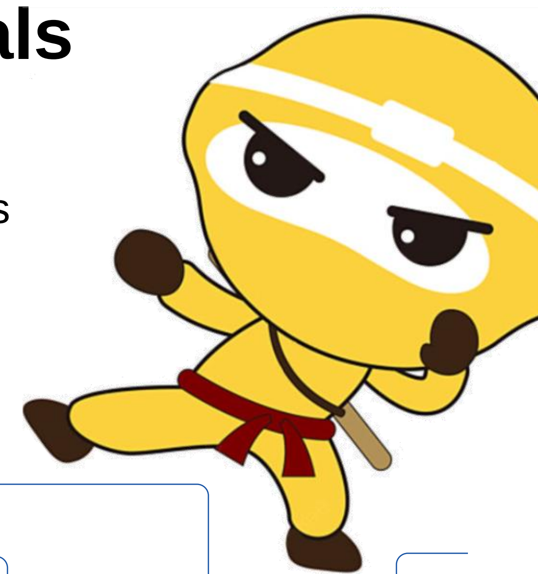
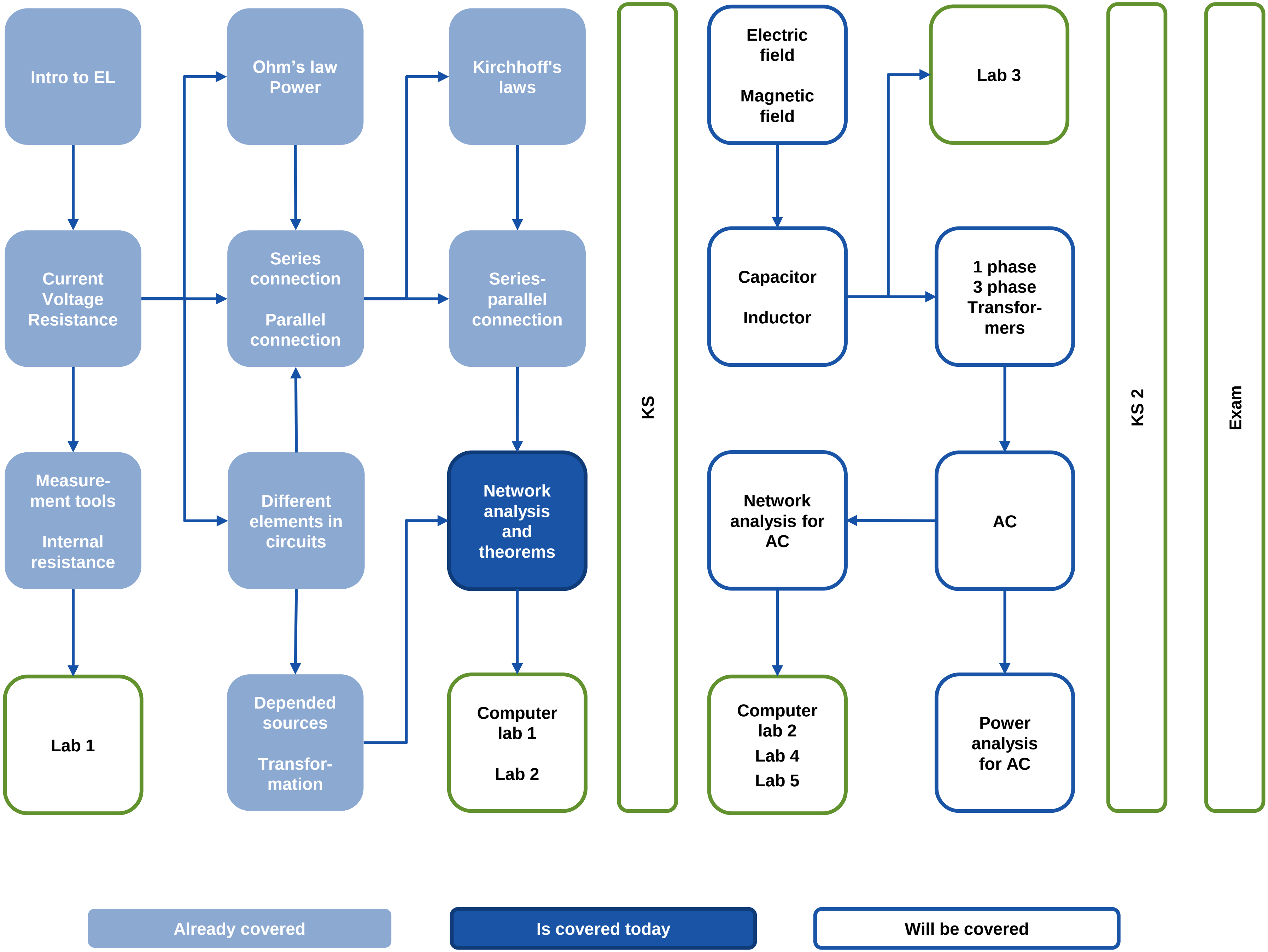
HE1027 Electrical Principals

Lectures 3-5: Network analysis and theorems

OR

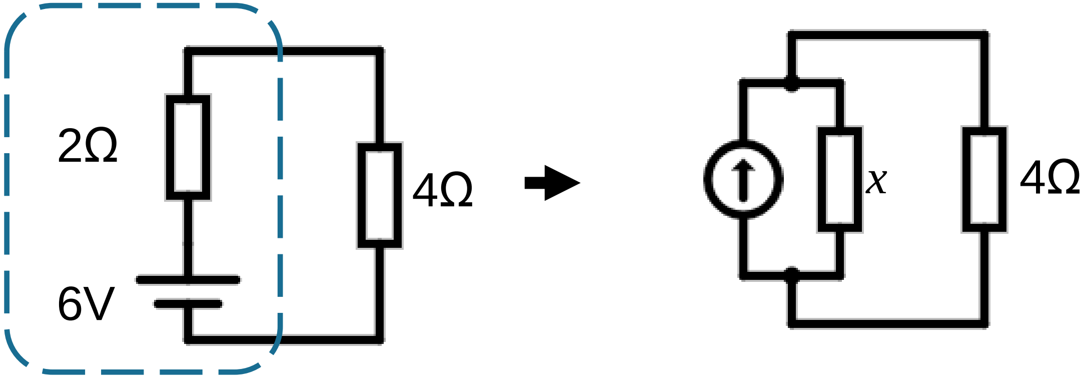
“HOW TO BE ELECTRIC NINJA”





Intro to EL
Ohm's law
Power
Kirchhoff's laws
KS
Electric field
Magnetic field
Lab 3
KS 2
Exam
Current
Voltage
Resistance
Series connection
Parallel connection
Series-parallel connection
Capacitor
Inductor
1 phase
3 phase
Transfor-mers
Measure-ment tools
Internal resistance
Different elements in circuits
Network analysis and theorems
Network analysis for AC
AC
Lab 1
Depended sources
Transfor-mation
Computer lab 1
Lab 2
Computer lab 2
Lab 4
Lab 5
Power analysis for AC
Already covered
Is covered today
Will be covered

Example
Convert to current source



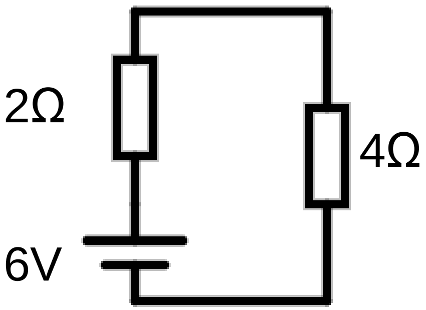
Internal resistance is the same for both circuit, so x=2Ω

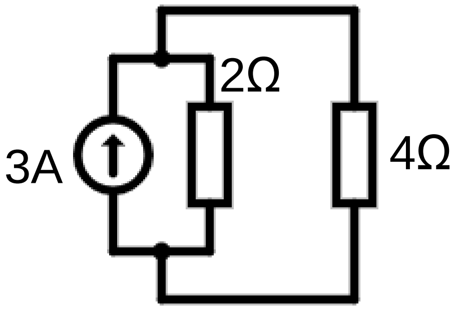
Current is equal to voltage source divided by internal resistance, so I=6/2=3A

Polarity of current source matches the polarity of voltage source

Example

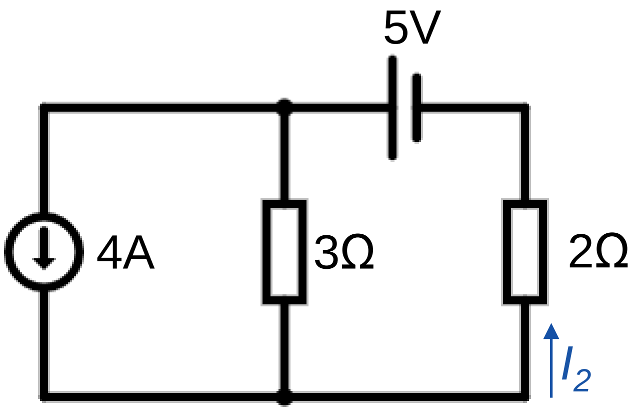
Calculate current and voltage over the 4Ω-resistor



