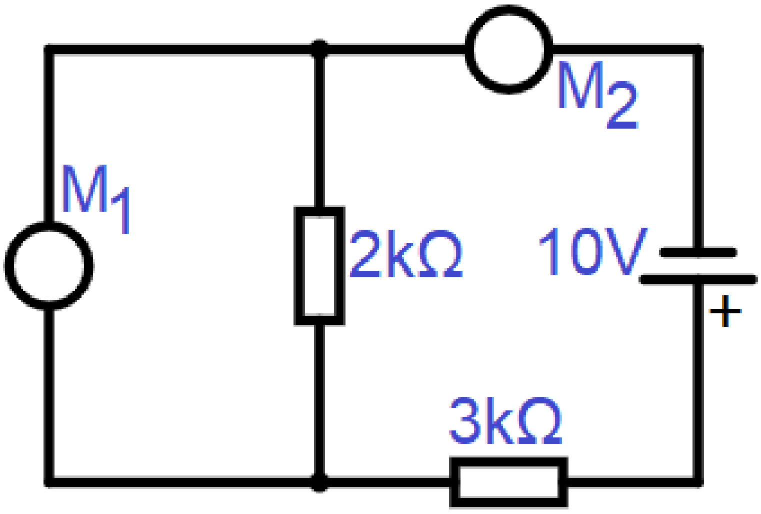
Kontrollskrivning KS1 i Ellära (HE1027)

Uppgift 1 [0.5p]

Två multimetrar (M_1 och M_2) är korrekt anslutna till kretsen nedan. Vad mäter dessa multimetrar och vilka värden läser de av?

Two multimeters (M_1 and M_2) are properly connected to the circuit below. What measure these multimeters and what values do they read?



Svar

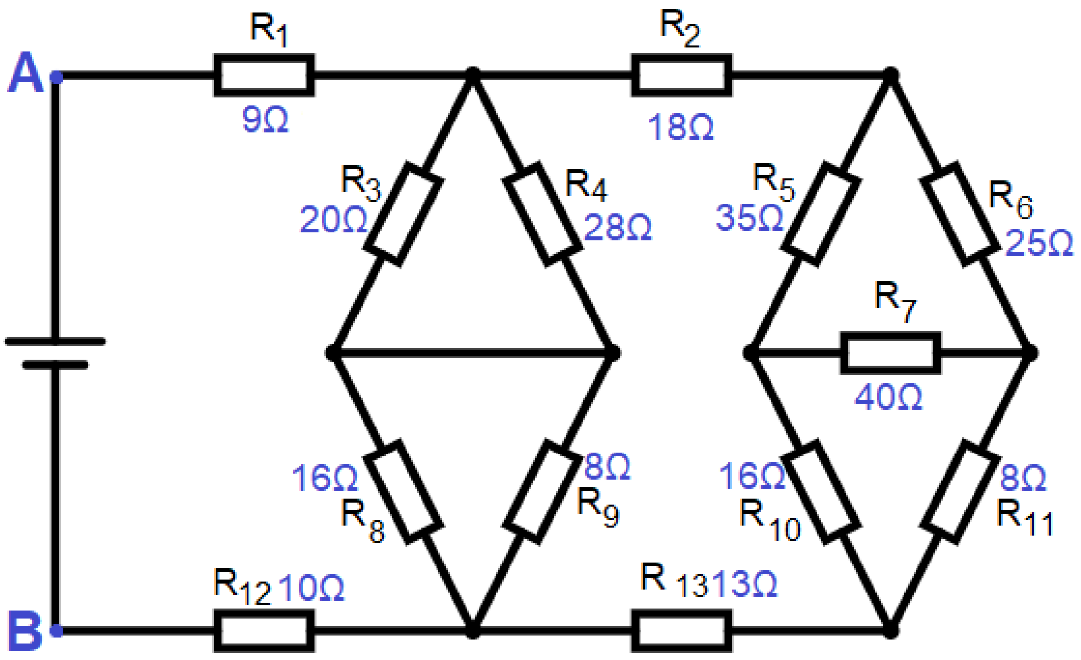
Since M1 is connected parallel to 2kΩ resistor, it can be voltmeter or ohmmeter. But the network has a power supply, it cannot be ohmmeter. M2 is connected in series to the network, so it is ammeter. RT is 2kΩ+3kΩ=5kΩ. IT=V/RT=10/5k=2mA. Voltage consumption by the 2kΩ resistor is IT*R=2m*2k=4V.

