Tentamen med svar

Kurs: **Kursnummer:**	Ellära HE1027
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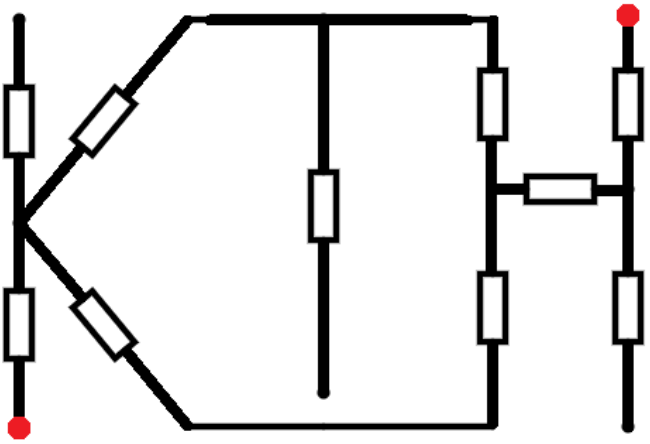
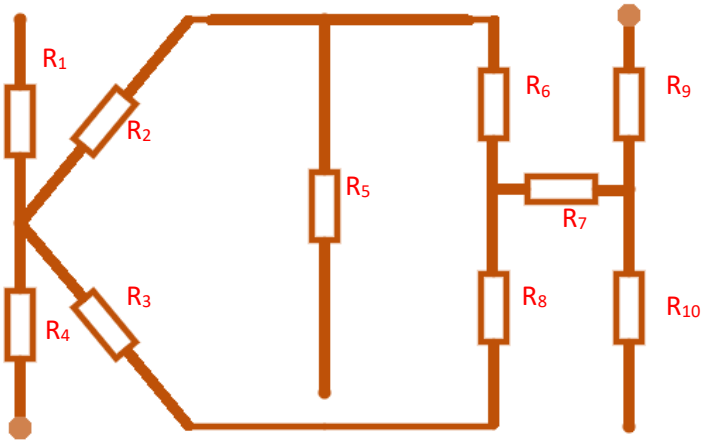
Datum: **Tid:**	2021-05-31 kl. 14:00 – 18:00
Hjälpmedel:	Räknare, utdelat formelblad och 2 sidor (ett blad) med anteckningar.
Omfattning och betygsgränser	Tentamen består av 7 uppgifter. För godkänt krävs 10 poäng av totalt 24 möjliga poäng. Betygsgränser: För betyg A, B, C, D, E krävs 22, 19, 16, 13 respektive 10 poäng. För betyget Fx krävs 9 poäng. *Fx är underkänt betyg, men med möjlighet till komplettering. Om kompletteringen är godkänd rapporteras betyget E, annars rapporteras F.*
Övrig information:	-Namn och personnummer skall anges på varje inlämnat skrivpapper. - Blad- och uppgiftsnummer skall anges på varje inlämnat skrivpapper. - Lämna endast in en uppgift på varje papper. - Redovisade lösningar skall vara fullständiga och lätta att följa. - Tydliga kommentarer.

Uppgift 1 [2p]

Beräkningar skall utföras på nedanstående krets. Bestäm resistansen mellan röda punkter om alla resistorna är 1kΩ.

Analyse a circuit below. Determine the resistance between red points if all resistors are 1kΩ.



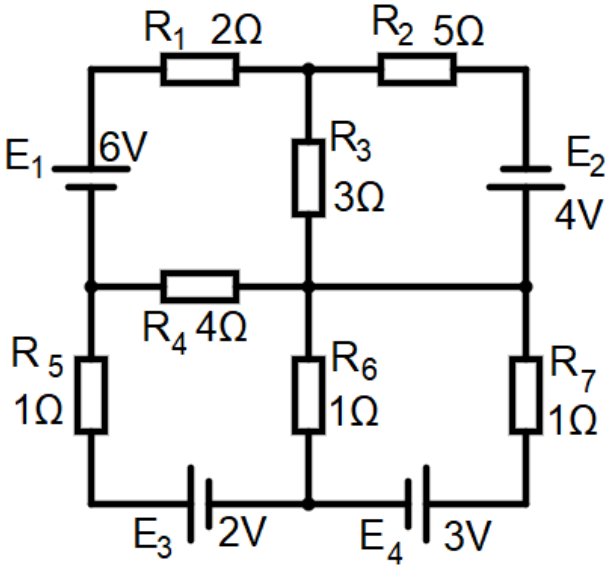


Resistors R1, R5 and R10 are open circuited, so we can ignore them. Then we have that $R_{total}=R_4+(R_2+R_6)//(R_3+$ $+R_8)+R_7+R_9=1+2//2+2=$ $=1+1+2=4k\Omega$