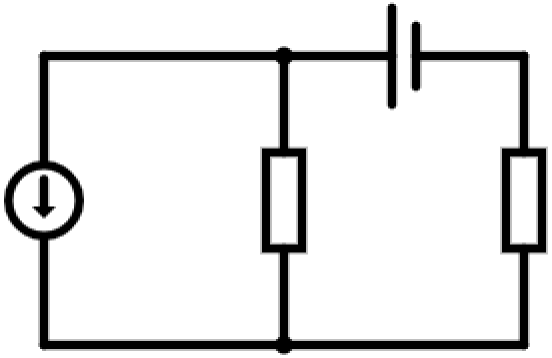


Example

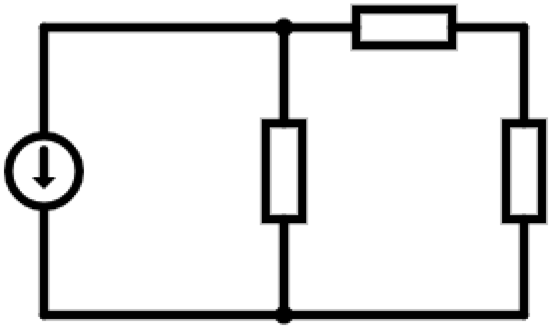
Determine current I_2



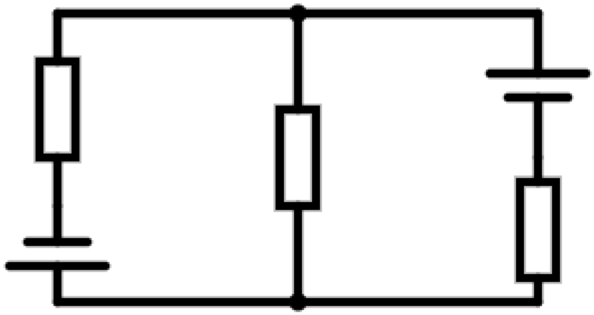
Network analysis



Source transformation

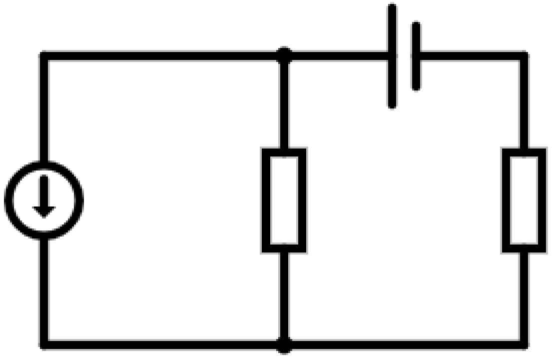


Serial-parallel resistors

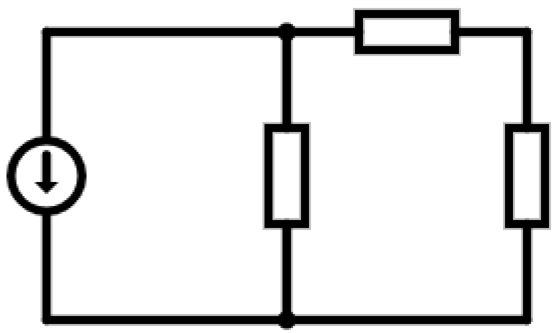


?

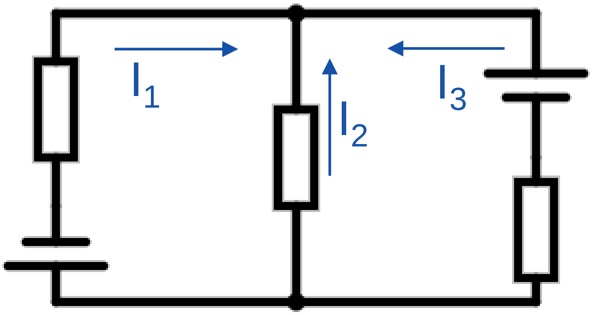
Network analysis



Source transformation



Serial-parallel resistors

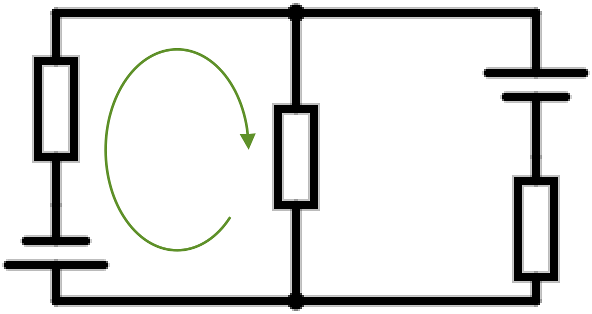


Kirchhoff's current law

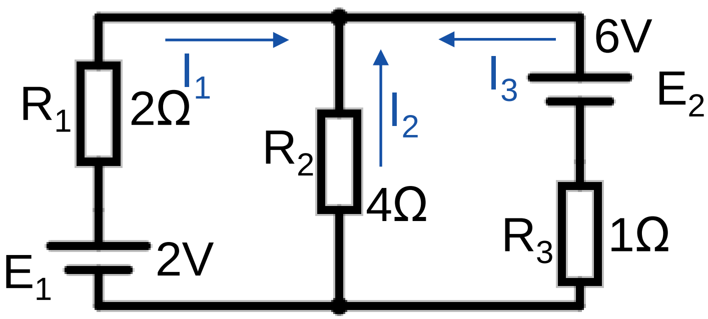
$I_1+I_2+I_3=0$

Kirchhoff's voltage law

$E_1+V_{R1}+V_{R2}=0$



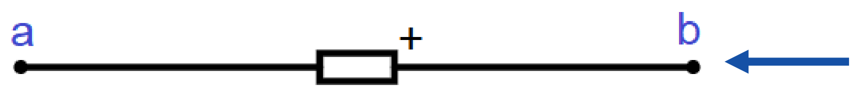
Network analysis based on Kirchhoff's current law



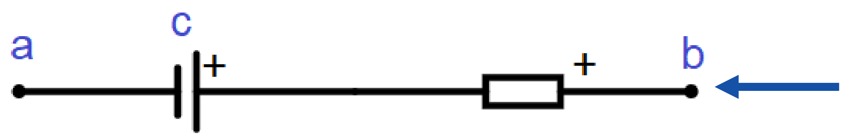
$I_1+I_2+I_3=0$, where I is current over a resistor

$$\mathrm{I} = \frac{\mathrm{V}}{\mathrm{R}}$$

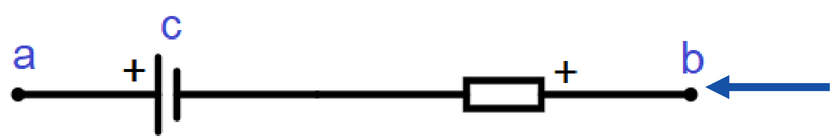
But we do not know what is V



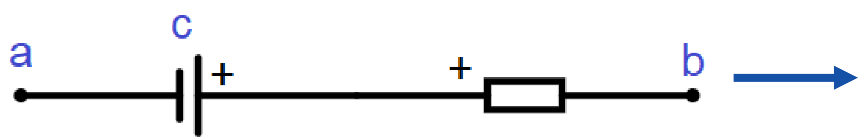
$$I = \frac{b-a}{R}$$



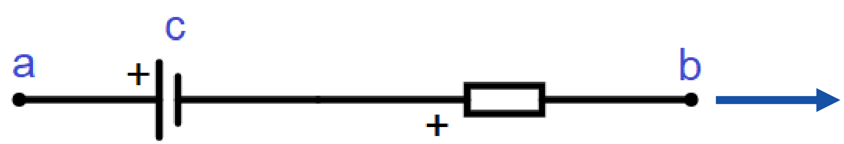
$$I = \frac{b-c-a}{R}$$



$$I = \frac{b+c-a}{R}$$

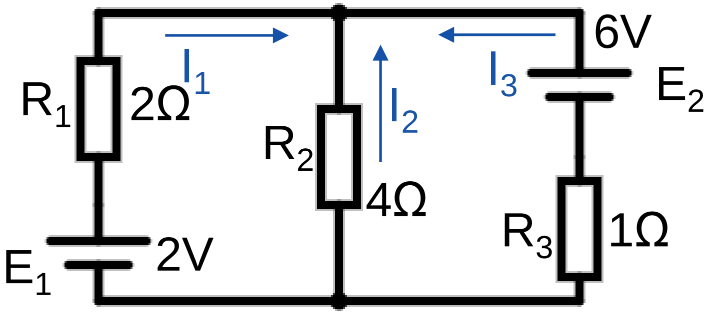


$$I = \frac{a+c-b}{R} = -\frac{b-c-a}{R}$$



$$I = \frac{a-c-b}{R}$$

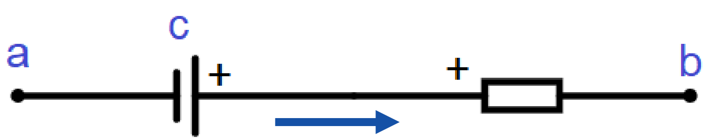
Network analysis based on Kirchhoff's current law



$I_1+I_2+I_3=0$, where I is current over a resistor

$$I = \frac{V}{R}$$

But we do not know what is V



$$I_1 = \frac{a+c-b}{R} = \frac{0+2-b}{2}$$

$$I_2 = \frac{0-b}{4}$$

$$I_3 = \frac{0+6-b}{1}$$

$$\frac{2-b}{2} - \frac{b}{4} + \frac{6-b}{1} = 0$$

$$\frac{2(2-b)}{4} - \frac{b}{4} + \frac{4(6-b)}{4} = 0$$

$$\frac{4-2b}{4} - \frac{b}{4} + \frac{24-4b}{4} = 0$$

$$\frac{28-7b}{4} = 0$$

$$28 = 7b$$

$$b = 4\text{v}$$

Nodal Analysis *(nodanalys)*

1. Determine the number of nodes within the network

2. Pick a reference node (earth where V=0), and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on

3. Apply Kirchhoff's current law at each node except the reference

   $I_1+I_2+I_3+..+I_n=0$

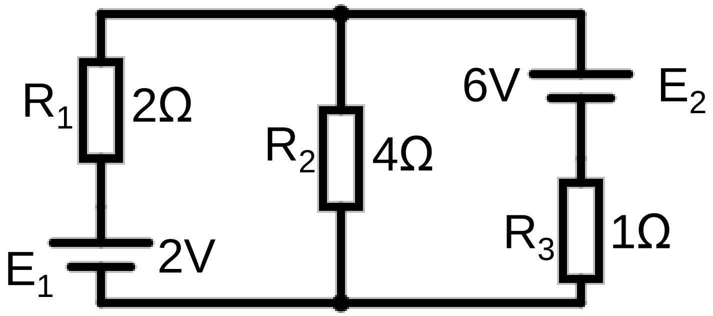
   $$I_1 = \frac{\text{Voltage difference between a node } V_1 \text{ and an other/reference node}}{\text{Total resistance between a node } V_1 \text{ and an other/reference node}}$$

4. Solve the resulting equations for the nodal voltages

   Number of unknowns is the same as number of equations!