So M1 is a voltmeter and it shows 4V and MS is a ammeter and it shows 2mA

Uppgift 2 [1.5p]

Bestäm resistansen R_{AB} mellan A och B i kretsen nedan.

Determine resistance R_{AB} between A and B in the following circuit.



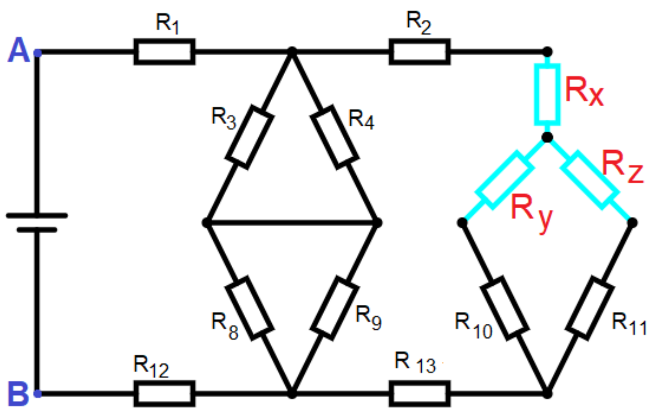
Svar:

We need to transform resistors R_5, R_6 and R_7 from the triangle to the Y (there are several options what we can transform).

$$R_x = \frac{R_5 \cdot R_6}{R_5 + R_6 + R_7} = \frac{35 * 25}{35 + 25 + 40} = \frac{875}{100} = 8.75\Omega$$

$$R_y = \frac{35 * 40}{35 + 25 + 40} = 14\Omega$$

$$R_z = \frac{25 * 40}{35 + 25 + 40} = 10\Omega$$

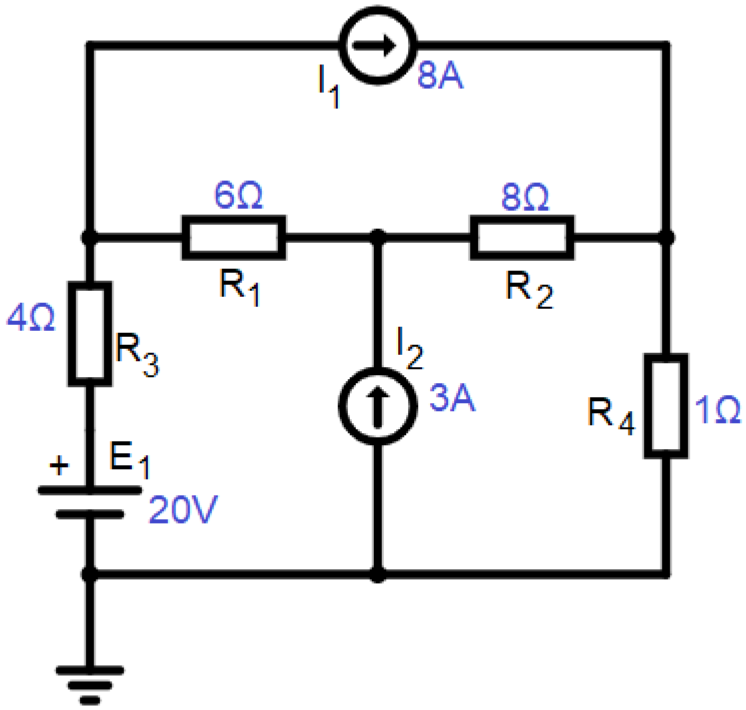


$R_{AB}=R_1+((R_3//R_4)+(R_8//R_9))//(R_2+R_x+(R_y+R_{10})//(R_z+R_{11})+R_{13})+R_{12}=31.75\Omega$

Uppgift 3 [2p]

Bestäm strömmarna genom varje resistor genom att använda maskanalys.

Determine current through all resistors using mesh analysis.



Svar:

Since we need to use mesh analysis, we need to assign currents: I_a in top loop, I_b in bottom left loop and I_c in bottom right loop. All currents flow clockwise.

