e^{-2\tau/\tau}=0.135*(E/R)$ |
| $t=5\tau$ | $v_c=E(1-e^{-5\tau/\tau})=E(1-e^{-5})=$ <br> $=E(1-0.007)=0.993E\approx E$ | $i_c=(E/R)*e^{-5\tau/\tau}=0.007*(E/R)\approx 0$ |

During charging, the major change in voltage and current happens during the first time constant

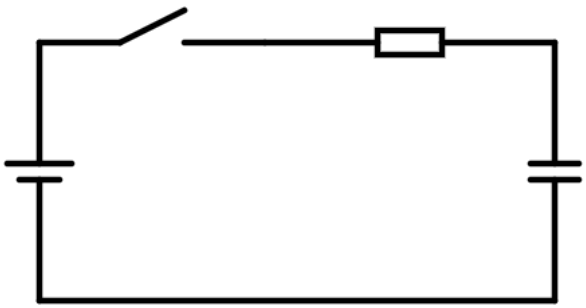
The charging phase of a capacitor has essentially ended after 5 time constants

The current is essentially 0A after 5 time constants

A capacitor is a open-circuit at the moment when it is fully charged

**Example:**

*Find voltage of capacitor 50ms after the connection if E=20V, C=4μF and R=5kΩ*



**Determine time τ**

$\tau = RC = 4\mu F * 5k\Omega = 0.02s$

**Determine $v_c$**

$$v_c(t) = V_{final} + (V_{initial} - V_{final})e^{-\frac{t}{\tau}}$$

$$v_c(t) = 20 + (0 - 20)e^{-\frac{t}{\tau}}$$

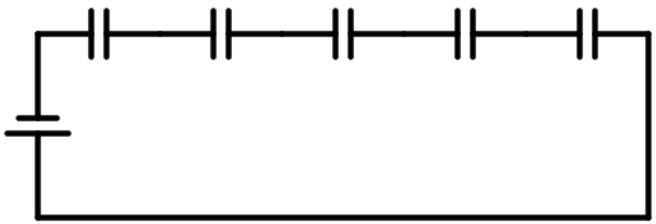
$$v_c(0.05) = 20 - 20e^{-\frac{0.05}{0.02}}$$

$$v_c(0.05) = 20 - 20e^{-2.5}$$

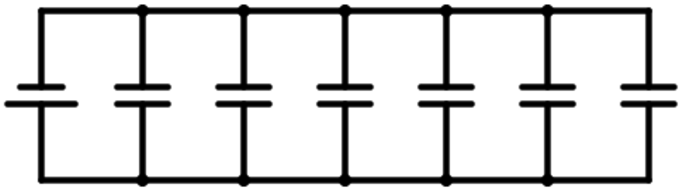
$$v_c(0.05) = 20 - 1.64$$

$$v_c(0.05) = 18.36V$$

# Capacitors in Series and Parallel



$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \ldots + \frac{1}{C_n}$$



$$C_T = C_1 + C_2 + C_3 + C_4 + \ldots + C_n$$

# Inductor (*spole*)

- Current flowing through a conductor generates a magnetic field
- The magnetic field starts out small, as current yet flows in only part of the conductor. Once steady current is established, magnetic field quantity will be stable
- Magnetic field stores charge
- Faraday's law of induction says that we should have very long conductor as a coil for best result. A magnet inside the coil will help. Such element is called inductor
- Inductance L
- Used for signal filtering, sensors, dynamics
- Two inductors form a transformer