Example

Apply nodal analysis



1. *Number of nodes*

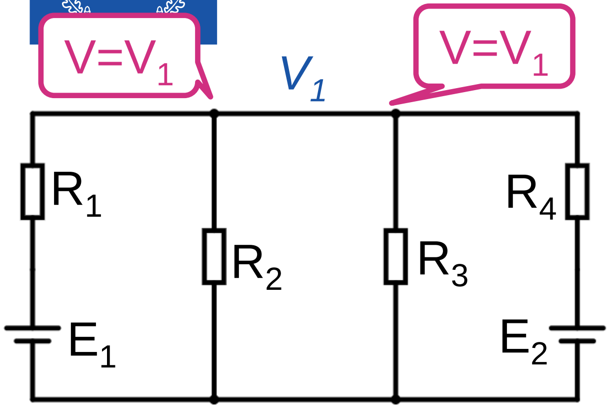
2a. *Reference node*

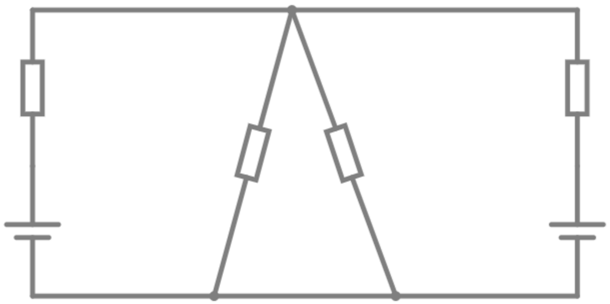
2b. *Label other nodes*

3. *Apply Kirchhoff's current law*

4. *Solve equations*

Supernode



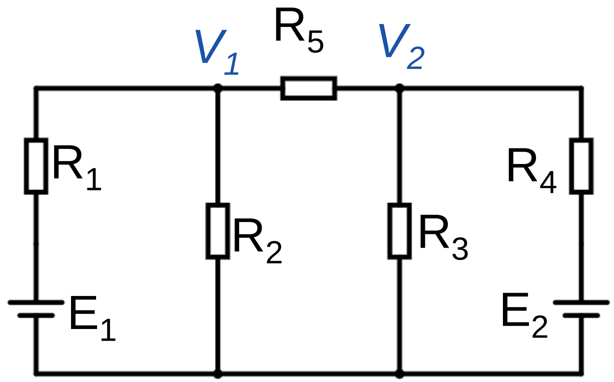


node V_1

$-I_1-I_2-I_3-I_4=0$

$I_1=(V_1-E_1)/R_1$
$I_2=V_1/R_2$
$I_3=V_1/R_3$
$I_4=(V_1-E_2)/R_4$

(one equation)



node V_1

$-I_1-I_2-I_3=0$

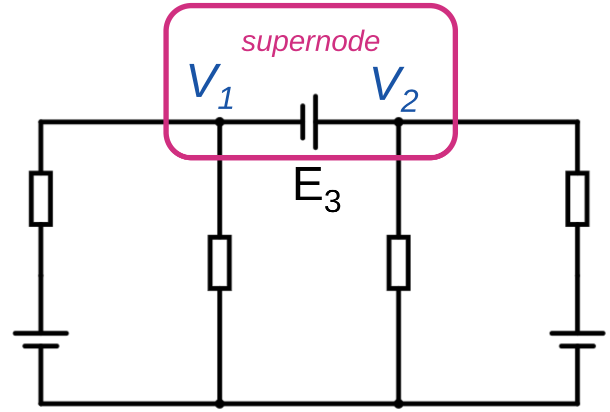
$I_1=(V_1-E_1)/R_1$
$I_2=(V_1-V_2)/R_5$
$I_3=V_1/R_3$

node V_2

$-I_3-I_4-I_5=0$

$I_3=(V_2-V_1)/R_5$
$I_4=V_2/R_3$
$I_5=(V_2-E_2)/R_4$

$$\begin{cases} -I_1-I_2-I_3=0 \\ -I_3-I_4-I_5=0 \end{cases}$$



Supernode case:

Direct connection instead of voltage source: $\mathbf{E_3=V_2-V_1}$

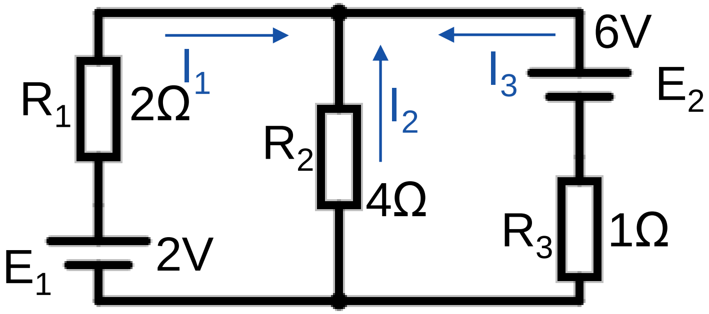
supernode

$-I_1-I_2-I_3-I_4=0$

$I1=(V_1-E_1)/R_1$
$I_2=V_1/R_2$
$I_3=V_2/R_3$
$I_4=(V_2-E_2)/R_4$

$$\begin{cases} -I_1-I_2-I_3-I_4=0 \\ E_3=V_2-V_1 \end{cases}$$

Network analysis based on Kirchhoff's voltage law



Kirchhoff's current law: $I_2=-I_1-I_3$

First loop/mesh:

$E_1+V_1+V_2=0$

$E_1-I_1*R_1+I_2*R_2=0$

$E_1-I_1*R_1+(-I_1+-I_3)*R_2=0$

$2-2*I_1-4*I_1-4*I_3=0$

$\mathbf{2-6*I_1-4*I_3=0}$

Second loop/mesh:

$R_2*I_2+E_2-R_3*I_3=0$

$(-I_1-I_3)*R_2+E_2-R_3*I_3=0$

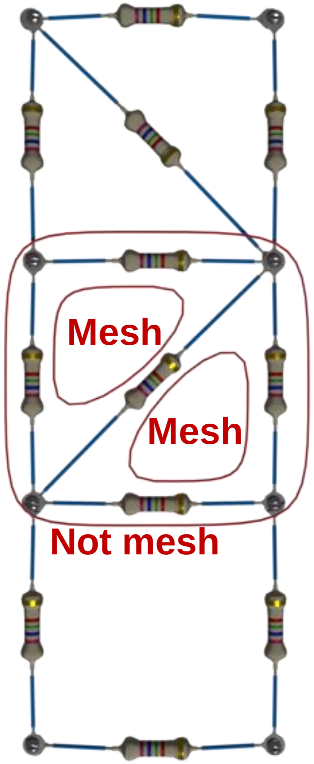
$-4*I_1-4*I_3+6-1*I_3=0$

$\mathbf{-4*I_1-5*I_3+6=0}$

$$\begin{cases} 2-6*I_1-4*I_3=0 \\ -4*I_1-5*I_3+6=0 \end{cases}$$

$I_1=-1A$ and $I_3=2A$

Mesh Analysis *(maskanalys)*



1. Assign a distinct current in the clockwise direction to each independent, closed loop of the network
2. Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop

   Note the requirement that the polarities be placed within each loop

   Resistors can have two sets of polarities across it
3. Apply Kirchhoff's voltage law around each closed loop in the clockwise direction

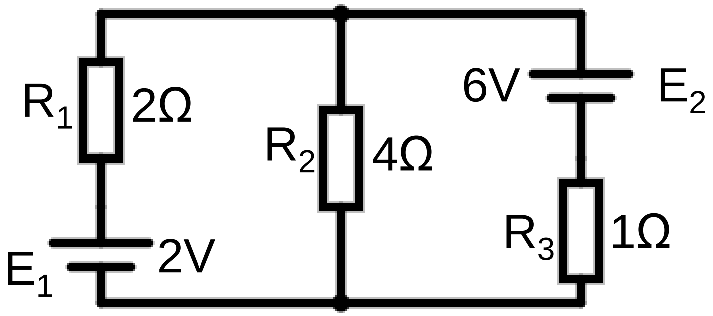
   If a resistor has several assumed currents:

   $I_{Total} = I_{current\ of\ the\ loop} + I_{same\ directions} - I_{opposite\ direction}$
4. Solve the resulting simultaneous linear equations for the assumed loop currents

Number of unknowns is the same as number of equations!

