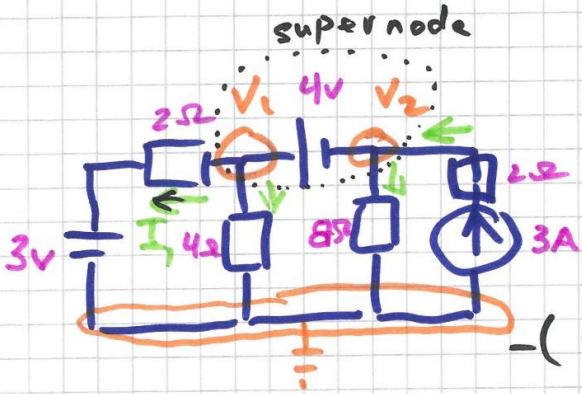
Supernode example



$V_2 + 4 = V_1$

$\dots -(-3) = 0$

$$-\left(\frac{V_1 - 3}{2}\right) - \left(\frac{V_1}{4}\right) - \left(\frac{V_2}{8}\right) + \left(\frac{3}{0}\right) = 0$$

$$\frac{-V_1 + 3}{2} - \frac{V_1}{4} - \frac{V_2}{8} + 3 = 0$$

$$\frac{-4V_1 + 12}{8} - \frac{2V_1}{8} - \frac{V_2}{8} + \frac{24}{8} = 0$$

$$-4V_1 + 12 - 2V_1 - V_2 + 24 = 0$$

$$-6V_1 - V_2 = -36$$

$$6V_1 + V_2 = 36$$

$V_2 + 4 = V_1 \Rightarrow V_2 = V_1 - 4$

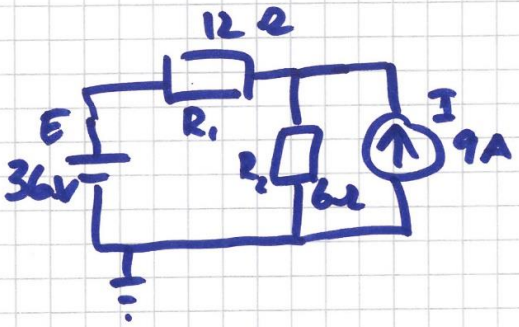
$$6V_1 + V_1 - 4 = 36$$

$$7V_1 = 40$$

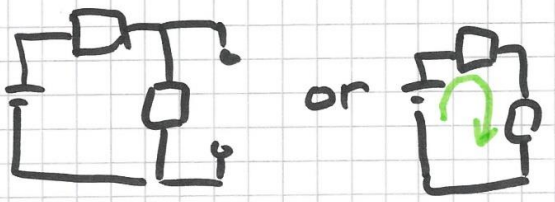
$$V_1 = \frac{40}{7} = 5.7\,V$$

$V_2 = 5.7 - 4 = 1.7\,V$

Superposition Theorem



1)

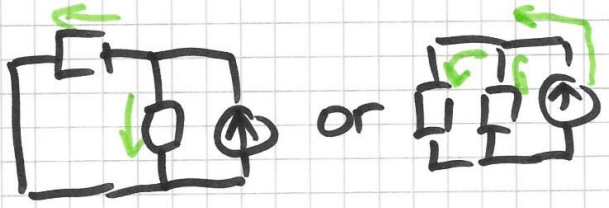


$R_T = 12 + 6 = 18\,\Omega$

$I_T = \frac{36}{18} = 2A$

$\rightarrow I'_{R_1} = 2A \qquad \downarrow I'_{R_2} = 2A$

2)



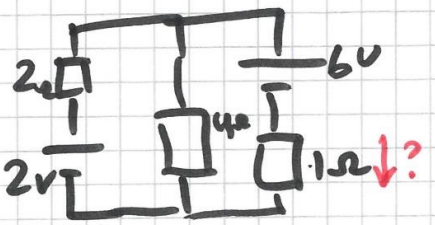
$R_T = \frac{12 \cdot 6}{12+6} = 4\,\Omega \qquad V_T = 4\Omega \cdot 9A = 36V$