Uppgift 2 [4p] (Denna uppgift kan du som är godkänd på KS-1 hoppa över)

Bestäm strömmarna genom alla spänningskällor.

Determine current through all voltage sources.



Since we need to find current, mesh analysis is the most logical choice.

Equation for the first loop (top left):

$$E_1 - R_1 * I_1 - R_3 * (I_1 - I_2) - R_4 * (I_1 - I_3) = 0$$
$$6 - 2I_1 - 3(I_1 - I_2) - 4(I_1 - I_3) = 0$$
$$-9I_1 + 3I_2 + 4I_3 = -6$$

Equation for the second loop (top right):

$$-R_3 * (I_2 - I_1) - R_2 * I_2 + E_2 = 0$$
$$-3(I_2 - I_1) - 5I_2 + 4 = 0$$
$$3I_1 - 8I_2 = -4$$

Equation for the third loop (bottom left):

$$-R_5 * I_3 - R_4 * (I_3 - I_1) - R_6 * (I_3 - I_4) + E_3 = 0$$
$$-I_3 - 4(I_3 - I_1) - (I_3 - I_4) + 2 = 0$$
$$4I_1 - 6I_3 + I_4 = -2$$

Equation for the fourth loop (bottom right):

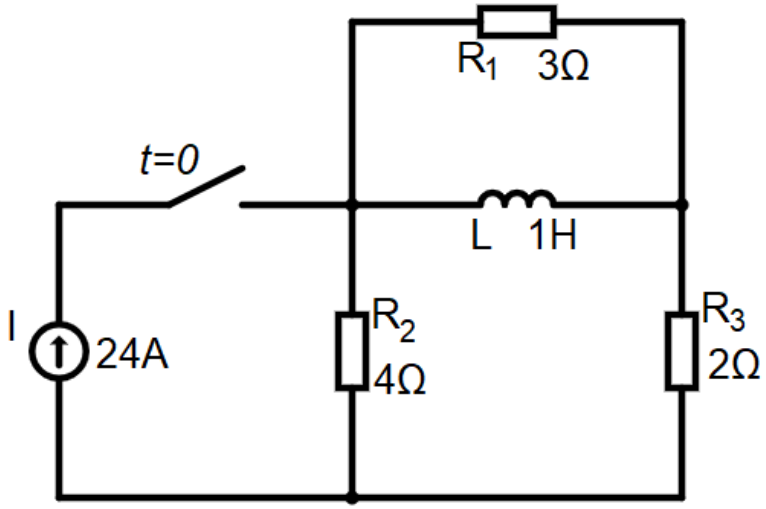
$$-R_6 * (I_4 - I_3) - R_7 * I_4 - E_4 = 0$$
$$-(I_4 - I_3) - I_4 - 3 = 0$$
$$I_3 - 2I_4 = 3$$

Solving the system of the equations, we are getting the final answers: I_1=1.584A, I_2=1.094A, I_3=1.243A, I_4=0.879A

Uppgift 3 [2p]

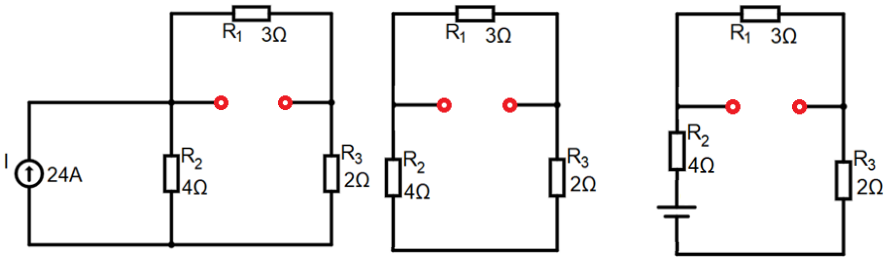
Vid tiden *t=0* sluts kontakten i kretsen bredvid. Spolen L är initialt oladdad. Ställ upp uttrycket för strömmen $i_L(t)$ genom spolen.