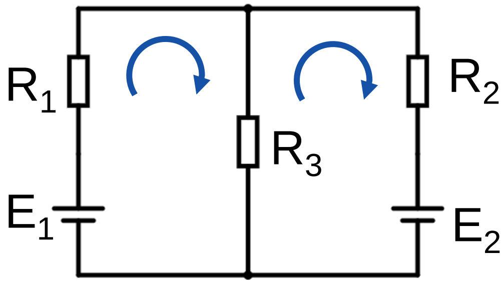
Example

Apply mesh analysis



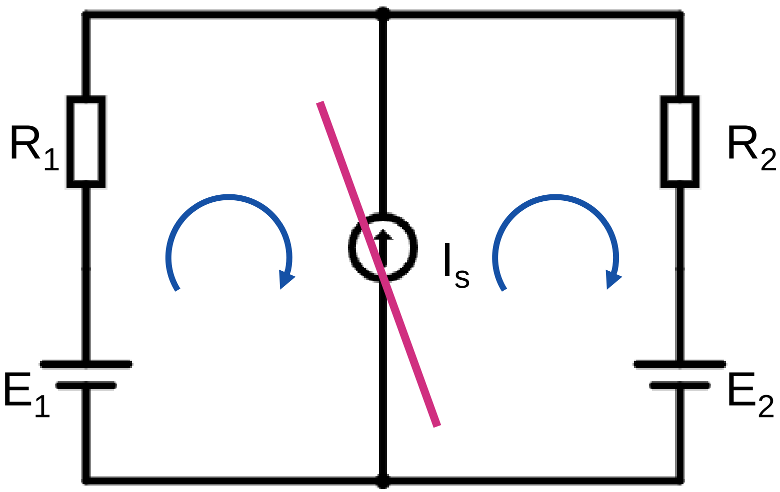
1. *Assign a current direction*
2. *Indicate the polarities*
3. *Apply Kirchhoff's voltage law*
4. *Solve equations*

Supermesh



Mesh method:

$$\begin{cases} E_1 - R_1 * I_1 - R_3(I_1 - I_2) = 0 \\ -R_3(I_2 - I_1) = -R_2 * I_2 - E_2 = 0 \end{cases}$$



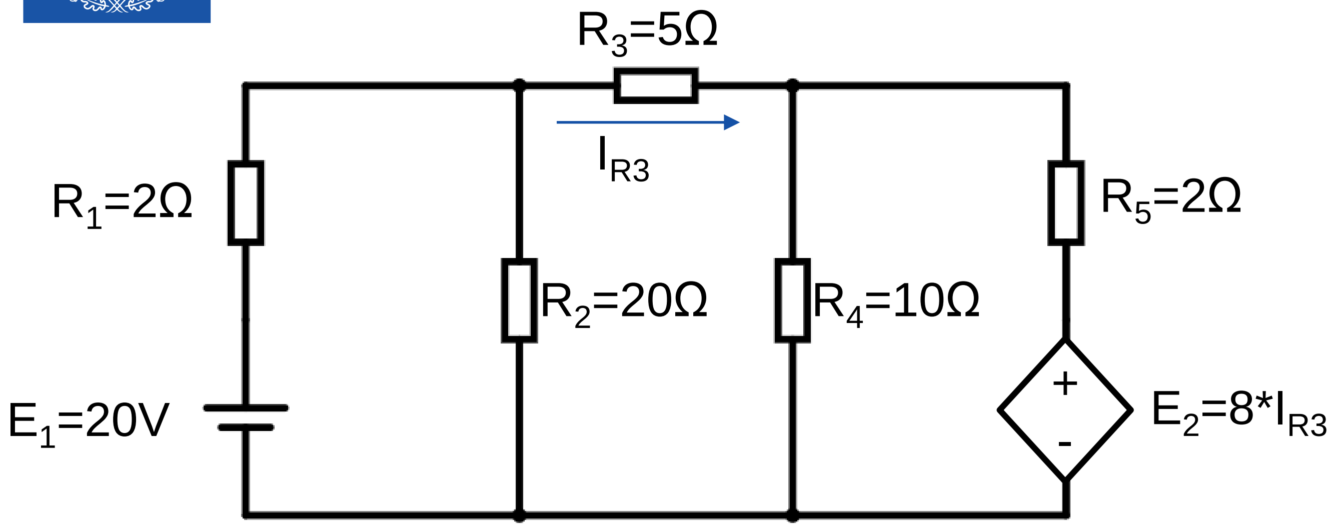
Supermesh method:

Open connection instead of current source: $I_1 - I_2 + I_s = 0$

$$\begin{cases} E_1 - R_1 * I_1 - R_2 * I_2 - E_2 = 0 \\ I_1 - I_2 + I_s = 0 \end{cases}$$

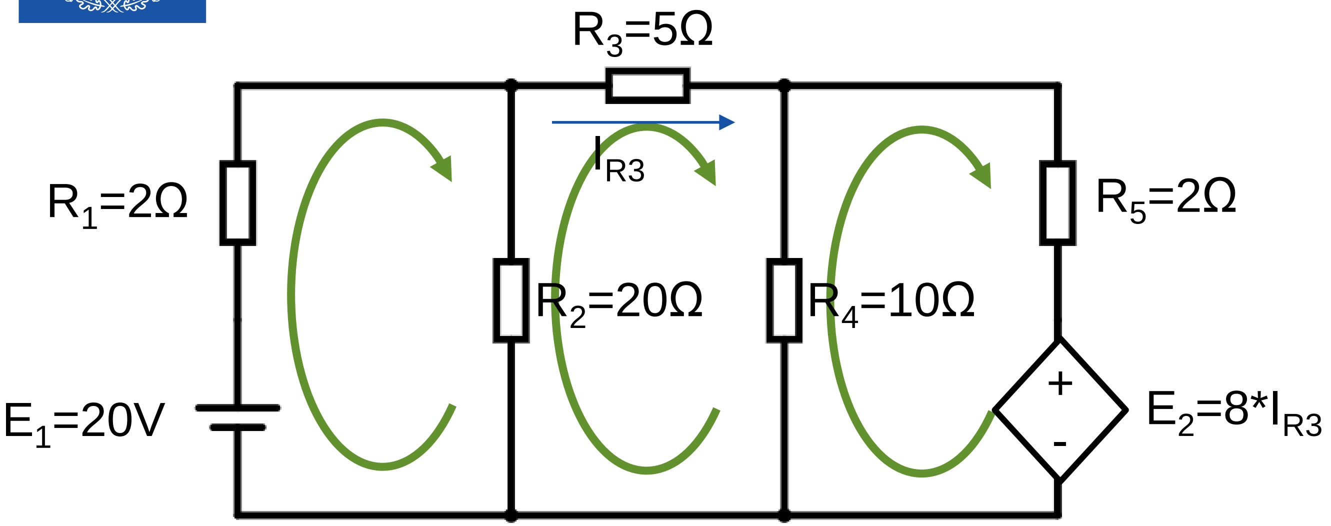
Exercise

Find voltage across R_3 in circuit with depended source



Exercise – Mesh analysis

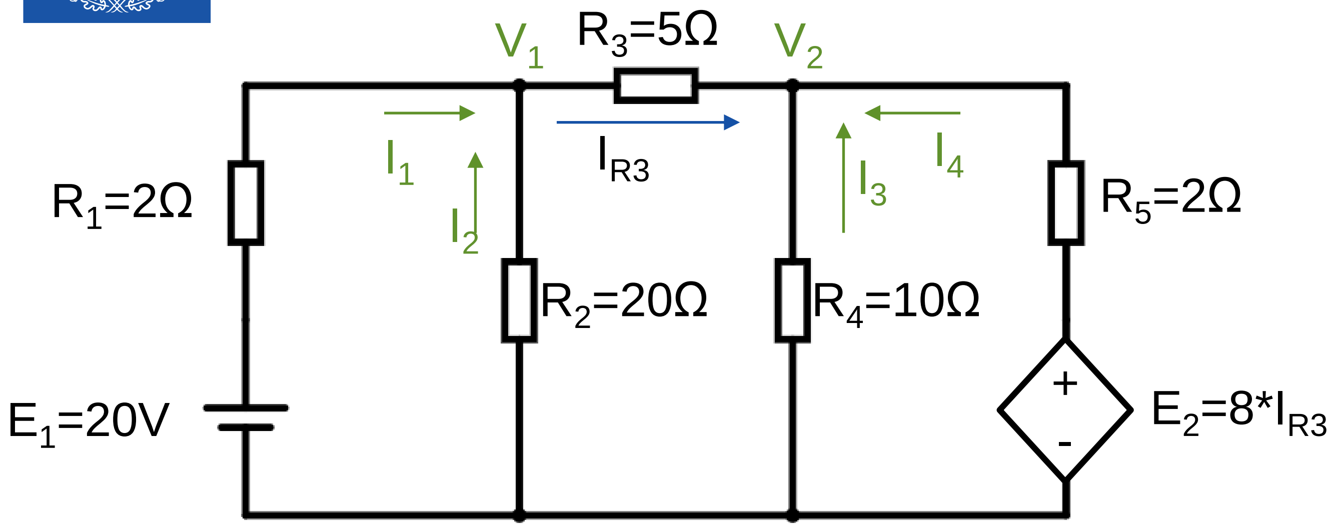
Find voltage across R_3 in circuit with depended source



$$
\begin{cases}
20-2*I_1-20(I_1-I_2)=0 \\
-20(I_2-I_1)-5*I_2-10(I_2-I_3)=0 \\
-10(I_2-I_3)-2*I_3-\mathbf{8*I_2}=0
\end{cases}
$$

Exercise – Nodal analysis

Find voltage across R_3 in circuit with depended source



$I_1+I_2-I_{R3}=0$

$I_1=(V_1+20)/2$
$I_2=V_1/20$
$I_{R3}=(V_1-V_2)/5$

$I_{R3}+I_3+I_4=0$

$I_{R3}=(V_1-V_2)/5$
$I_3=V_2/10$
$I_4=(V_2+8*I_{R3})/2=(V_2+\mathbf{8*[(V_1-V_2)/5]})/2$

