Kontrollskrivning i HE1027 Ellära
Måndag 2 februari 2020, kl. 8:00 – 10:00

Ansvarig lärare: Maksims Kornevs
Ansvarig examinator: Elias Said

Hjälpmedel: Endast utdelat formelblad samt miniräknare.
För godkänt krävs 5 poäng av 9 möjliga poäng. Godkänd kontrollskrivning ger bonus enligt kurs PM.
Fullständiga lösningar skall presenteras på alla uppgifter.

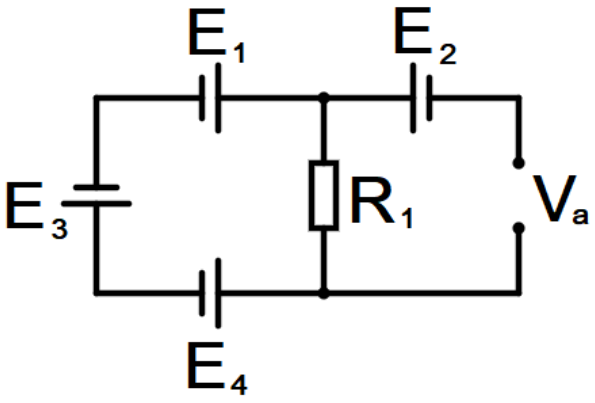
Börja varje ny uppgift på ett nytt blad, detta gör att rättningen blir säkrare.Skriv endast på en sida av papperet. Skriv namn och personnummer på varje blad.Inlämnade uppgifter skall markeras med kryss på omslaget.

Uppgift 1 [1p]

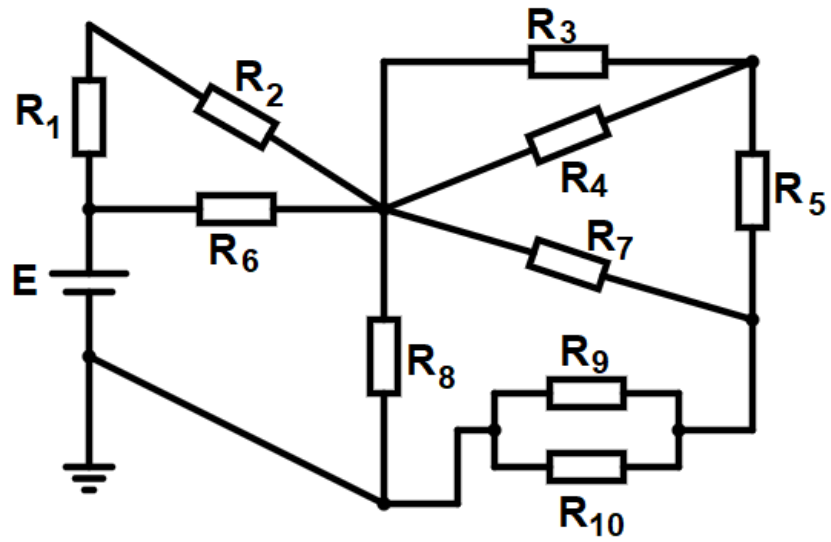
Bestäm spänningsfallet V_a och ange dess polaritet.

Determine the voltage for V_a and indicate its polarity.

E_1=10V, E_2=5V, E_3=2V och E_4=5V



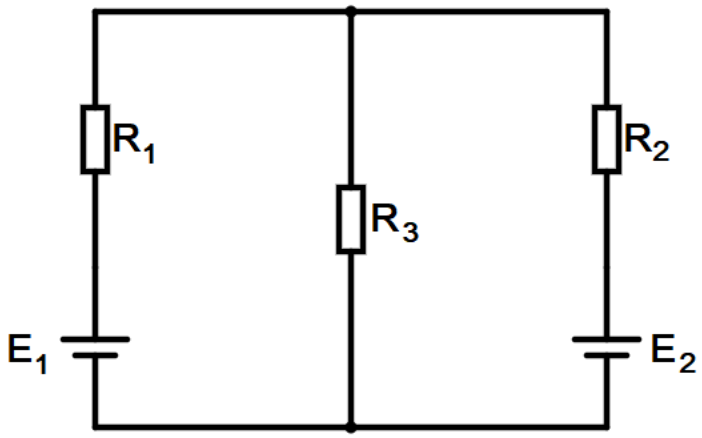
Uppgift 2 [2p]



Fyll i tabellen. Fill the table.

