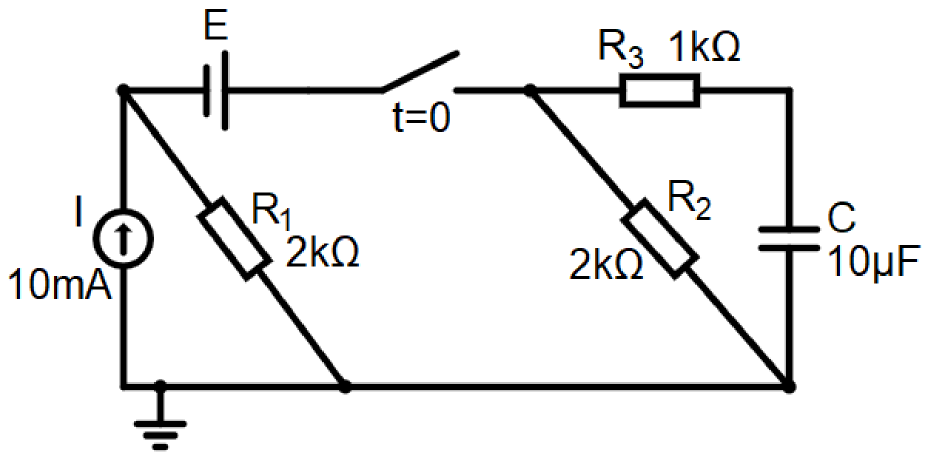
Kontrollskrivning 2 i HE1027 Ellära med svar
Fredag 20 maj 2021, kl. 10:00 – 12:00

Uppgift 1 [3p]

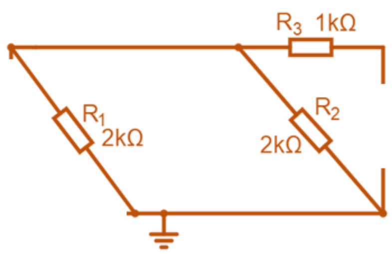
Spänningskällan *E* ger likspänningen 10V och strömkällan *I* ger strömmen 10mA. Kondensatorn är energitom för t < 0. Kontakten sluts vid tiden t=0. Bestäm spänningen över kondensatorn $v_C(t)$ och strömmen genom denna $i_C(t)$ för t=20ms.

The voltage source *E* gives the direct voltage 10V and the current source *I* gives the current 10mA. The capacitor is empty of energy for t <0. The contact closes at time t=0. Determine the voltage v*C*(t) and the current i*C*(t) for t=20ms.



Svar

The circuit is too complex to solve in standard way, especially because we have some parallel connection to the capacitor. It means we need to find an equal circuit using Thevenin theorem.



R_{Th}=R1//R2+R1=2kΩ

Next to calculate E_{Th}. We can transform current source to the voltage source with value 10mA*2kΩ=20V, and then using voltage division rule E_{Th}=(20+10)/2=15V.

Now we simplified circuit to just voltage source (15V), a resistor (2kΩ) and a capacitor (still 10μF).

$\tau = RC = 2k * 10\mu = 20ms$

$$v_c(t) = \mathrm{E}\left(1 - \mathrm{e}^{-\frac{t}{\tau}}\right) = 15\left(1 - \mathrm{e}^{-\frac{t}{0.02}}\right)$$

$$v_c(0.02) = 15\left(1 - \mathrm{e}^{-\frac{0.02}{0.02}}\right) = 9.48\mathrm{V}$$

The current over the capacitor will be equal with to voltage difference at this time point divided by the total resistance. Voltage difference is how much the source produces minus the voltage of the capacitor. The total resistance is 2kΩ.

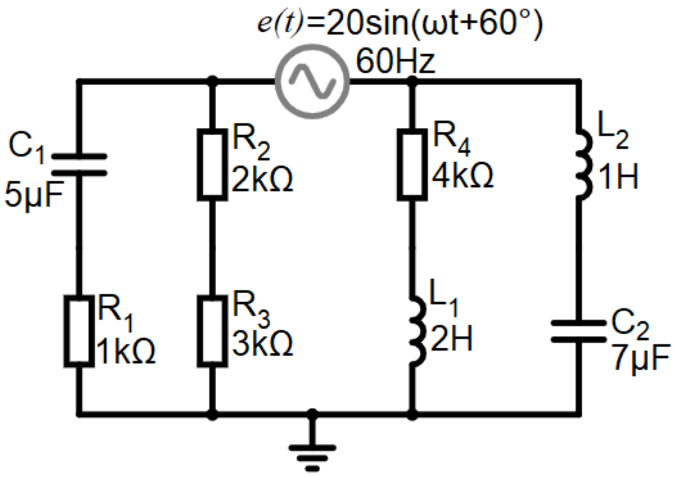
$$i_C(t) = \frac{\mathrm{E} - 15\left(1 - \mathrm{e}^{-\frac{t}{0.02}}\right)}{2000} = \frac{15 - 15 + 15\mathrm{e}^{-\frac{t}{0.02}}}{2000} = 7.5\mathrm{e}^{-\frac{t}{0.02}}\mathrm{mA}$$

$$i_C(0.02) = 2.7\mathrm{mA}$$

Uppgift 2 [3p]

Bestäm spänningen över kondensatorn C_1 $v_{C_1}(t)$.

Determine voltage over capacitor C_1 $v_{C_1}(t)$.



Svar

It is important to remember that all branches are not in the parallel: $Z_{first}//Z_{second}+Z_{third}//Z_{fourth}$. So we cannot assume that voltage is the same everywhere.