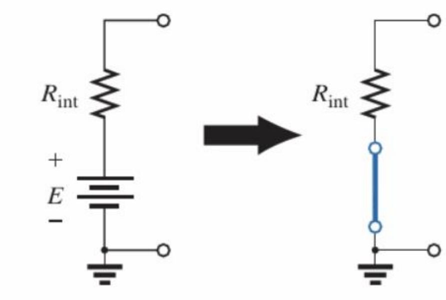
Superposition Theorem

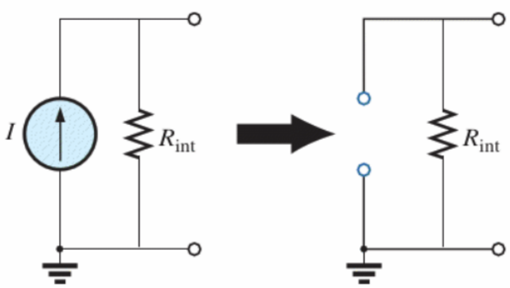
- Used to analyse networks that have two or more sources that are not in series or parallel

- The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source

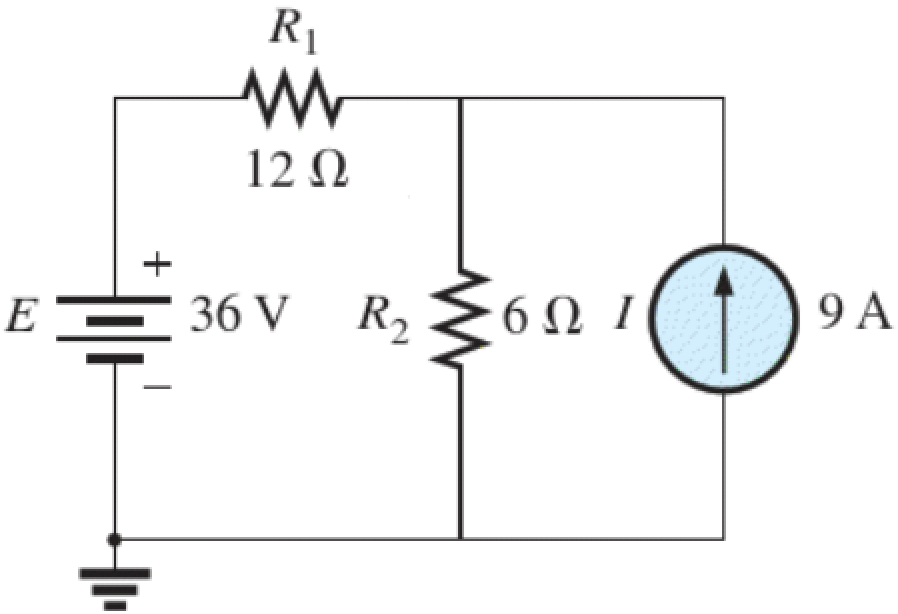
- This theorem allows us to find a solution for a current or voltage using only one source at a time
  - by replacing voltage sources with direct connection (zero ohms)
  - by replacing current sources with open connection (infinite ohms)

- Since the effect of each source will be determined independently, the number of networks to be analysed will equal the number of sources



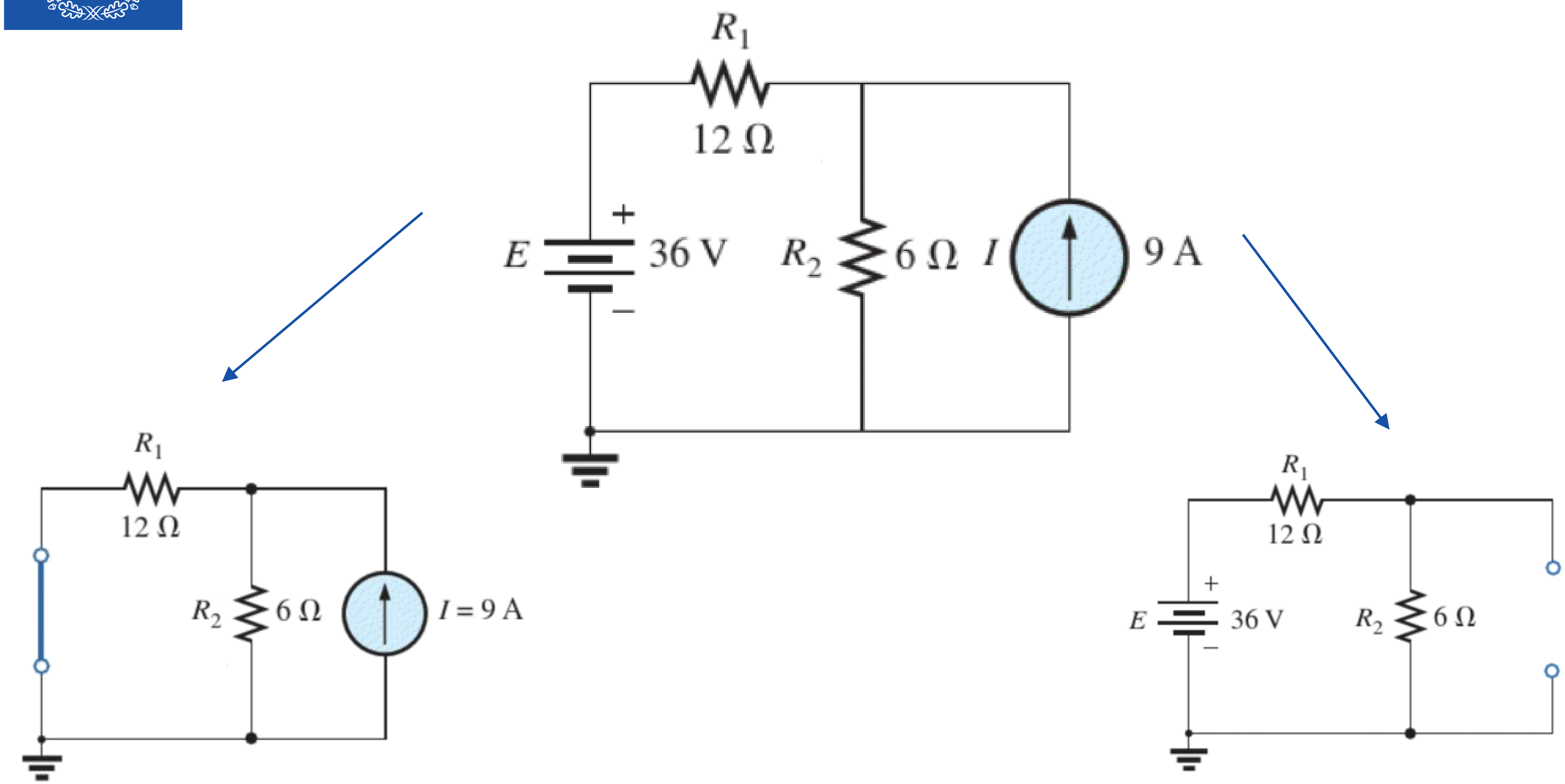


Example (Superposition Theorem)



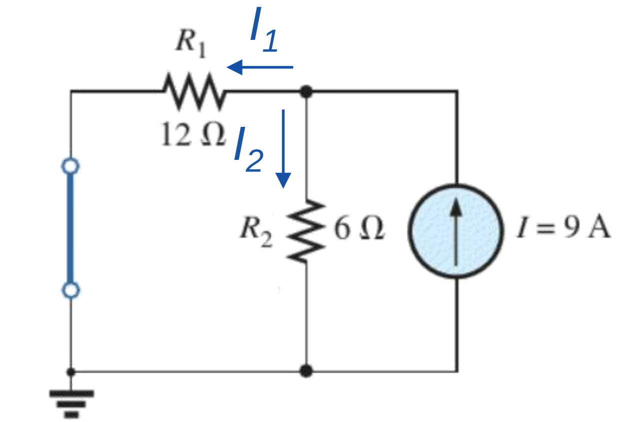
Determine current through R_1 and R_2

Example (Superposition Theorem)



Determine current through R_1 and R_2

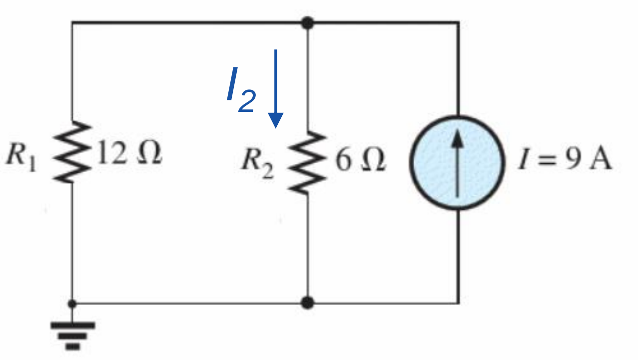
Example (Superposition Theorem)



$I = I_1 + I_2 \quad V = V_1 = V_2 \quad R_T = (R_1 * R_2)//(R_1 + R_2)$

$R'_T = (12*6)//(12+6) = 72/18 = 4$

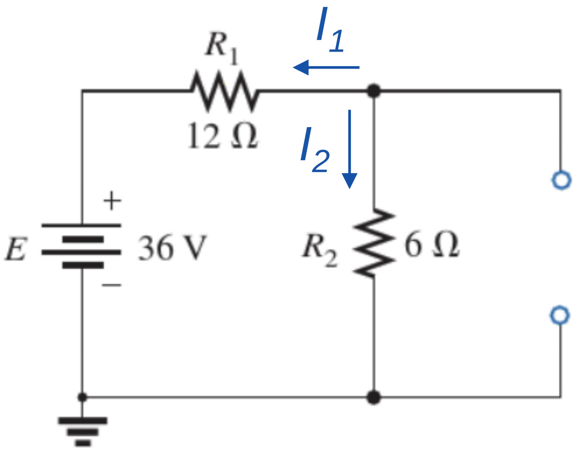
$V' = I * R_T = 9*4 = 36V$



$I'_1 = V/R_1 = 36/12 = 3A$

$I'_2 = v/R_2 = 36/6 = 6A$

Example (Superposition Theorem)