	E	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
V (V)	27										
I (A)											
R (Ω)		7	5	2	2	7	6	8	8	10	15

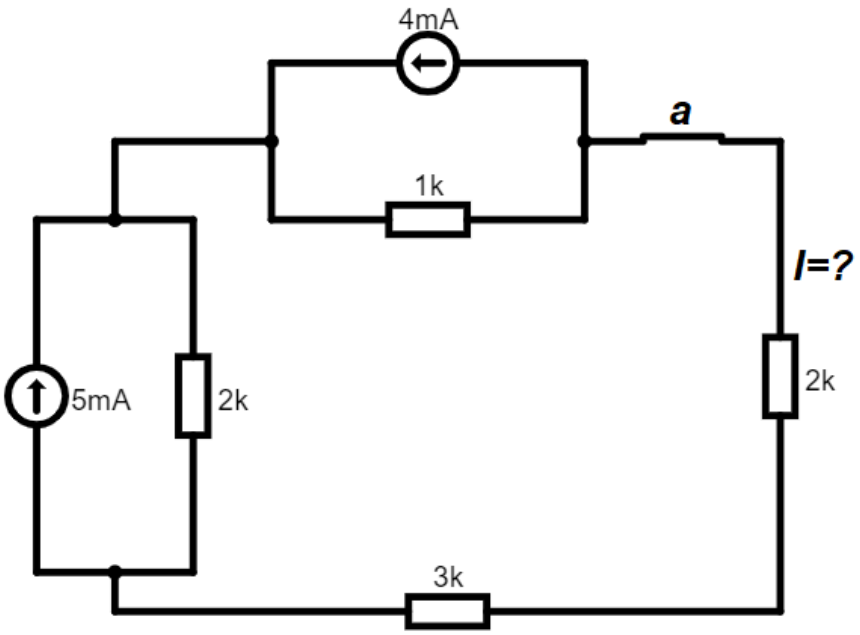
Uppgift 3 [2p]



Fyll i tabellen. Fill the table.

	R1	R2	R3	E1	E2
V (V)				1	2
I (A)					
R (Ω)	1	2	3		

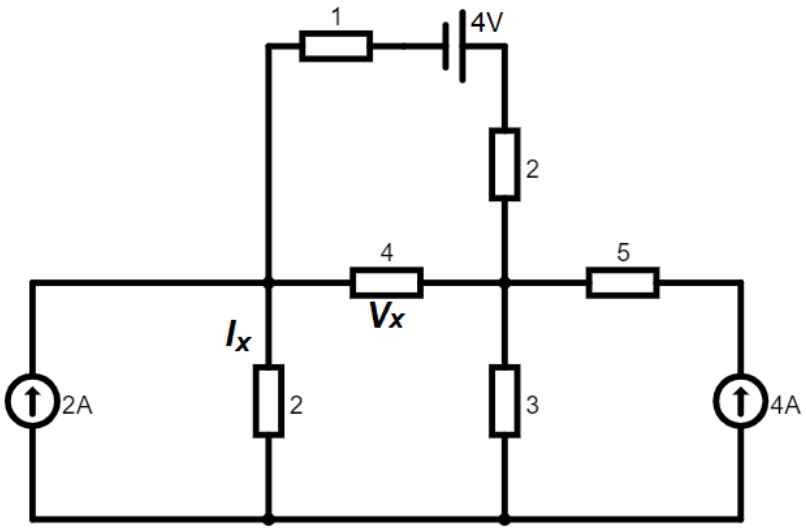
Uppgift 4 [2p]



Bestäm strömmen *I* och ange dess riktning. Var ska man sätta jordpunkten så att det inte påverkar nätverket? Hur ström och spänning över motstånd kommer att förändras om omkopplare *a* skulle vara öppen?