Lets transform all values to phasor form:

$$\mathrm{E} = \frac{20}{\sqrt{2}}\angle 60° = 14.14\angle 60°$$

$$X_{C_1} = \frac{1}{2\pi f C} = \frac{1}{2\pi * 60 * 5\mu} = 530.52\Omega$$

$$X_{L_1} = 2\pi f L = 2\pi * 60 * 2 = 753.98\Omega$$

$$X_{C_2} = \frac{1}{2\pi f C} = \frac{1}{2\pi * 60 * 7\mu} = 378.94\Omega$$

$$X_{L_2} = 2\pi f L = 2\pi * 60 * 1 = 376.99\Omega$$

Now, we can assume that X_{C2} is the same as X_{L2} (there difference is too small comparing to other numbers), so they cancel each other and it means that the branch has no resistance. Then voltage on the first branch is the same as the voltage source. Hence,

$$v_{C_1} = E\frac{Z_{C_1}}{Z_{C_1} + R_1} = 14.14\angle 60° \frac{530.52\angle -90°}{1000 - 530.52i} = 14.14\angle 60° \frac{530.52\angle -90°}{1132.02\angle 27.95°}$$

$$v_{C_1} = \frac{14.14 * 530.52}{1132.02}\angle(60 - 90 - 27.95) = 6.63\angle -57.95° = 9.38\sin(\omega t - 57.95°)$$

If we did not assumed that X_{C2} is the same as X_{L2}, then based on voltage division rule

$$V_{\text{first branch}} = V_{\text{second branch}} = E\frac{Z_{\text{first}}//Z_{\text{second}}}{Z_{\text{first}}//Z_{\text{second}} + Z_{\text{third}}//Z_{\text{fourth}}}$$

$$V_{\text{first}} = 14.14\angle 60° \frac{(1000 - 530.52i)//(2k + 3k)}{(1000 - 530.52i)//(2k + 3k) + (4000 + 753.98i)//(376.99i - 378.94i)}$$

$$V_{\text{first}} = 14.14\angle 60° \frac{865.66 - 365.57i}{865.66 - 365.57i + 0.001 - 1.95i} = 14.14\angle 60° \frac{865.66 - 365.57i}{865.66 - 367.52i}$$

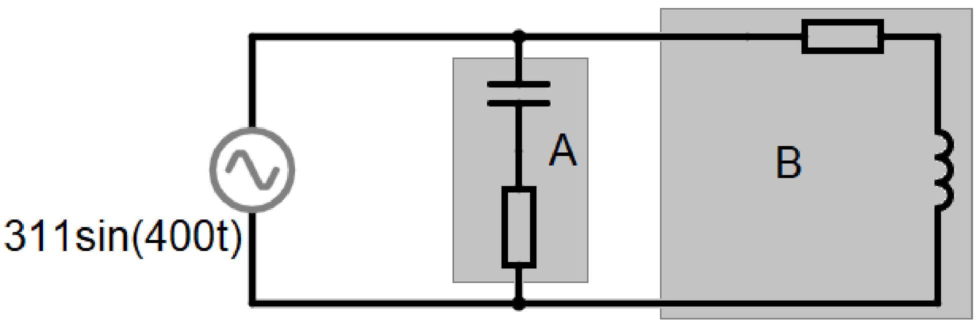
$$V_{\text{first}} = 14.14\angle 60° \frac{939.69\angle 22.89°}{940.42\angle 23°} = 14.13\angle 59.89°$$

$$v_{C_1} = 14.13\angle 59.89° \frac{530.52\angle -90°}{1132.02\angle 27.95°} = 6.62\angle -58.06° = 9.36\sin(\omega t - 58.06°)$$

Uppgift 3

Beräkningar skall utföras på nedanstående krets. Aktiva effekt i del A är 10kW och reaktiva effect är 5kVAR. Aktiva effekt i del B är 20kW och effektfaktorn cosθ är 0.6.

Calculations must be performed on the circuit below. Active power in part A is 10kW and reactive power is 5kVAR. Active power in part B is 20kW and the power factor cosθ is 0.6.



1) Bestäm aktiva, reaktiva och skenbara effekten och effektfaktorn i del 1 och del 2 [1p]
2) Bestäm totala aktiva, reaktiva och skenbara effekten. [1p]
3) Bestäm ett element och dess värde för att höja effektfaktorn till 1. [1p]

1) Determine the active, reactive and apparent powers and power factors in part 1 and part 2 [1p]
2) Determine the total active, reactive and apparent powers and power factor [1p]
3) Determine an element and its value to raise power factor to 1. [1p]

Svar

Del A:

P=10kW

Q=5kVAR (capac)

$S = \sqrt{P^2 + Q^2} = 11.18\text{kVA}$

cosθ=P/S=0.89 (capac)

Del B:

P=20kW

cosθ=0.6 (induct)

S=P/cosθ=33kVA

$Q = S \cdot \sin(\cos^{-1}(\cos\theta)) = 26.6\text{kVAR}$ or

$Q = \sqrt{S^2 - P^2} = 26.6\text{kVAR}$

Total:

P=PA+PB=10+20=30kW

Q=-QA+QB=-5+26.6=21.6kVAR (minus because QA is capacitive)

$S = \sqrt{P^2 + Q^2} = 37\text{kVA}$

cosθ=P/S=0.81 (induct)

Power factor correction:

Effective voltage is $\frac{311}{\sqrt{2}} = 220\text{V}$

Since power factor is 1, it means we need to remove all Q_{Total}

$$X_C = \frac{V^2}{Q} = \frac{220^2}{21.6\text{k}} = 2.23\Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{400 \cdot 2.23} = 1.12\text{mF}$$