$I=I_1=I_2 \quad V=V_1+V_2 \quad R_T=R_1+R_2$

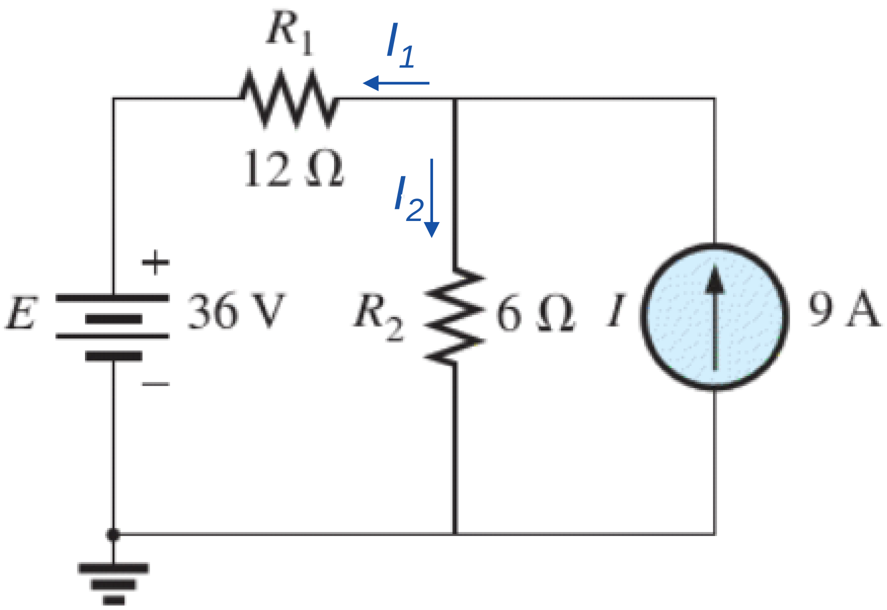
$R''_T=12+6=18$

$I''=36/18=2A$

$I''_1=2A$

$I''_2=2A$

Example (Superposition Theorem)

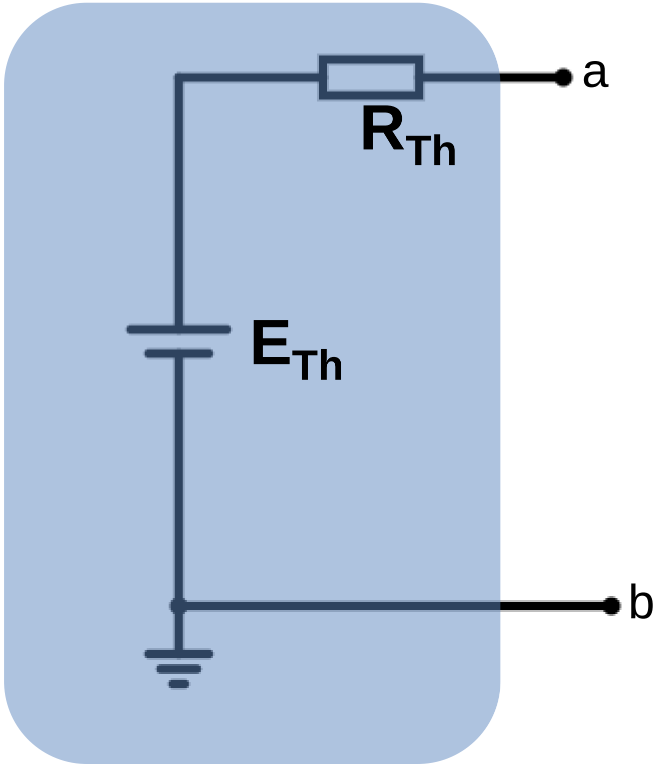


$I_1 = I'_1 + I''_1 = 3 - 2 = 1A$ (different polarity)

$I_2 = I'_2 + I''_2 = 6 + 2 = 8A$

Thévenin's Theorem

- Any two-terminal network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor



Thévenin's Theorem Procedure

Preliminary

1. Remove that portion of the network where the Thévenin equivalent circuit is found
2. Mark the terminals of the remaining two-terminal network

R_{Th}

3. Set all sources to zero: voltage sources are replaced by short circuits and current sources by open circuits
4. Calculate R_{Th} by finding the resultant resistance between the two marked terminals

E_{Th}

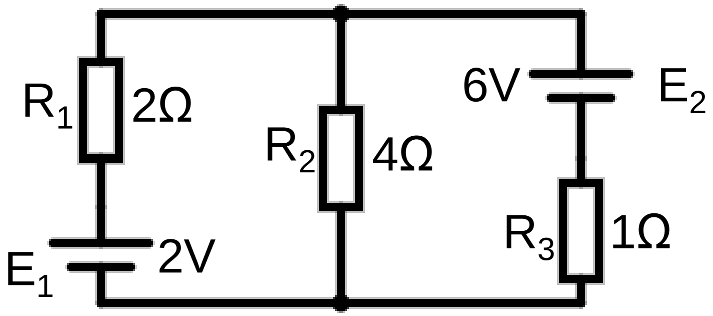
5. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals

Conclusion

6. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

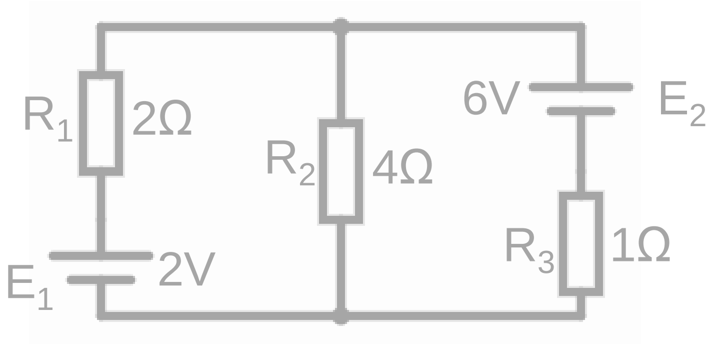
Example

Find current I_3 using Thévenin's Theorem - ***Preliminary***



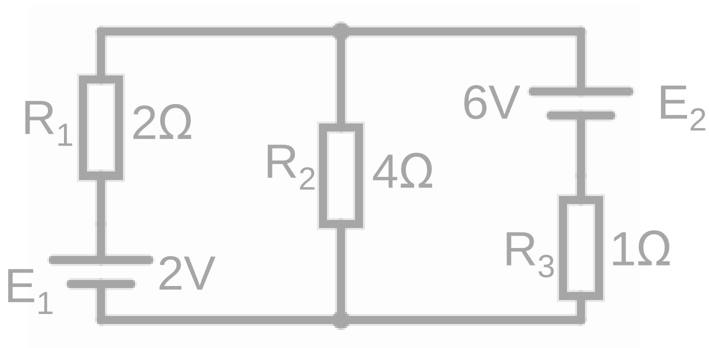
Example

Find current I_3 using Thévenin's Theorem - $\boldsymbol{R_{Th}}$



Example

Find current I_3 using Thévenin's Theorem - $\boldsymbol{E_{Th}}$

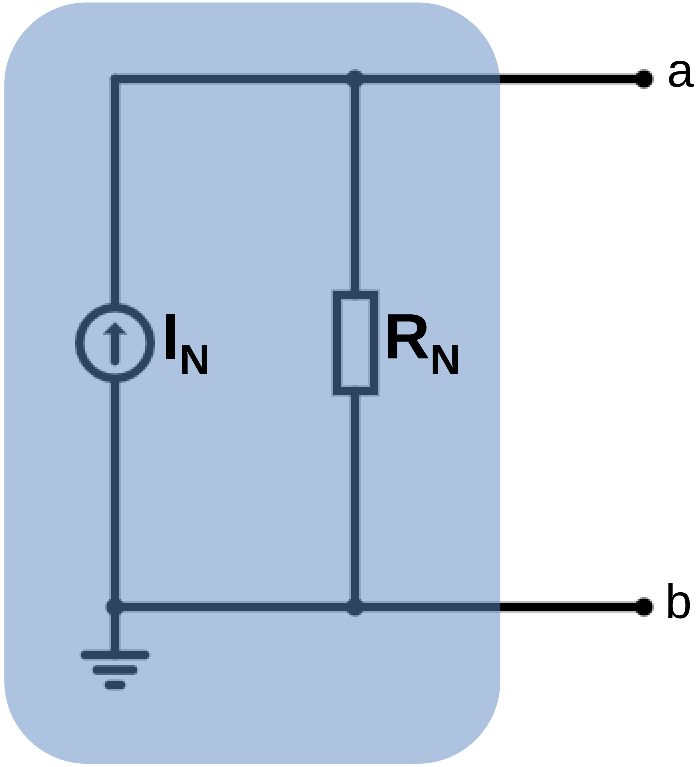


Example

Find current I_3 using Thévenin’s Theorem - ***Conclusion***

Norton's Theorem

- Any two-terminal linear network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor



Northon’s Theorem Procedure

Preliminary

Remove that portion of the network across which the Norton equivalent circuit is found

Mark the terminals of the remaining two-terminal network

R_N

Calculate R_N by first setting all sources to zero and then finding the resultant resistance between the two marked terminals

I_N

Calculate I_N by first returning all sources to their original position and then finding **the short-circuit current** between the marked terminals