$\leftarrow I''_{R_1} = \frac{36}{12} = 3A \qquad \downarrow I''_{R_2} = \frac{36}{6} = 6A$

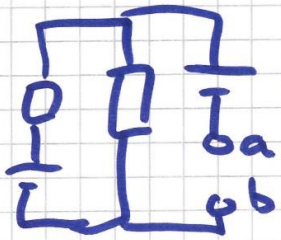
3) $\leftarrow I_{R_1} = I'_R + I''_{R_1} = -2 + 3 = 1A$

$\downarrow I_{R_2} = I'_{R_2} + I''_{R_2} = 2A + 6A = 8A$

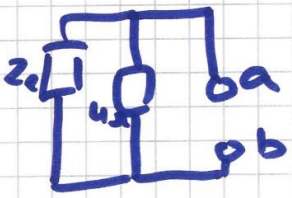
Tevenin Theorm



1)



2) R_{TH}



$$R_{TH} = R_{ab} = \frac{4 \cdot 2}{4+2} = \frac{8}{6} = 1.333\ \Omega$$

3) E_{TH}

$$E_{TH} = E_{ab} = E_a - E_b = E_a$$

$$E_a = E_c - 6$$

$$R_T = 2 + 4 = 6\ \Omega$$

$$I = 2/6 = 0.333\ A$$

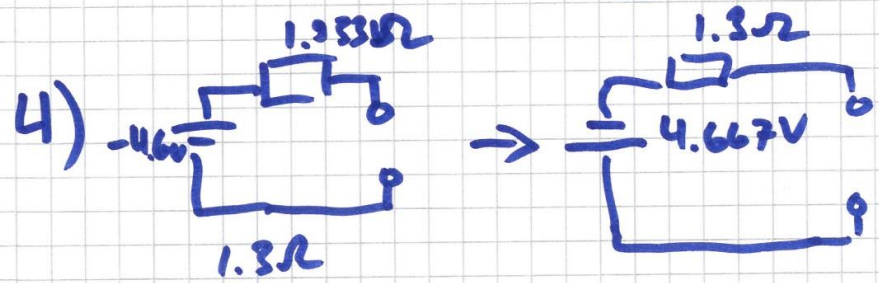
$$V_{R_1} = 2 \cdot 0.333 = 0.666\ V$$

$$V_c = 2 - 0.666 = 1.333 V$$

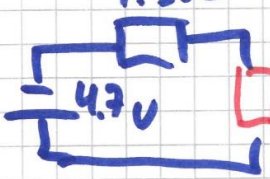
$$V_a = 1.333 - 6 = -4.667\ V$$

$$E_{TH} = V_a = -4.667 V$$

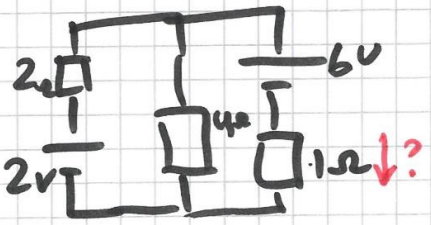
4)

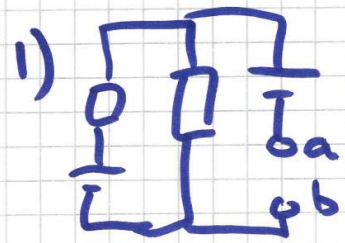


5)



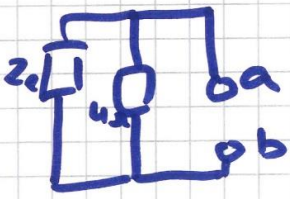
$$I_T = \frac{4.7}{1.3+1} = 2A \qquad I_{R_3} = 2A \uparrow$$