At time t=0, the switch in the adjacent circuit is closed. The inductor L is initially uncharged. Set up the expression for the current iL (t) through the inductor.



The circuit is too complex to solve as it is, and we need to find Thévenin equivalent circuit. We start by removing the inductor and determining R_{Th}. R_{Th}=R1//(R2+R3)=3//(4+2)=2. To find E_{Th}, we can covert current source to voltage source, with the value E=24A*4Ω=96V. Then E_{Th} is $E-E_{R2}+E_{R3}$:

$$E_{Th} = E - E_{R_2} + E_{R_2} = 96 - E\frac{R_2}{R_{total}} - E\frac{R_3}{R_{total}} = 96 - 96\frac{4}{9} + 96\frac{2}{9} = 32V$$



Now we can put all together:

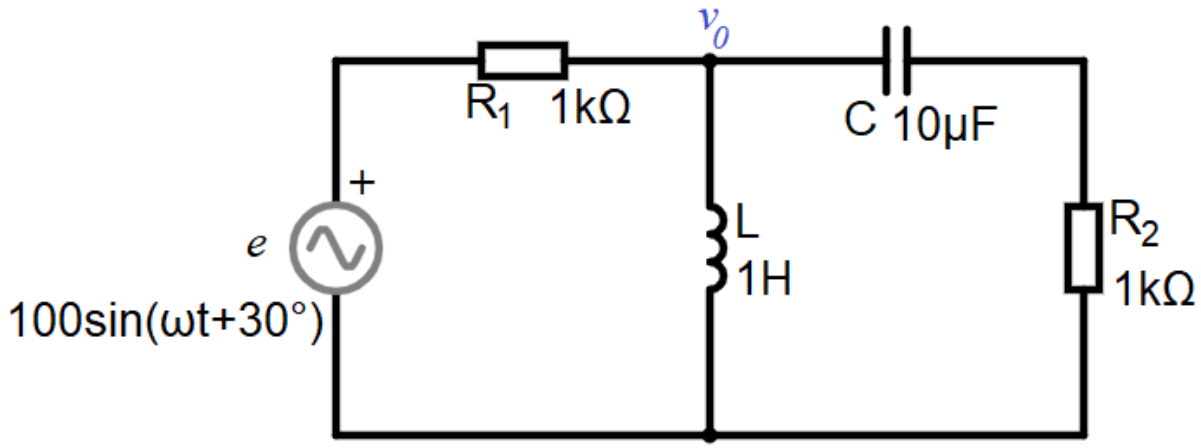
$$\tau = \frac{L}{R} = \frac{1}{2} = 0.5s$$

$$i_L(t) = I\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{E_{Th}}{R_{Th}}\left(1 - e^{-\frac{t}{0.25}}\right) = 16\left(1 - e^{-\frac{t}{0.5}}\right)$$

Uppgift 4 [4p] (Denna uppgift kan du som är godkänd på KS-2 hoppa över)

Bestäm spänningen v_0 (markerad i kretsen) med frekvensen f=$\frac{1}{\pi}$Hz, f=50Hz och f=50kHz.

Determine the voltage v0 (marked in the circuit) with the frequency f = $\frac{1}{\pi}$Hz, f = 50Hz and f = 50kHz.



First, let's find phasor form of voltage, and impedance of all components:

$$E = \frac{100}{\sqrt{2}}\angle 30° = 70.7\angle 30°$$

$$Z_{L_{f=\frac{1}{\pi}}} = 2\pi fL\angle 90° = 2\angle 90°,\ Z_{L_{f=50}} = 314\angle 90°,\ Z_{L_{f=50k}} = 314k\angle 90°$$

$$Z_{C_{f=\frac{1}{\pi}}} = \frac{1}{2\pi fC}\angle -90° = 50k\angle -90°,\ Z_{C_{f=50}} = 318.3\angle -90°,\ Z_{C_{f=50k}} = 0.3\angle -90°$$