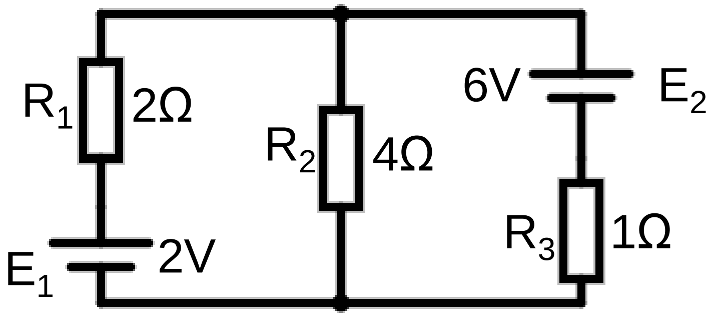
Conclusion

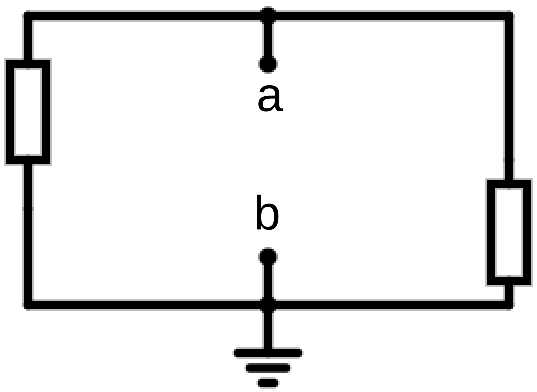
Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit

Example

Find current I_3 using Northon's Theorem - ***Preliminary***

Same as Thévenin's theorem

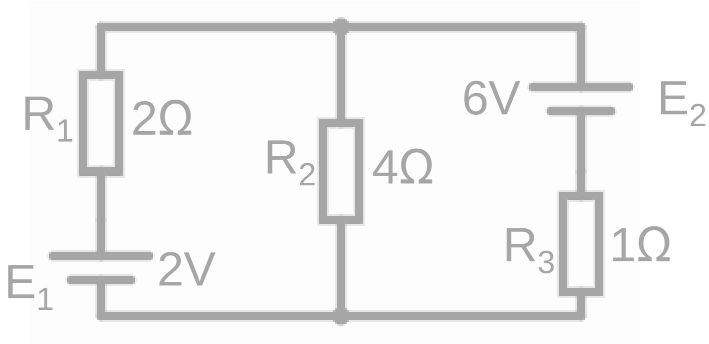


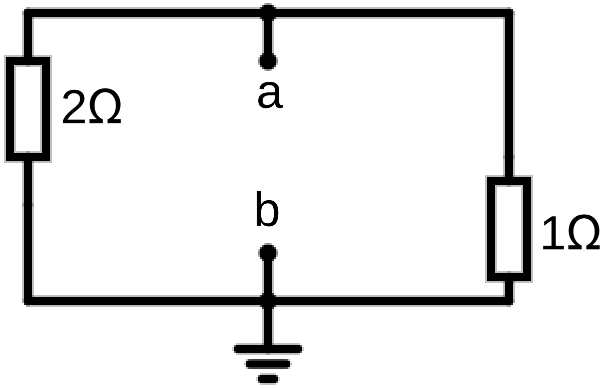


Example

Find current I_3 using Northon's Theorem - $\boldsymbol{R_N}$

Same as Thévenin's theorem

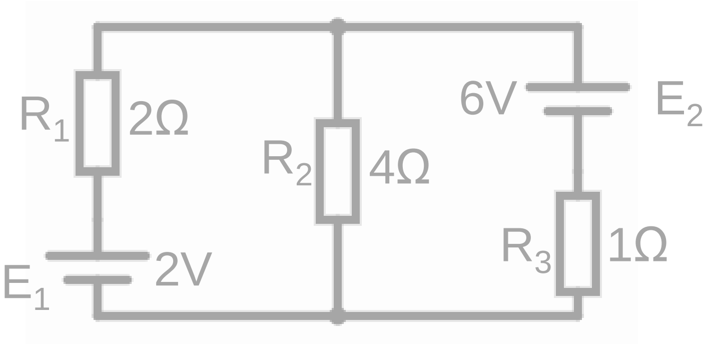




$R_N = R_1 // R_2 = 2//1 = 0.667\Omega$

Example

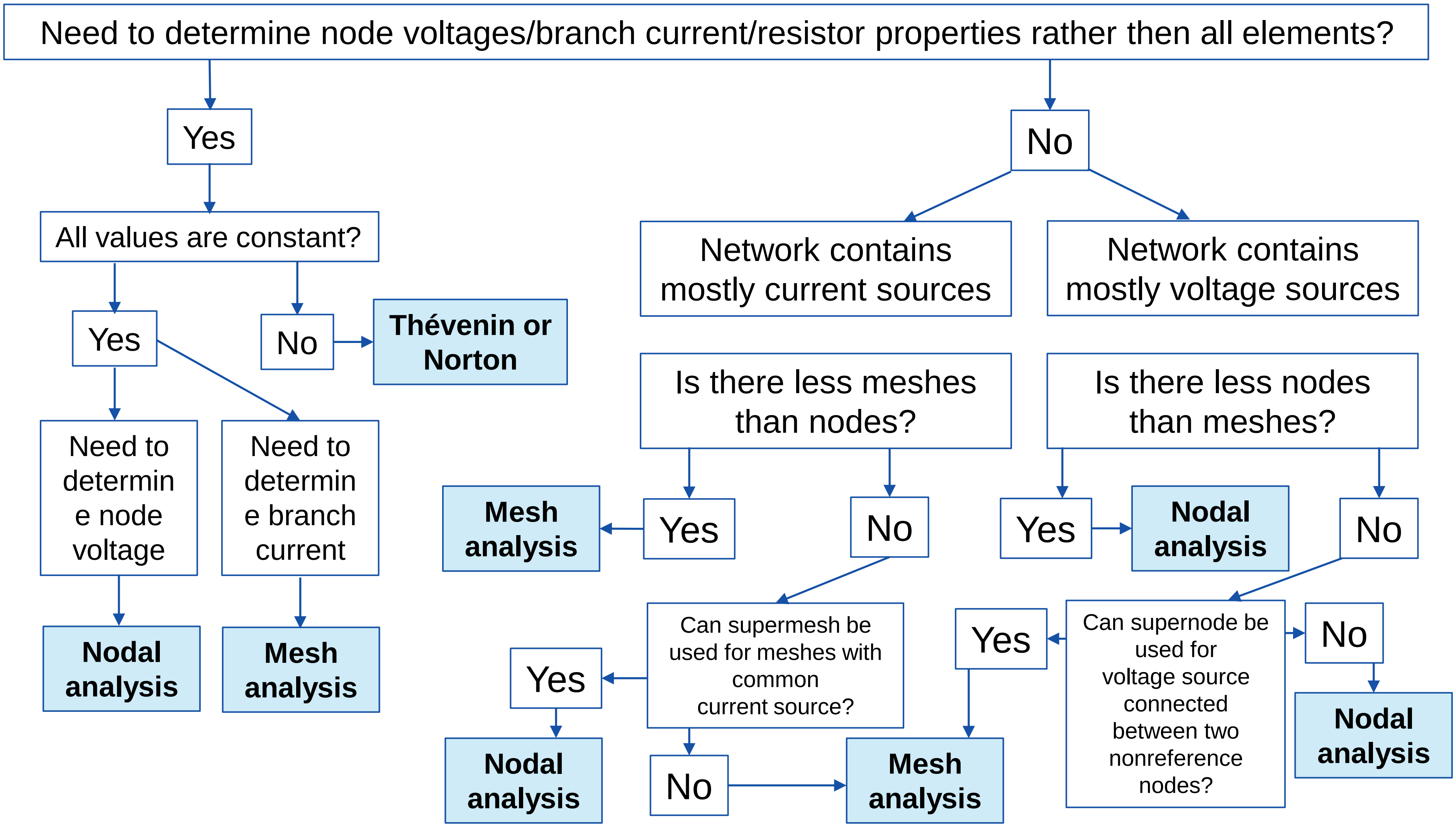
Find current I_3 using Northon's Theorem - $\boldsymbol{I_N}$



Example

Find current I_3 using Northon's Theorem - ***Conclusion***

Choosing Analysis Method



Thévenin’s theorem with depended source

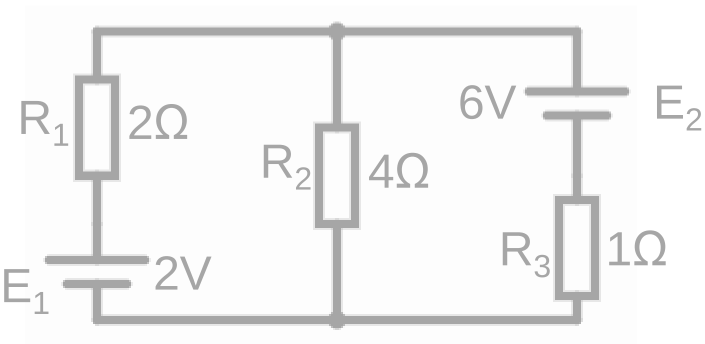
- Depended source cannot be ignored in calculating R_{Th} or R_N because the source affects the network!