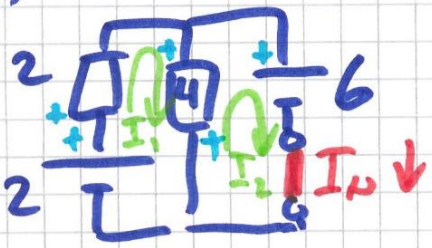
Norton Theorem

2) R_{TH}



$$R_{TH} = R_{ab} = \frac{4 \cdot 2}{4+2} = \frac{8}{6} = 1.333\ \Omega$$

3) $I_N = I_2$



$$L_1: \quad +2 - 2I_1 - 4(I_1 - I_2) = 0$$
$$2 - 2I_1 - 4I_1 + 4I_2 = 0$$
$$-6I_1 + 4I_2 + 2 = 0$$

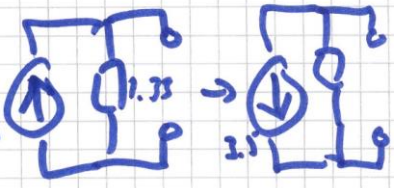
$$L_2: \quad -4(I_2 - I_1) - 6 = 0$$
$$4I_1 - 4I_2 - 6 = 0$$

$$\begin{cases} -6I_1 + 4I_2 + 2 = 0 \\ 4I_1 - 4I_2 - 6 = 0 \end{cases}$$

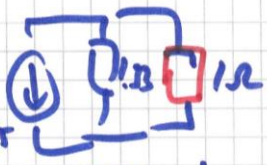
$$-2I_1 - 4 = 0$$
$$I_1 = -2A$$
$$-6(-2) + 4I_2 + 2 = 0$$
$$12 + 4I_2 + 2 = 0$$
$$4I_2 = -14$$
$$I_2 = -3.5A$$

$$R_N = 1.333\ \Omega$$

4)



5)