Find current *I* and its direction. Where to put the ground to not affect the network? How current and voltage across resistors will change if switch *a* would be open?

Uppgift 4 [2p]

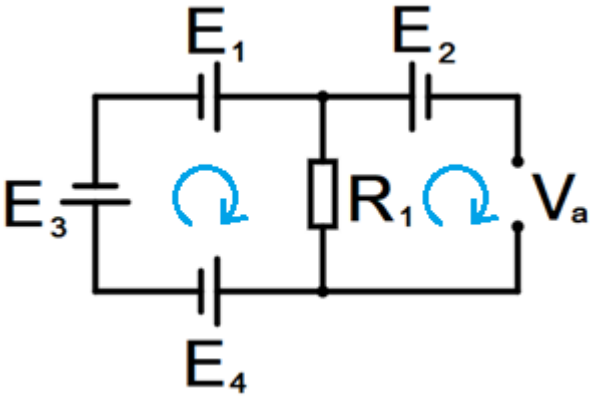


Bestäm strömmen I_x och spänningsfallet V_x (markerade i kretsen).

Find current I_x and voltage V_x.

Uppgift 1

Apply Kirchhoff's voltage law for both loops:



The first loop:
$-E_3+E_1-V_{R1}-E_4=0$
$-2+10-V_{R1}-5=0$
$3-V_{R1}=0$
$V_{R1}=-3V$

The second loop:
$-R_1-R_2+V_2=0$
$-(-3)-5+V_2=0$
$3-5+V_2=0$
$V_2=-2V$

Answer: voltage Va is 2V and it goes ‘up’ towards E_2

Uppgift 2

Start by replacing pairs of resistors in series or in parallel:

$R_a=R_1+R_2=7+5=12\Omega$

$R_b=R_a//R_6=(12*6)/(12+6)=4\ \Omega$

$R_c=R_3//R_4=(2*2)/(2+2)=1\ \Omega$

$R_d=R_c+R_5=1+7=8\ \Omega$

$R_e=R_d//R_7=(8*8)/(8+8)=4\ \Omega$

$R_f=R_9//R_{10}=(10*15)/(10+15)=6\ \Omega$

$R_g=R_e+R_f=4+6=10\ \Omega$

$R_h=R_g//R_8=(10*8)/(10+8)=4{,}444\ \Omega$

$R_{Total}=R_b+R_h=4+4.444=8{,}444\ \Omega$

$I_{Scource}=E/R_{Total}=27/8{,}444=3{,}197A$

$R_{Total}=R_b+R_h \rightarrow I_b=I_h=I_{Scource}=3{,}197A$

$V_b=I_b*R_b=3{,}197*4=12{,}788V$

V_h=3,197*4,444= 14,207V

R_h=Rg//R8-> V_g=V_8=V_h=14,207V

I_g=V_g/R_g=14,207/10=1,421A

I_8=14,207/8=1,776A

R_g=Re+Rf - > I_e=I_f=1,421A

V_e=I_e*R_e=1,421*4=5,684V

V_f=1,421*6=8,526V

R_f=R_9//R_{10} -> V_9=V_{10}=8,526V

I_9=V_9/R_9=8,526/10=0,853A

I_{10}=8,526/15=0,568A

R_e=R_d//R_7 - > V_d=V_7=5,684V

I_d=V_d/R_d=5,684/8=0,711A

I_7=5,684/8=0,711A

R_d=R_c+R_5 -> I_c=I_5=0,711A

V_c=I_c*R_c=0,711*1=0,711V

V_5=0,711*7=4,977V

R_c=R_3//R_4 ->V_3=V_4=0,711V

I_3=V_3/R_3=0,711/2=0,356A

I_4=0,711/2=0,356A

R_b=R_a//R_6 - > V_a=V_6=12,788V

I_a=V_a/R_a=12,788/12=1,066A

I_6=12,788/6=2,131A

R_a=R_1+R_2 -> I_1=I_2=1,066A

V_1=I_1*R_1=1,066*7=7,460V

V_2=1,066*5=5,330V

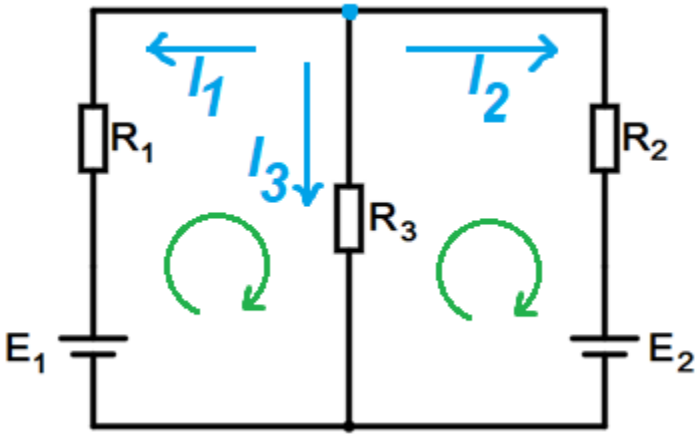
Answer:

	E	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
V	27	**7,460**	**5,330**	**0,711**	**0,711**	**4,977**	**12,788**	**5,684**	**14,207**	**8,526**	**8,526**
I	**3,197**	**1,066**	**1,066**	**0,356**	**0,356**	**0,711**	**2,131**	**0,711**	**1,776**	**0,853**	**0,568**
R	**8,444**	7	5	2	2	7	6	8	8	10	15

Uppgift 3 – Branch-current method

Assign direction of flow for each branch.

Apply Kirchhoff's voltage law for both loops:



$E_1+V_{R1}-R3=0$
$E_1+I_1*R_1-I_3*R_3=0$
$1+1*I_1-3*I_3=0$
$I_1-3*I_3=-1$

$V_{R3}-V_{R2}-E_2=0$
$I_3*R_3-I_2*R_2-E_2=0$
$3*I_3-2*I_2-2=0$
$-2*I_2+3*I_3=2$

Apply Kirchhoff's current law:

$-I_1-I_2-I_3=0$

Solve system of equation:

$$\begin{cases} I_1-3*I_3=-1 \\ -2*I_2+3*I_3=2 \\ -I_1-I_2-I_3=0 \end{cases}$$

I_1=0,091A
I_2=-0,455A
I_3=0,364A

$V_1= I_1*R_1=0,091*1=0,091V$

$V_2= I_2*R_2=-0,455*2=-0,910V$

$V_3= I_3*R_3=0,364*3=1,092V$

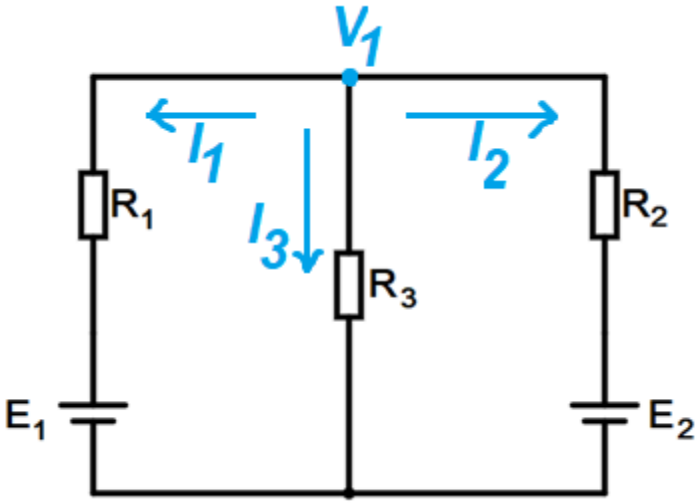
$R_{E1}=E_1/I_1=1/0,091=10,989\Omega$

$R_{E2}=E_2/I_2=2/(-0,455)=-4,396\Omega$

Answer

	R1	R2	R3	E1	E2
V (V)	**0,091**	**-0,910**	**1,092**	1	2
I (A)	**0,091**	**-0,455**	**0,364**	**0,091**	**-0,455**
R (Ω)	1	2	3	**10,989**	**-4,396**