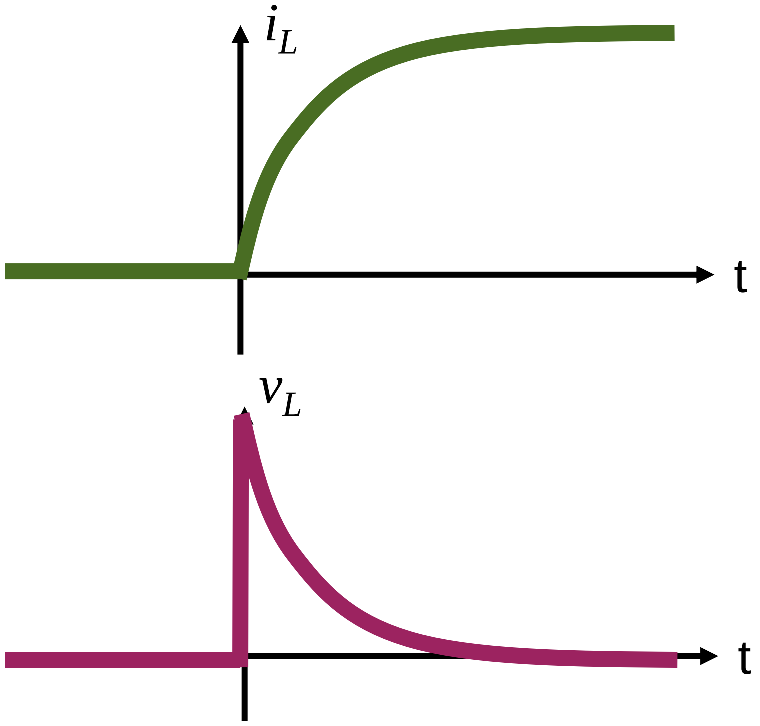
# R-L curcuit

Time constant $\tau = \frac{L}{R}$

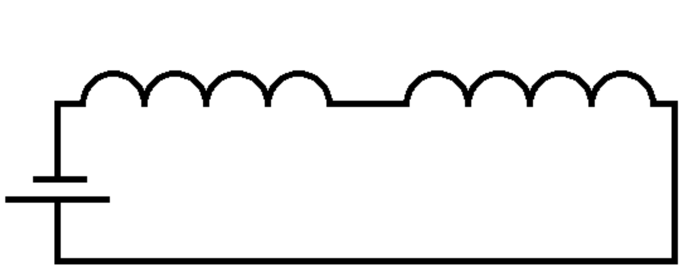
Current of an inductor over time with an initial value $I_{initial}$