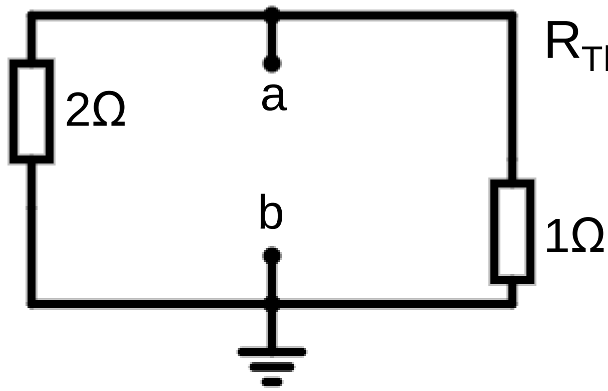
- Alternative 1: Since we still can calculate E_{Th} and I_N, $R_{Th}=E_{Th}/I_N$

- Alternative 2: Apply a current source or voltage source at the open terminal
  - If a voltage source is used, assume value of 1V for simple calculations and use mesh analysis. $R_{Th}=1V/I_0$
  - If a current source is used, assume value of 1A for simple calculations and use nodal analysis. $R_{Th}=V_0/1A$

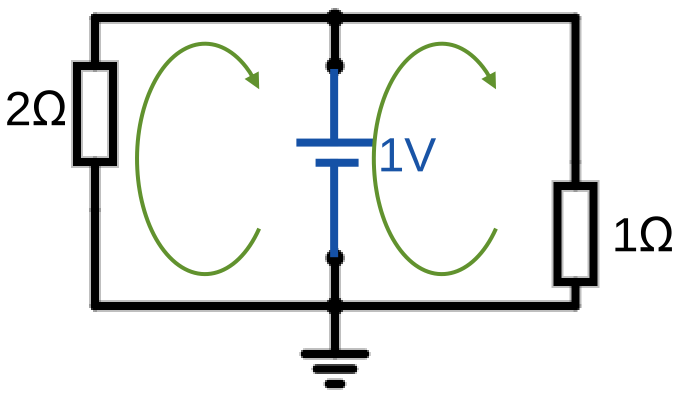
Example

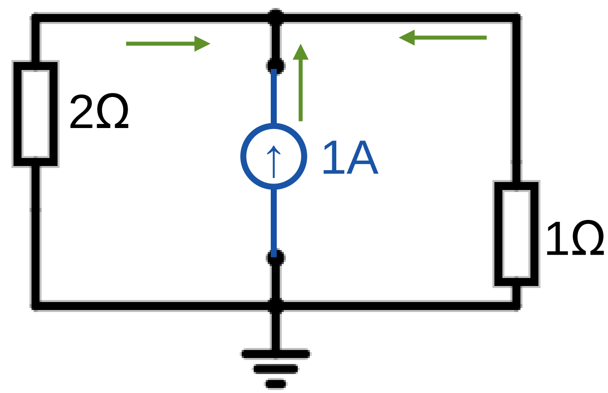
Testing alternative 2





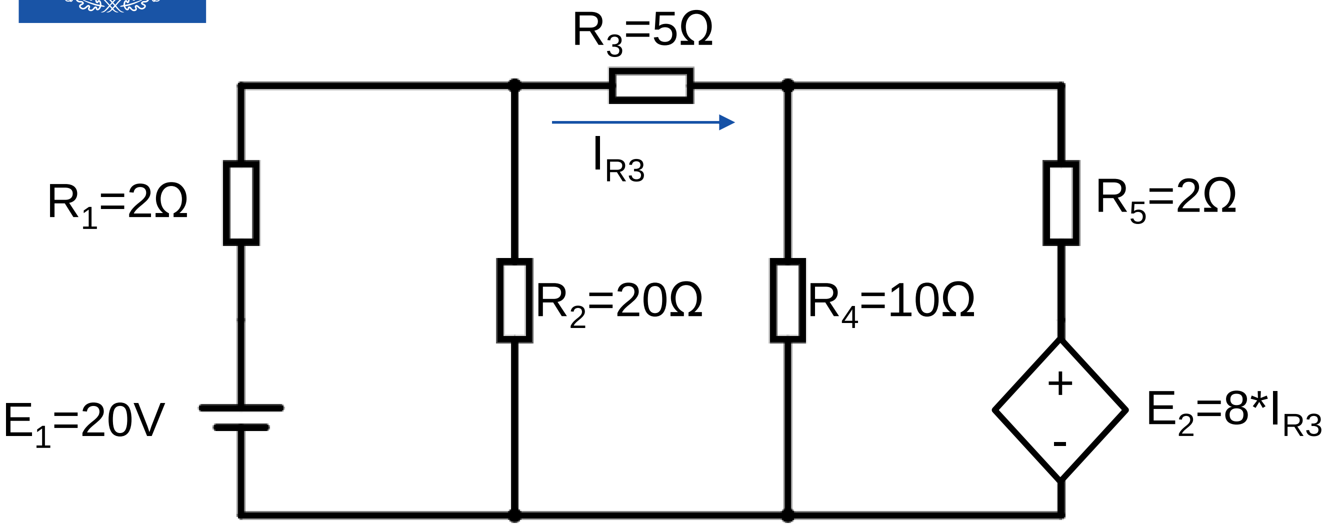
$R_{Th}=R_1//R_2=2//1=0.667\Omega$





Example

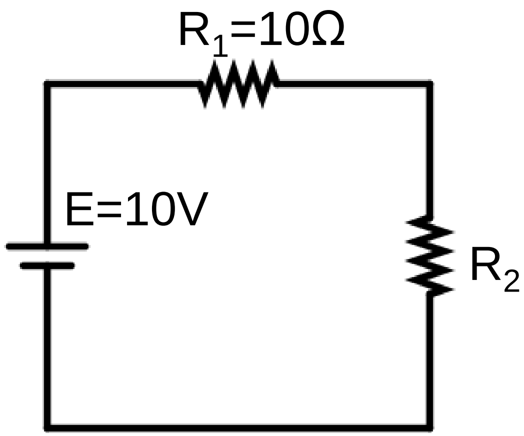
Create equivalent circuit for R_4 using Thévenin's or Northon's Theorem



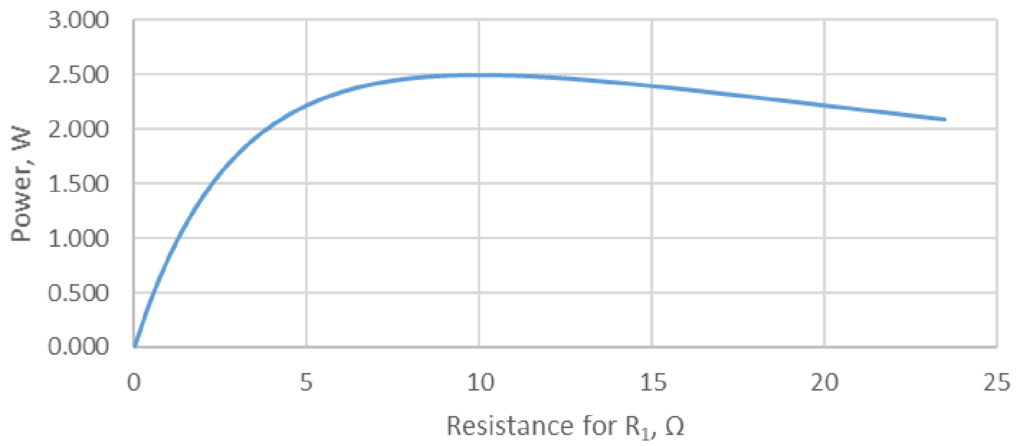
Example

Create equivalent circuit for R_4 using Thévenin’s or Northon’s Theorem

Power vs Resistance



R_2	$I=E/(R_1+R_2)$	$P_2=I^2R_2$
0.01Ω	0.999A	0.010W
1Ω	0.909A	0.826W
5Ω	0.667A	2.222W
10Ω	0.500A	2.500W
15Ω	0.400A	2.400W
1000Ω	0.010A	0.098W

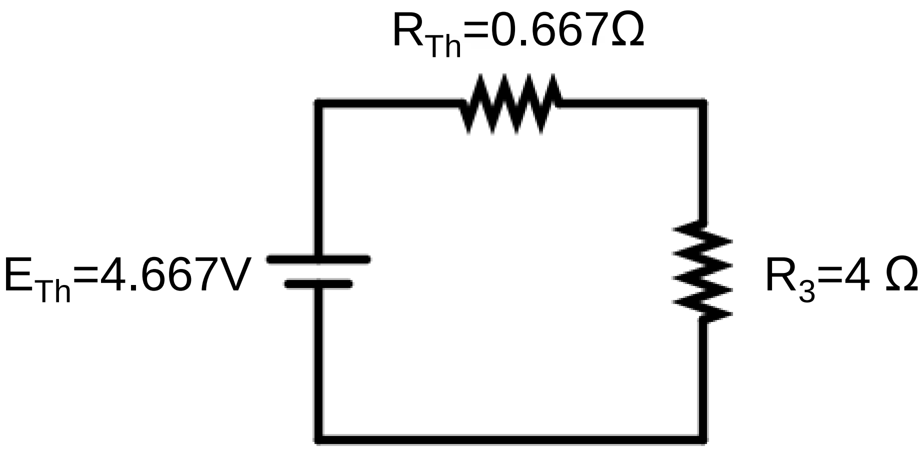


Maximum Power Transfer Theorem

A load will receive maximum power from a network when its resistance is exactly equal to the Thévenin resistance of the network applied to the load

- $I = \dfrac{E_{Th}}{R_{Th}+R_L} = \dfrac{E_{Th}}{R_{Th}+R_{Th}} = \dfrac{E_{Th}}{2*R_{Th}}$
- $P = I^2 R_L = \left[\dfrac{E_{Th}}{2*R_{Th}}\right]^2 R_{Th} = \dfrac{E^2_{Th}*R_{Th}}{4*R^2_{Th}} = \dfrac{E^2_{Th}}{4*R_{Th}}$

Maximum Power Transfer Theorem



Power with R_3=4Ω is 4W

Maximum power is when R_3=0.6667Ω

Maximum power = $4.667^2/(4*0.667)=21.781/2.668=8.164$W

Efficiency $\eta=P/P_{max}=4/8.164=0.489=48.9\%$

Suggested reading

Introductory Circuit Analysis

- Kap 8: **8.4**, 8.5, **8.6**, 8.7, 8.10 - 8.11
- Kap 9: **9.2 - 9.5**

*suggestions in bold are more important

Suggested exercises

- Mesh analysis (kapital 8): 21, 23, 30, 31
- Nodal analysis (kapital 8): 41, 43, 50, 51
- Superposition Theorem (kapital 9): 1, 3, 7
- Thévenin’s Theorem (kapital 9): 9, 11, 15, 21
- Norton’s Theorem (kapital 9): 23, 25, 27, 29
- Maximum Power Transfer Theorem (kapital 9): 31, 33, 35