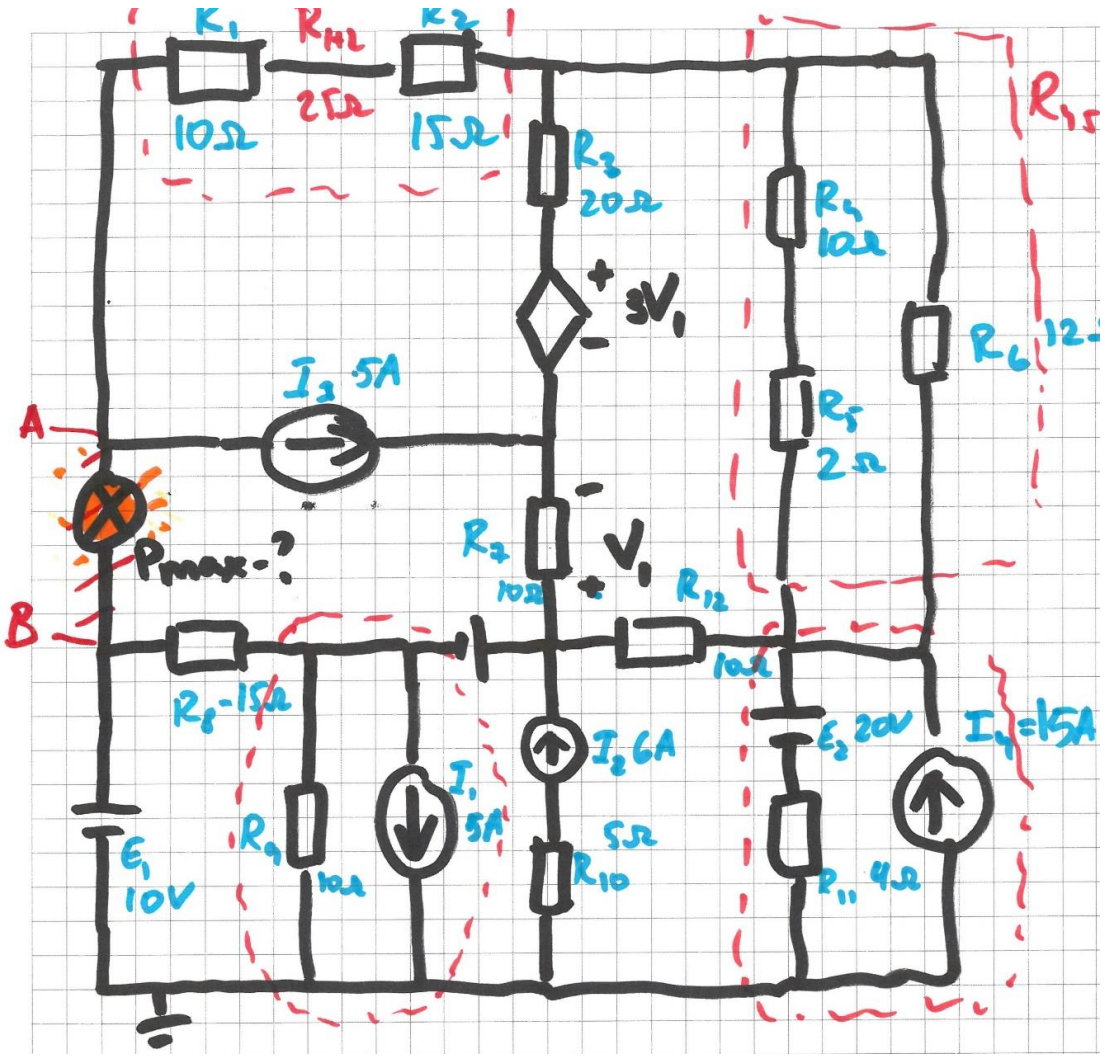


$$R_T = \frac{1.333 \cdot 1}{2.333} = 0.57\ \Omega$$

$$V_T = 0.57 \cdot 3.5 = 2V$$

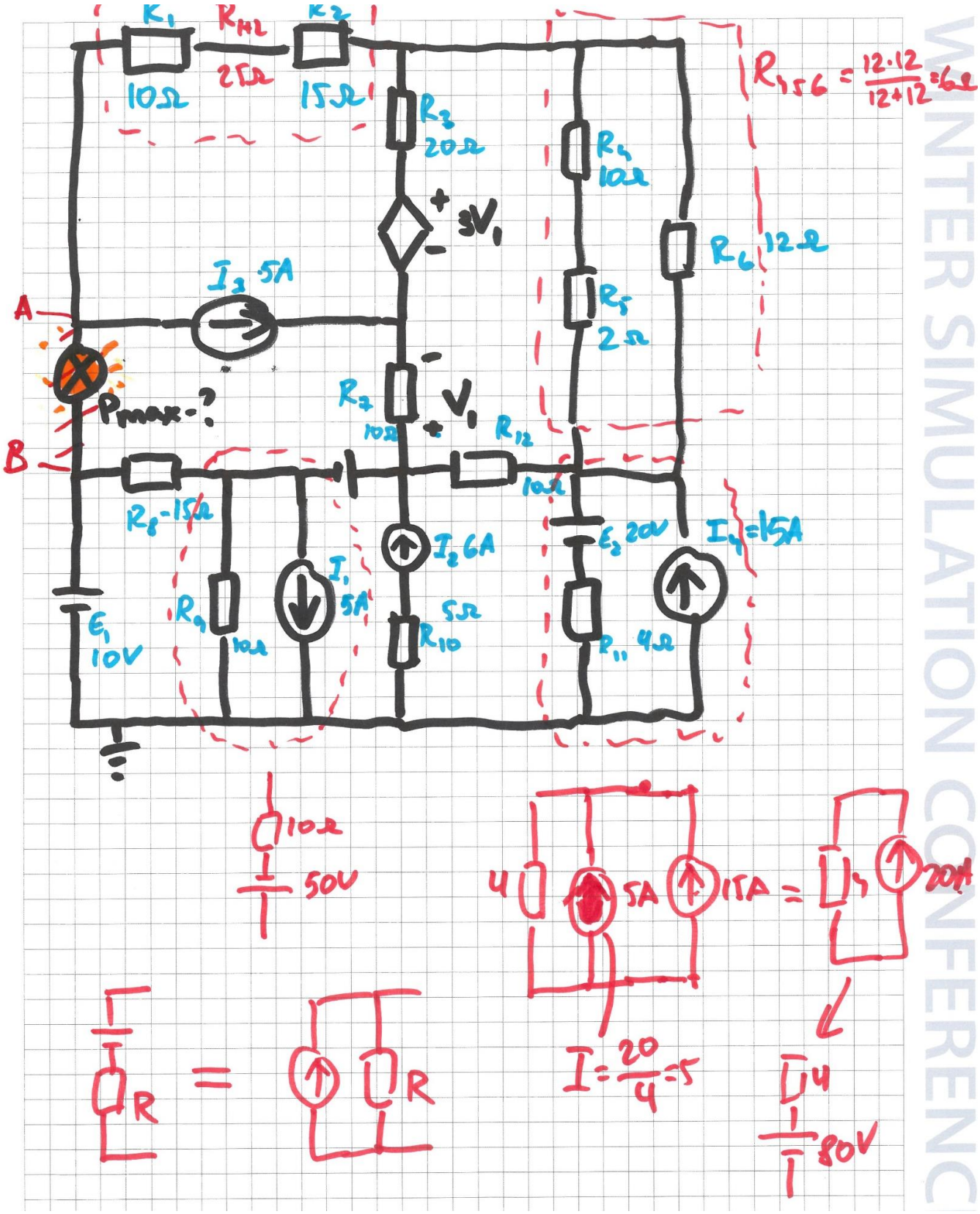
$$\uparrow I_{R_1} = \frac{2}{1} = 2V$$

Find what is maximum power of a light bulb

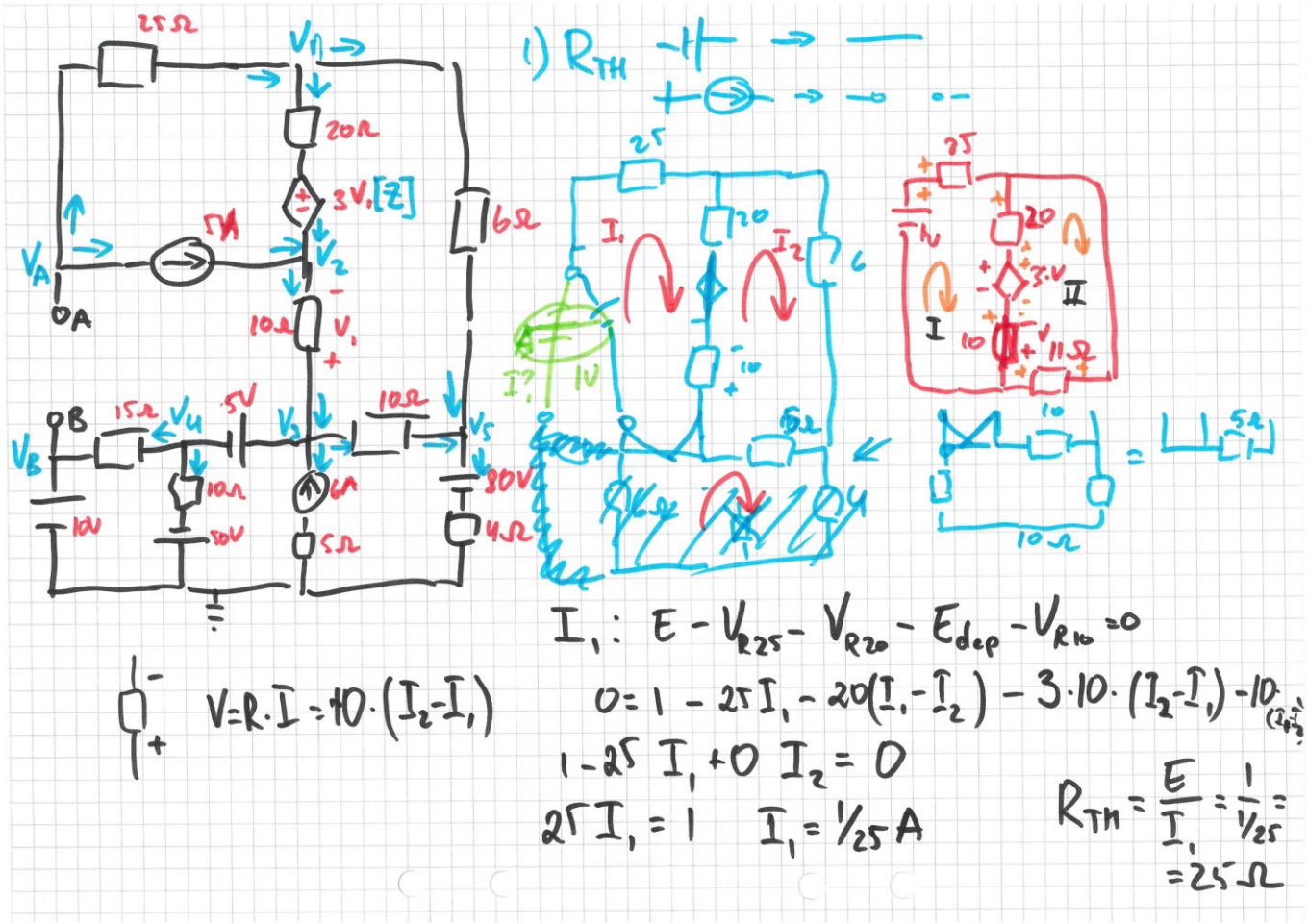


Since we need to find P_{max} it means we need to use maximum power theorem and Thevenin theorem

Optional: first we simplify circuit using serial-parallel technique and source transormation