$$i_L(t) = I_{final} + \left(I_{initial} - I_{final}\right)e^{-\frac{t}{\tau}}$$

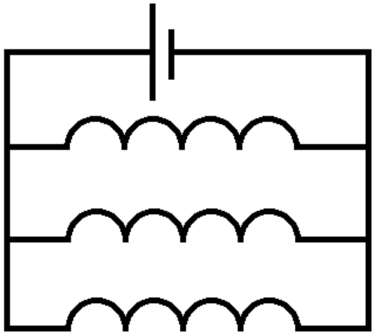
$$I_{final} = \frac{E}{R_{Total}}$$



# Inductors in Series and Parallel

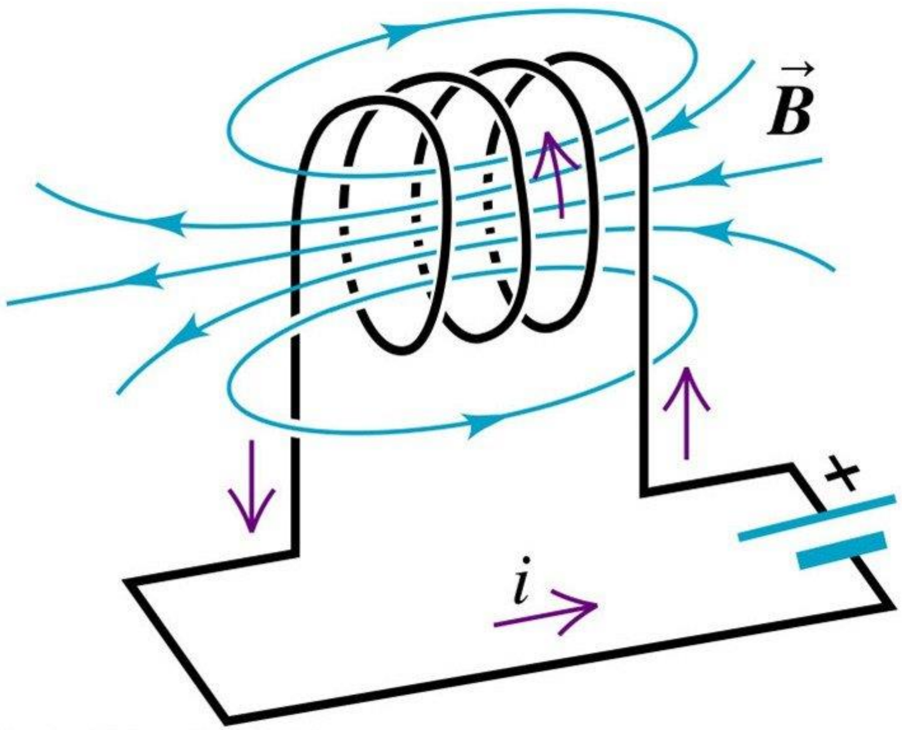


$L = L_1 + L_2 + L_3$



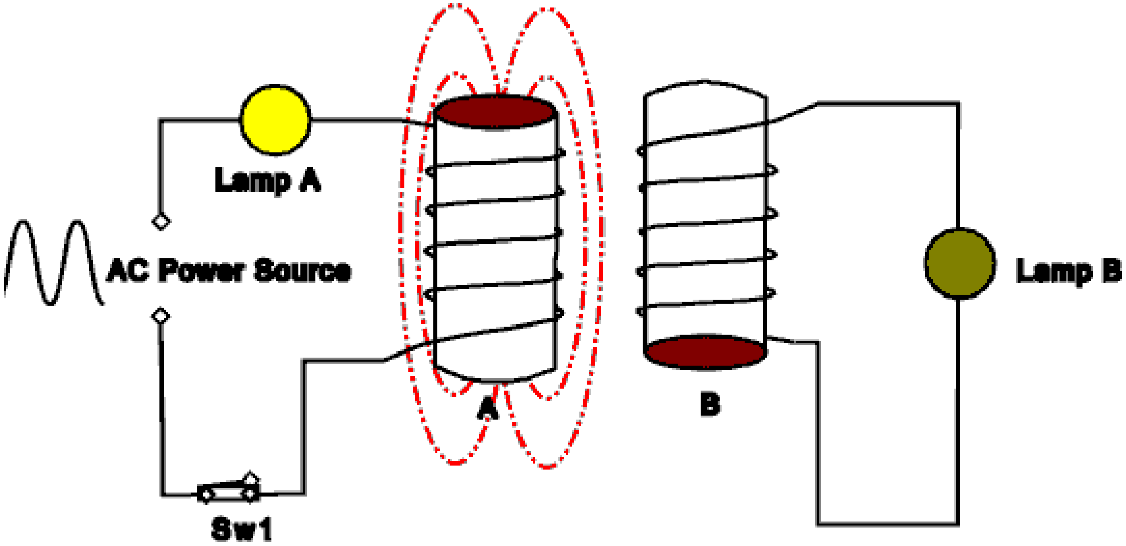
$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