I_a: I_a is equal to I_1, so I_a=8A

I_b and I_c is a supermesh because they have only one current source in between, and $I_b=I_c-I_2$ or $I_b=I_c-3$

I_b+I_c: $E_1-R_3 \cdot Ib-R_1(I_b-I_a)-R_2(I_c-I_a)-R_4 \cdot I_c=0$

$20-4I_b-6I_b+6 \cdot 8-8I_c+8 \cdot 8-1I_c=0$

Now I will substitute I_b with I_c-3 (since $I_b=I_c-3$)

$20-4I_c+12-6I_c+18+48-8I_c+64-1I_c=0$

$-19I_c+162=0$

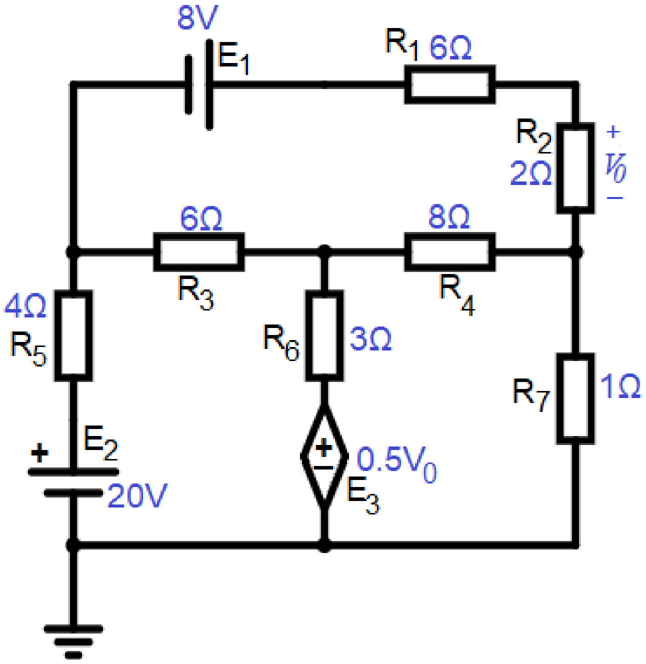
$I_c=8.526A$

Hence, $I_{R4}=I_c=8.526A$, $I_{R3}=I_b=I_c-3=5.526A$, $I_{R1}=I_a-I_b=2.474A$ and $I_{R2}=I_a-I_c=0.526A$

Uppgift 4 [2p]

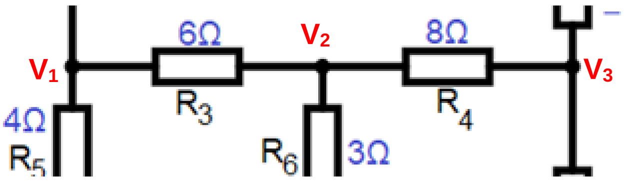
Bestäm spänningen V_o genom att använda nodanalys.

Determine the voltage V_o using node analysis.



Svar:

The circuit has 3 nodes, all currents will be leaving the nodes.



V1: $-\frac{V_1-20}{4}-\frac{V_1-V_2}{6}-\frac{V_1+8-V_3}{6+2}=0$

We have calculated current over R2 ($I_{R_2}=\frac{V_1+8-V_3}{6+2}$), so we can get its voltage: $V_0=V_{R_2}=I_{R_2}\cdot R_2=2\cdot\frac{V_1+8-V_3}{6+2}$ V.

V2: $-\frac{V_2-V_1}{6}-\frac{V_2-0.5V_0}{3}-\frac{V_2-V_3}{8}=0$

I will keep V_0 as a fourth unknown and I will have a system of 4 equations.