We find R_{Th}. Important, that the circuit has a depended source!



We are calculating E_{Th}. The final answer is not correct, since we did not complete solving equations

$E_{TH} = V_a - V_B = V_a - 10$

V_a: $(5A) + \left(\frac{V_a - V_1}{25}\right) = 0$

$\frac{V_a}{25} - \frac{V_1}{25} = -5$

V_1: $+\left(\frac{V_a - V_1}{25}\right) - \left(\frac{V_1 - V_2 - 3Z}{20}\right) - \left(\frac{V_1 - V_5}{6}\right) = 0$

$Z = V_3 - V_2$

$\frac{V_a}{25} - \frac{V_1}{25} - \frac{V_1}{20} + \frac{V_2}{20} + \frac{3V_3}{20} - \frac{3V_2}{20} - \frac{V_1}{6} + \frac{V_5}{6} = 0$

V_2: $(5) + \left(\frac{V_1 - V_2 - 3V_3 + 3V_2}{20}\right) - \left(\frac{V_2 - V_3}{10}\right) = 0$

$V_3 + V_4$ - supernode $V_4 + 5 = V_3$

$V_3 + V_4$: $-\left(\frac{V_4 - V_2}{15}\right) - \left(\frac{V_3 - 0 + 50}{10}\right) - (-6) + \left(\frac{V_2 - V_3}{10}\right) - \left(\frac{V_3 - V_5}{10}\right) = 0$

V_5: $+\left(\frac{V_1 - V_5}{6}\right) + \left(\frac{V_3 - V_5}{10}\right) - \left(\frac{V_5 - 80}{4}\right) = 0$