Uppgift 3 - Nodal method



Assign direction of flow for each branch

Find each current based on node V1

$I_1=(V_1-E_1)/R_1$ $I_1=(V_1-1)/1$ $I_1=V_1-1$	$I_2=(V_1-E_2)/R_2$ $I_2=(V_1-2)/2$ $I_2=0,5V_1-1$	$I_3=V_1/R3$ $I_3=V_1/3$ $I_3=0,333V_1$

Apply Kirchhoff's current law at node V1

$-I_1-I_2-I_3=0$

$-V_1+1-0,5V_1+1-0,333V_1=0$

$-1,833V_1+2=0$

$V_1=2/(1,833)=1,091V$

$I_{E1}=I_1= V_1-1=1,091-1=0,091A$

$I_{E2}=I_2=0,5V_1-1=-0,455A$

$I_3=0,333V_1=0,363A$

$V_{R1}= V_1-E_1=1,091-1=0,091V$

$V_{R2}=V_1-E_2=1,091-2=-0,909V$

$V_{R3}=V_1=1,091V$

$R_{E1}=E_1/I_{E1}=1/0,091=10,989\Omega$

$R_{E2}=E_2/I_{E2}=2/(-0,455)= -4,396\Omega$

Answer:

	R1	R2	R3	E1	E2
V (V)	**0,091**	**-0,909**	**1,091**	1	2
I (A)	**0,091**	**-0,455**	**0,363**	**0,091**	**-0,455**
R (Ω)	1	2	3	**10,989**	**-4,396**

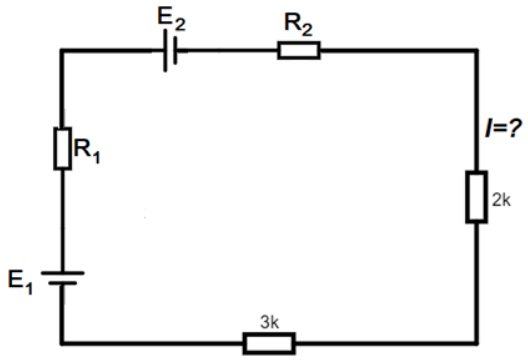
Uppgift 4 – Replacing sources

Current source with resistor in parallel can be replaced with voltage source and resistor in series