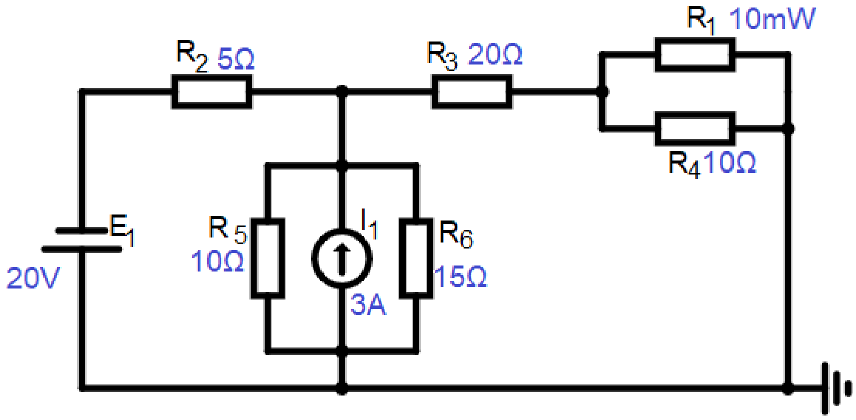
V3: $-\frac{V_3-8-V_1}{6+2}-\frac{V_3-V_2}{8}-\frac{V_3}{1}=0$

Solving our system, we get values: V_1= 9.04348V, V_2= 3.82609V, V_3= 2.08696V and V_0= 3.73913V.

Uppgift 5 [3p]

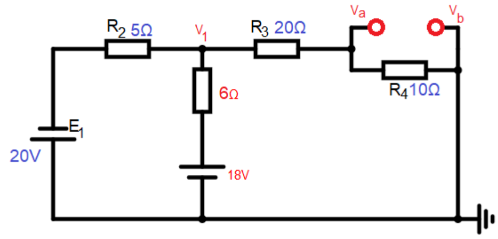
Bestäm driftseffektiviteten η (kvoten mellan den nuvarande effekten och maximal möjlig effekt) för R_1. [2.5p]
Bestäm nuvarande resistansen R_1. [0.5p]

Determine operating efficiency η (ratio of the actual power to maximum possible power) of R_1. [2.5p]
Determine current resistance of R_1. [0.5p]



Svar:

We need to find maximum possible power, and the formula for it is $P_{max} = \frac{E_{Th}^2}{4 \cdot R_{Th}}$. Hence, we will need to solve using Thevenin theorem. Before doing we can simplify the network a little bit. R_5 and R_6 are in parallel, so their combine resistance is 10//15=6Ω. And the current source with the parallel 6Ω resistor we can transform into voltage source 3A*6Ω=18V with a 6Ω resistor in series.
We can calculate R_{Th} by as resistance between terminals a and b with short-circuited all voltage sources.
$R_{Th}=R_4//(R_3+R_2//6Ω)=6.944Ω$.



E_{Th} is $V_a - V_b$ and since V_b is grounded and equal zero, $E_{Th}=V_a$. We will use nodal analysis to solve it. While there is only one real node V_1, I will use two nodes: V_1 and V_a. All currents are leaving nodes.

V_1: $-\frac{V_1+20}{5} - \frac{V_1-18}{6} - \frac{V_1-V_a}{20} = 0$ and V_a: $-\frac{V_a-V_1}{20} - \frac{V_a}{10} = 0$

Solving two equations we get an answer: V_1=-2.5V and V_a=-0.833V. Hence E_{Th}=-0.833V.

$P_{max} = \frac{E_{Th}^2}{4 \cdot R_{Th}} = \frac{(-0.833)^2}{4 \cdot 6.944} = 0.025W$ and $\eta = \frac{P_{now}}{P_{max}} = \frac{0.01}{0.025} = 0.4$ or 40%.

To find resistance of R1, we will work in Thevenin equivalent circuit: 0.833V voltage source, 6.944Ω resistor and R1 in series. We know that PR1=VI=I2R. I is the same all over the circuit and equal to voltage source divided by total resistance: $I = \frac{0.833}{6.944+R_1}$. Now combining these, we get that

$$P_{R_1} = (\frac{0.833}{6.944 + R_1})^2 \cdot R_1 = \frac{0.833^2 \cdot R_1}{(6.944 + R_1)^2} = 0.01$$

$$0.833^2 \cdot R_1 = 0.01 \cdot (6.944 + R_1)^2 = 0.01 \cdot (6.944^2 + 2 \cdot 6.944 \cdot R_1 + {R_1}^2)$$

$$-0.01 \cdot {R_1}^2 + 0.833^2 \cdot R_1 - 0.01 \cdot 2 \cdot 6.944 \cdot R_1 - 0.01 \cdot 6.944^2 = 0$$

$$R_1^2 - 55.501R_1 + 48.219 = 0$$

R_1 is equal to 0.883Ω or 54.618Ω