Using nodal analysis:

$$-\frac{v_0 - E}{R_1} - \frac{v_0}{Z_L} - \frac{v_0}{Z_C + R_2} = -\frac{v_0 - 70.7\angle 30°}{1k} - \frac{v_0}{Z_L} - \frac{v_0}{Z_C + 1k} = 0$$

For f=$\frac{1}{\pi}$Hz: $-\frac{v_0 - 70.7\angle 30°}{1k} - \frac{v_0}{2\angle 90°} - \frac{v_0}{50k\angle -90° + 1k} = 0$, then v_0=0.071+0.122i=0.14∠59.80°

Alternatively, we can think that at low frequencies the inductor acts as a wire, and the capacitor is an open connection. So we can ignore capacitor and R_2. Hence v_0 is a ground point, so v_0=0V.

For f=50Hz: $-\frac{v_0 - 70.7\angle 30°}{1k} - \frac{v_0}{314\angle 90°} - \frac{v_0}{318.3\angle -90° + 1k} = 0$, then v_0=18.23+9.14i= 20.39∠26.62°

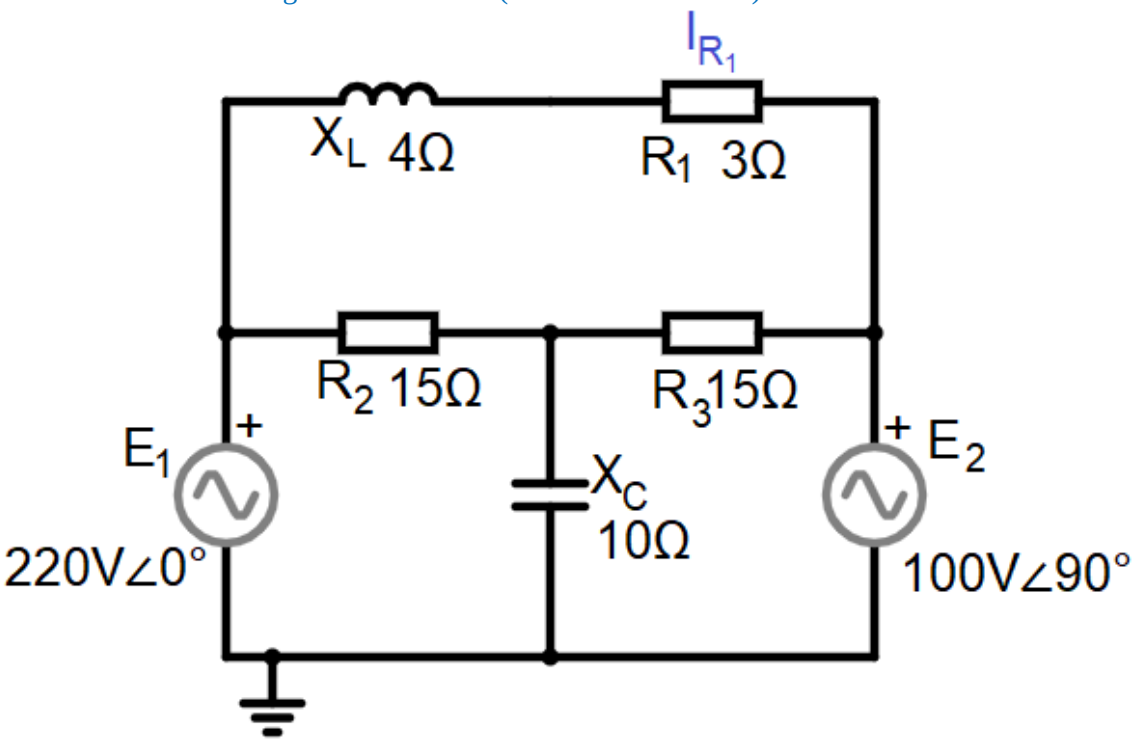
For f=50kHz: $-\frac{v_0 - 70.7\angle 30°}{1k} - \frac{v_0}{314k\angle 90°} - \frac{v_0}{0.3\angle -90° + 1k} = 0$, then v_0=30.64+17.63i=35.35∠29.92°

Alternatively, we can think that at low frequencies the inductor acts as an open connection, and the capacitor is a wire. So we can ignore the inductor, the only resistance are coming from two resistors with the same value, so v_0 is a half of voltage from the voltage source and v_0=35.35∠30°

Uppgift 5 [3p]

Bestäm strömmen genom resistoren R1 (markerad i kretsen).