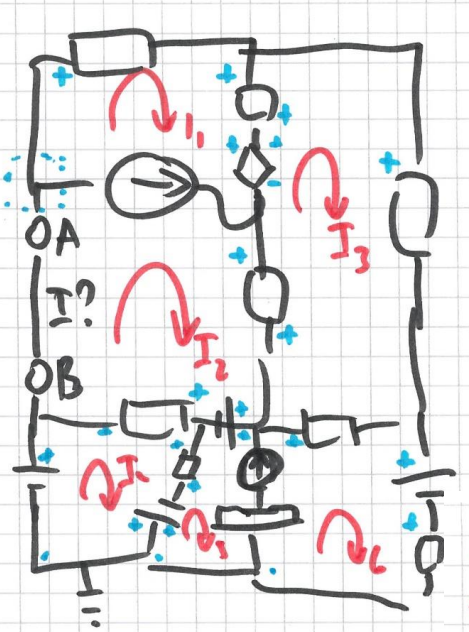
$V_B = 10\,V$

guess

$V_a = 20\,V$

$E_{TH} = 20 - 10 = 10\,V$

Optional: We are calculating I_N. The final answer is not correct, since we did not complete solving equations



$I_N = -I_2$

I_4: $10 - 15(I_4 - I_2) - 10(I_4 - I_1) + 50 = 0$

I_3: $10(I_3 - I_2) + 3(10(I_3 - I_2)) - 20(I_3 - I_1) - 6I_3 = 0$

$I_1 + I_2$: I_1 ↑, I_2 ↑, 5A $\Rightarrow$ $I_2 = I_1 + 5$

$I_1 + I_2$: $-25I_1 - 20(I_1 - I_3) - 3(10(I_3 - I_2)) - 10(I_3 - I_3) - 5V - 15(I_2 - I_4) = 0$

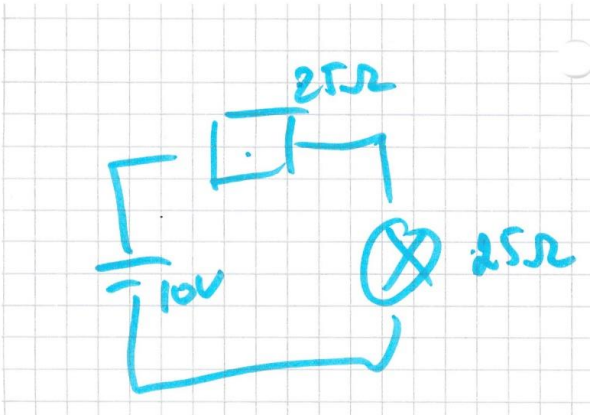
$I_5 + I_6$: 6A ↑, I_5 ←, I_6 ← $\Rightarrow$ $I_6 = I_5 + 6$

$-50 - 10(I_5 - I_4) + 5 - 10(I_6 - I_2) - 80 - 4I_6 = 0$

guess

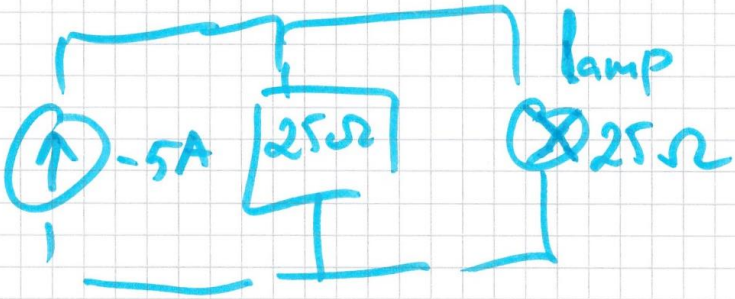
$I_2 = 5A$ $\quad I_N = -5A$

Finding final power



$$I_T = \frac{10}{25+25} = \frac{10}{50} = 0.2$$

$$P = V \cdot I = I \cdot R \cdot I = 0.2^2 \cdot 25 = 1\,W$$



$P = 1\,W$

$$R_{TH} = \frac{E_{TH}}{I_N} = \frac{10V}{-5} = 25\,\Omega$$