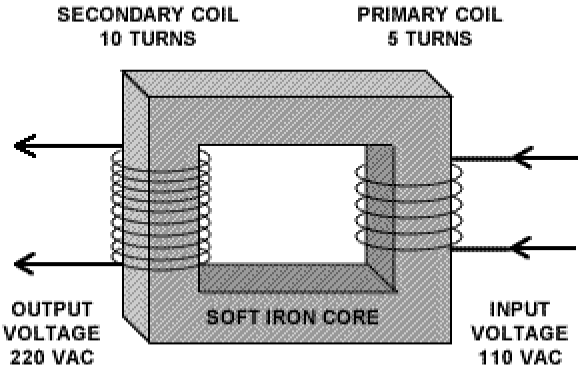
# Inductor and Magnetic Field



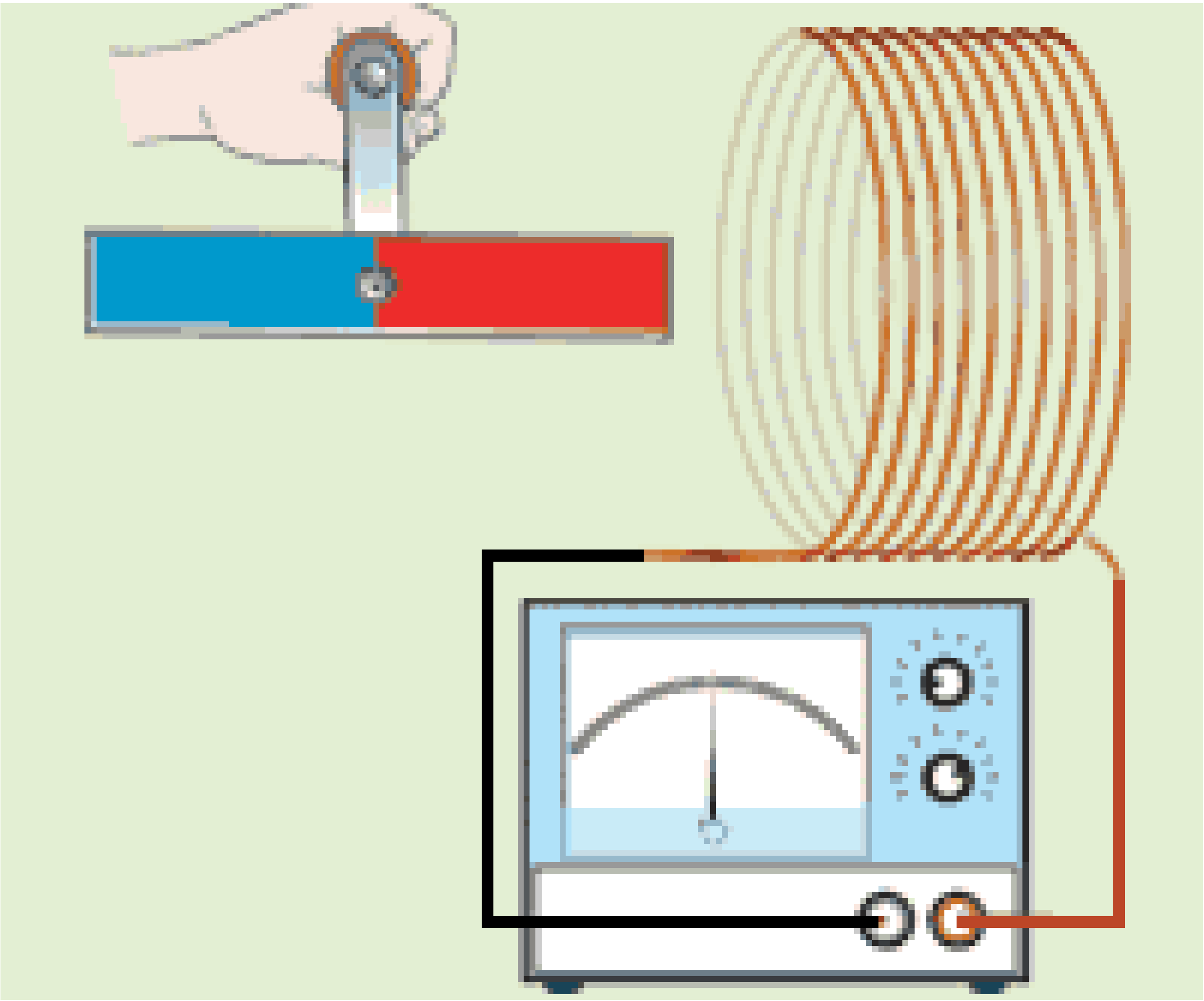

- Movement of electrons causes magnetic field
- BECAUSE electrons and magnetic field are friends
- SO moving magnetic field causes movement of electrons