Determine the current through the resistor R1 (marked in the circuit).



Since we need to find the current, the mesh method is the most logical.

For top loop: $-Z_L * I_1 - R_1 * I_1 - R_3 * (I_1 - I_3) - R_2 * (I_1 - I_2) = 0$

$$-4i * I_1 - 3 * I_1 - 15 * (I_1 - I_3) - 15 * (I_1 - I_2) = 0$$

For bottom left loop: $E_1 - R_2 * (I_2 - I_1) - Z_C(I_2 - I_3) = 0$

$$220 - 15(I_2 - I_1) + 10i(I_2 - I_3) = 0$$

For bottom right loop: $-Z_C * (I_3 - I_2) - R_3 * (I_3 - I_1) - E_2 = 0$

$$10i * (I_3 - I_2) - 15 * (I_3 - I_1) - 100i = 0$$

Solving the system of equations, we get that I_1=10.4-47.2i, I_2=18.7-45.81i, I_3=16.69-55.25i

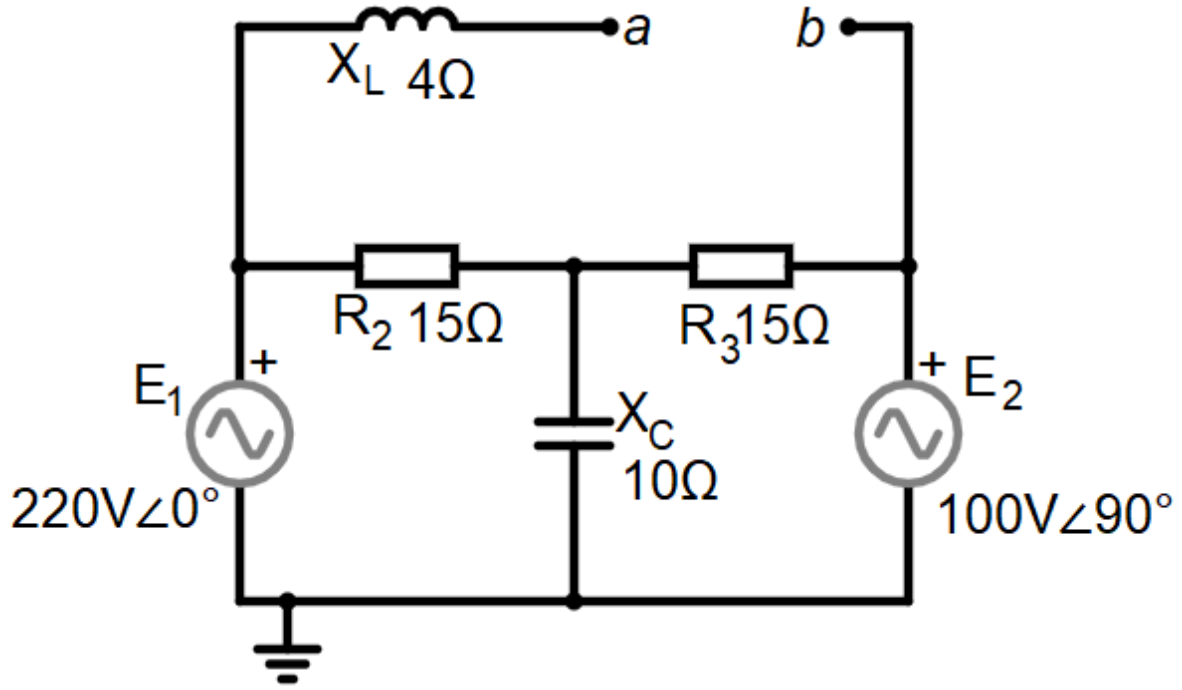
So, the current is 10.4-47.2i or 48.33∠-77.57°

Alternatively, voltage on the branch with the inductor and the resistor, is E1-E2=200-100i. The impedance on this branch is Z=4i+3. So the current is $\frac{220-100i}{4i+3} = \frac{241.66\angle -24.44°}{5\angle 53.13°} = 48.33\angle -77.57°$