$E_1=I_1*R_1=0,005*2000=10V$

$E_2=I_2*R_2=0,004*1000=4V$

Redraw network with new information



$E_{Total}=E_1-E_2=10-4=6V$

$R_{Total}=R_1+R_2+R_3+R_4=2k\Omega+1k\Omega+2k\Omega+3k\Omega=8k\Omega$

$I_{Total}=6/8000=0.00075A=750\mu A$

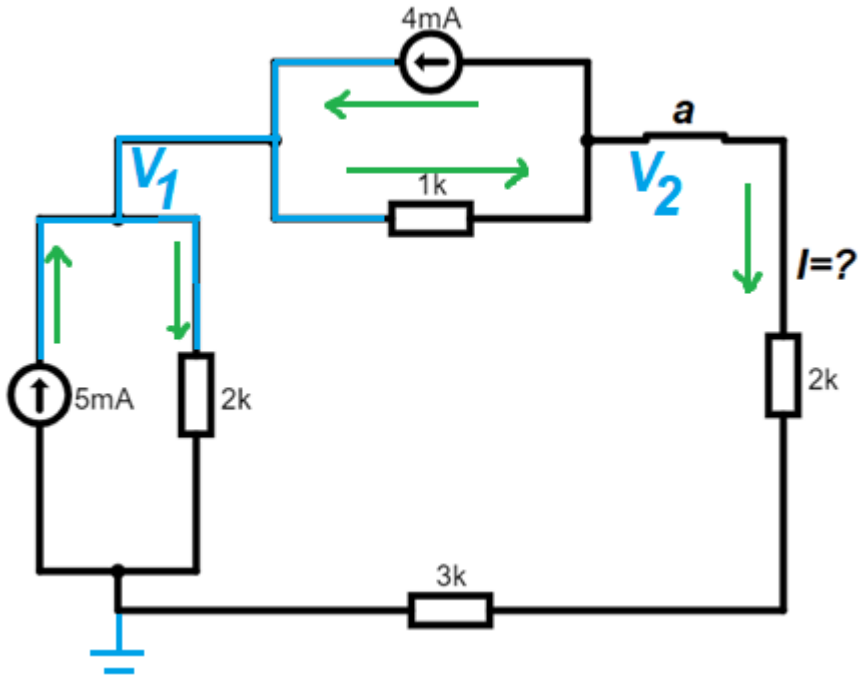
Answer: 750µA

Since E_1 is more powerful then E_2 it means that current flows clockwise. It also means that right before E_1 voltage is 0 and ground can be placed there.

To understand what will happen when switch *a* we need to look on original network. If switch *a* would be open, resistors with 2kΩ and 3kΩ (the ones that in series) will be 0A and 0V. For resistors that are in parallel to source current sources, the current will be the same as its source: current through resistor with 2kΩ is 5mA (its voltage will be 2kΩ*5mA=10V) and current through resistor with 1kΩ is 4mA (its voltage will be 1kΩ*4mA=4V).

Uppgift 4 – Nodal method

There are three nodes defined: V_1, V_2 and reference point (ground). Blue lines show where the V_1 is. Green lines show selected current flows.



Apply Kirchhoff's current law at nodes V_1 and V_2

5mA+4mA-V1/2kΩ-(V1-V2)/1kΩ=0	-4mA+(V1-V2)/1kΩ-V2/(2kΩ+3kΩ)=0
0.009-0.0005V1-0.001V1+0.001V2=0	-0.004+0.001V1-0.001V2-0.0002V2=0
-0015V1+0.001V2=-0.009	0.001V1-0.0012V2=0.004

Solving both equations as a system, V_1=8.5V and V_2 is 3.75V

I=V_2/(2kΩ+3kΩ)=3.75/5000=0.00075A=750μA

Answer: 750μA

Current goes the same way as shown in the network.

Ground can be anywhere between the resistor with 3kΩ and source with 5mA and its parallel resistor with 2kΩ. Example, ground shown in the network.

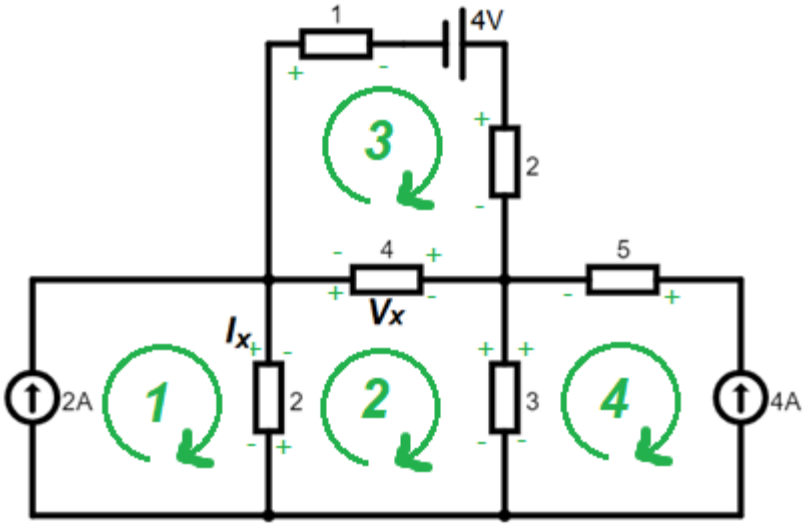
If switch *a* would be open, resistors with 2kΩ and 3kΩ (the ones that in series) will be 0A and 0V.