# Transformer

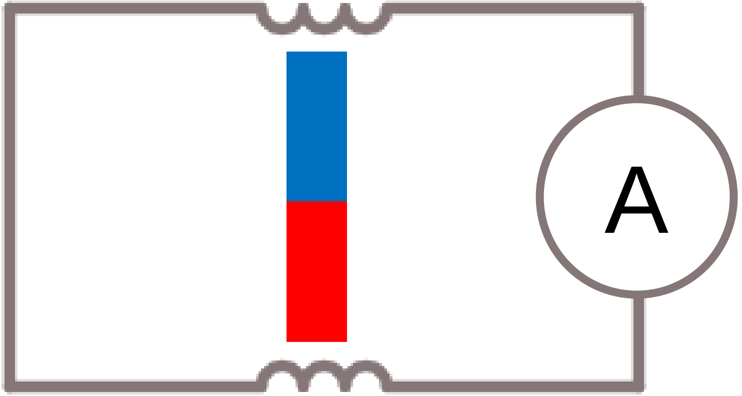


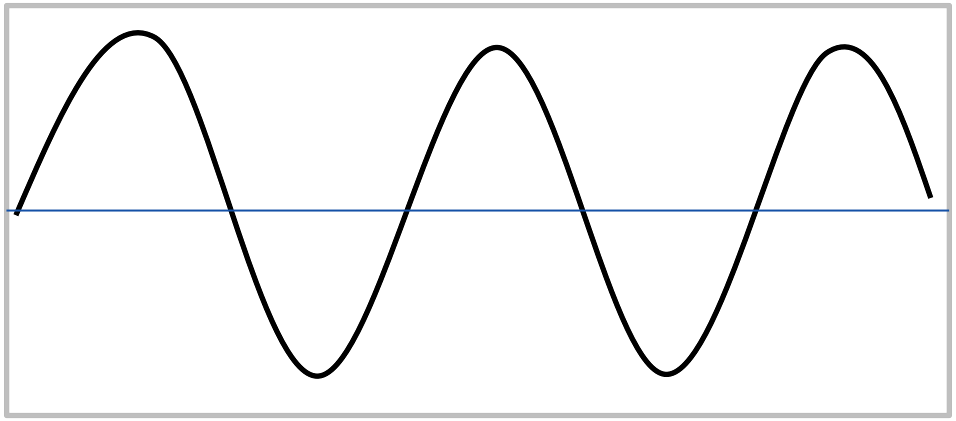



# Magnet Produces Electricity

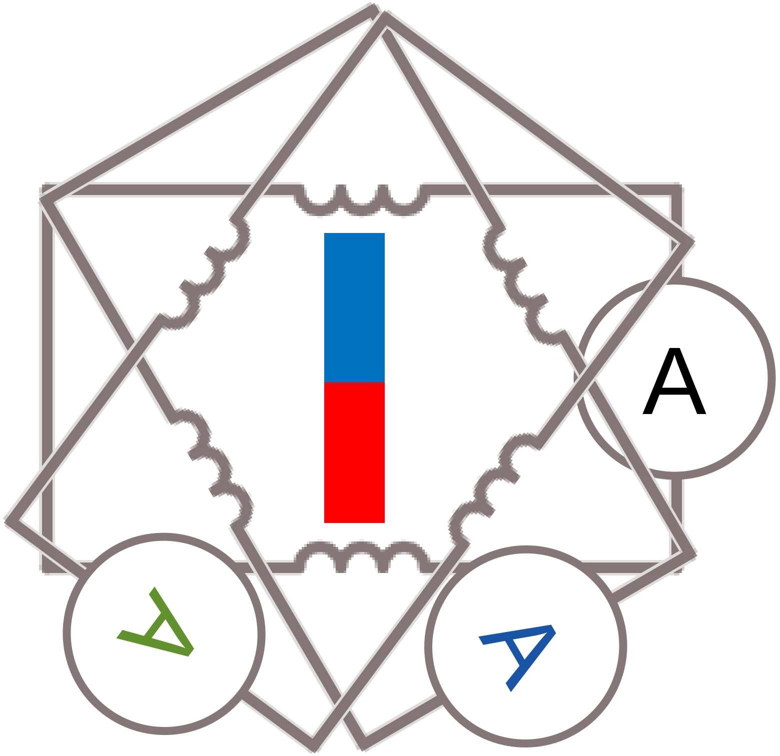


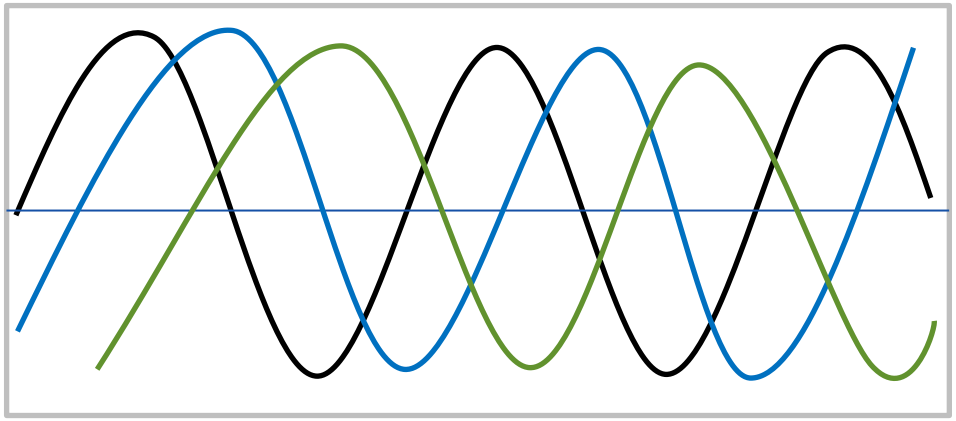
# One Phase Electricity



# Three Phase Electricity



## Suggested reading

# Introductory Circuit Analysis

- Kap 10: 10.2 - 10.4, **10.5 - 10.9**, 10.11 - 10.13
- Kap 11: 11.2 - 11.3, **11.4 - 11.8**, 11.9 - 11.12
- Kap 23: 23.1 - 23.3
- Kap 24: 2

*suggestions in bold are more important

## Suggested exercises

- Capacitors (kapital 10): 19, 21, 25, 29, 37, 42, 43
- Inductors (kapital 11): 11, 13, 15, 17, 21, 23, 24