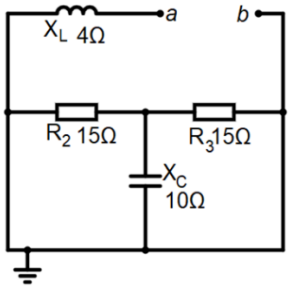
Uppgift 6 [3 p]

Bestäm Thévenins ekvivalent krets med avseende på terminalerna ***a*** och ***b.***

Find the Thévenin equivalent circuit with the respect to terminals ***a*** and ***b.***

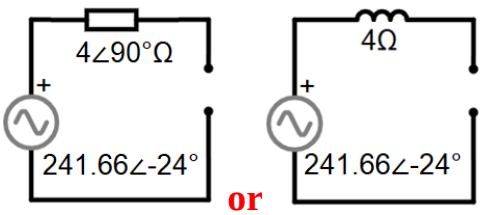


So first we are looking for R_{Th}. To do it, we remove voltage sources, then we see that R2, R3 and a capacitor are short-circuited, so we do not need to calculate them. So $R_{Th}=Z_L=4i$



To calculate E_{Th}, we need to find a voltage on node ***a*** and node ***b***. Since no current goes through the inductor, the voltage before the inductor is the same as voltage on node ***a***. And it is equal to E_1. And voltage on node ***b*** is the same as E_2. Hence $E_{Th}=220-100i=241.66\angle -24°V$.

Hence, the circuit looks like this:



Uppgift 7 [6p]

En anläggning som består av nedan parallellkopplade delbelastningar är ansluten till spänningen 220 V (effektivvärde), 50 Hz. Delbelastningar består av följande:p
Del 1: ström är 5A, effektfaktor är 0.7 (induktiv)
Del 2: har bara en 10kΩ resistor
Del 3: 1kVAR, effektfaktor är 0.6 (kapacitiv)
Del 4: 5kW och 7kVA (induktiv)
Del 5: skenbara effekten är 4000+3000*i* [VA]

a) Beräkna den totala aktiva, reaktiva och skenbara effekterna (2p)
b) Beräkna kapacitansen hos kondensatorn för att förbättra totala effektfaktor till 0,95 (2p)
c) Jämför nivåer av strömmar från skällan (original- och förbättrad ström) (2p)

A system consisting of sub-loads connected in parallel below is connected to the voltage 220 V (effective value), 50 Hz. Partial loads consist of the following:
Load 1: current is 5A, power factor is 0.7 (inductive)
Load 2: has only a 10kΩ resistor
Load 3: 1kVAR, power factor is 0.6 (capacitive)
Load 4: 5kW and 7kVA (inductive)
Load 5: apparent power is 4000 + 3000i [VA]

a) Determine total active, reactive and apparent powers (2p)
b) Determine the capacitive element required to raise total power factor to 0.95 (2p)
c) Compare levels of currents from the supply (original and improved current) (2p)

Del	P	Q	S	cosθ
1	P=V*I*cosθ=220*5 *0.7=770W	$Q = \sqrt{S^2 - P^2} = \sqrt{1100^2 - 770^2} = 785.5VAR$	S=V*I=220*5=1100 VA	0.7 – inductive
2	$P=V^2/R=4.84W$	0	S=P=4.84VA	1.0
3	P=S*cosθ=1250*0.6 =750W	1kVAR	$S = \frac{Q}{sin\theta} = \frac{1k}{0.8} = 1250VA$	0.6 – capacitive
4	5kW	$Q = \sqrt{S^2 - P^2} = 4.9kVAR$	7kVA	0.71- inductive
5	4kW	3kVAR	5kVA	0.8 - inductive (because imaginary part is positive)
Total	10524.8W	Q_1-Q_3+Q_4+Q_5= 7658VAR	$S = \sqrt{P^2 + Q^2} = 13032.2VA$	0.8 - inductive

Since we want cosθ of 0.95, it means that new S`=P/0.95=10524.8/0.95=11078.7VA. A new Q`=3459.35VAR (inductive). Hence, we need to remove ΔQ=7658-3459.35=4198.65VAR.
$X_C = \frac{E^2}{\Delta Q} = \frac{220^2}{4198.65} = 11.527\Omega$, $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi * 50 * 11.527} = 276\mu F$
Original current consumption is I=S/V=13032.2/220=59.23A, improved I`=11078.7/220=50.35A, hence current level is decreased by 8.88A or by 15%