For resistors that are in parallel to source current sources, the current will be the same as its source: current through resistor with 2kΩ is 5mA (its voltage will be 2kΩ*5mA=10V) and current through resistor with 1kΩ is 4mA (its voltage will be 1kΩ*4mA=4V).

Uppgift 5 – Mesh method

Assign current flow clockwise.

Determine polarity of all elements. Pay attention that current in loop 4 goes counter clockwise.



Apply Kirchhoff's voltage law for each loop.

Loop 1: current I1 is known from the current source: I_1=2A

Loop 4: current I4 is known from the current source: I_4=4A

Loop 2:
-2(I_2-I_1)-4(I_2-I_3)-3(I_2+I_4)=0
-2*I_2+2*2-4*I_2+4*I_3-3*I_2-3*4=0
-9*I_2+4*I_3-8=0

Loop 3:
-1*I_3+4-2*I_3-4*(I_3-I_2)=0
-1*I_3+4-2*I_3-4*I_3+4*I_2=0
4*I_2-7*I_3+4=0

Solving both equations as a system, I_2=-0.851A and I_3=0.085A

The current of any branch is equal to the algebraic sum of associated mesh currents. I_1 is in the opposite direction of I_x and I_2 is in the same direction as I_x. Therefore:

$I_x = I_2 - I_1 = -2.851A$

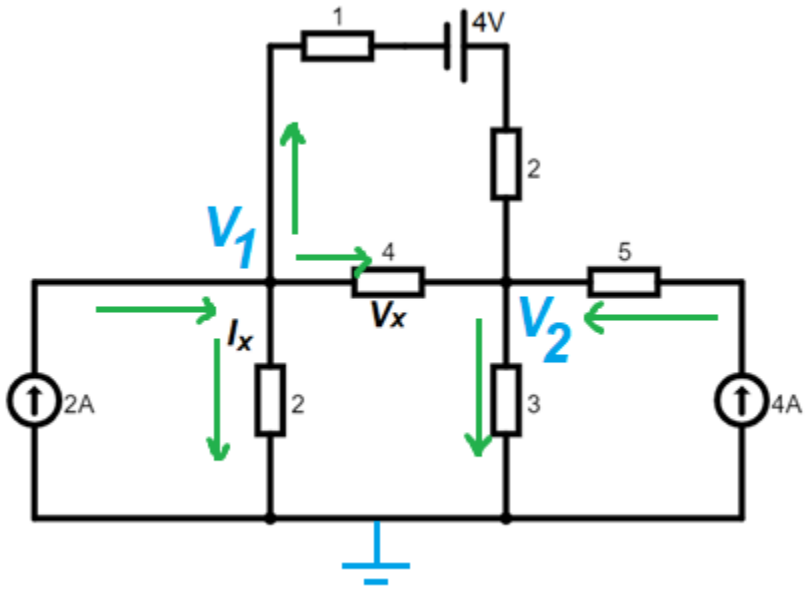
Voltage V_x can be found based on current through the resistor:

$I_{R2} = I_2 - I_3 = -0.936A$

$V_{R2} = I_{R2} \times R_2 = -0.9362 \times 4 = -3.744V$

Answer: current I_x=-2.851A and voltage V_x=-3.744V

Uppgift 5 – Nodal method



Apply Kirchhoff's current law at nodes V1 and V2

Node V1:

2-V1/2-(V1-V2)/4-(V1+4-V2)/(1+2)=0

2-0.5V1-0.25V1+0.25V2-0.333V1-1.333+0.333V2=0

-1.083V1+ 0.583V2+0.667=0

Node V2:

(V1-V2)/4+(V1+4-V2)/(1+2)-V2/3+4=0

0.25V1-0.25V2+0.333V1+1.333-0.333V2-0.333V2+4=0

0.583V1-0.916V2+5.333=0

Solving both equations as a system, V_1=5.704V and V_2= 9.453V

Vx=V1-V2=5.704-9.453=-3.749V

Ix=V1/2=5.704/2=2.852A

Answer: current I_x=2.852A and voltage V_x=3.749V