

Transmission imaging I

HLI 202

m colarieti tosti, mct@kth.se

Office hours (för frågor) denna vecka: <https://kth-se.zoom.us/j/63298493978>

Onsdag 8 feb 12:00-13:00

Fredag 10 feb 13:00-14:00

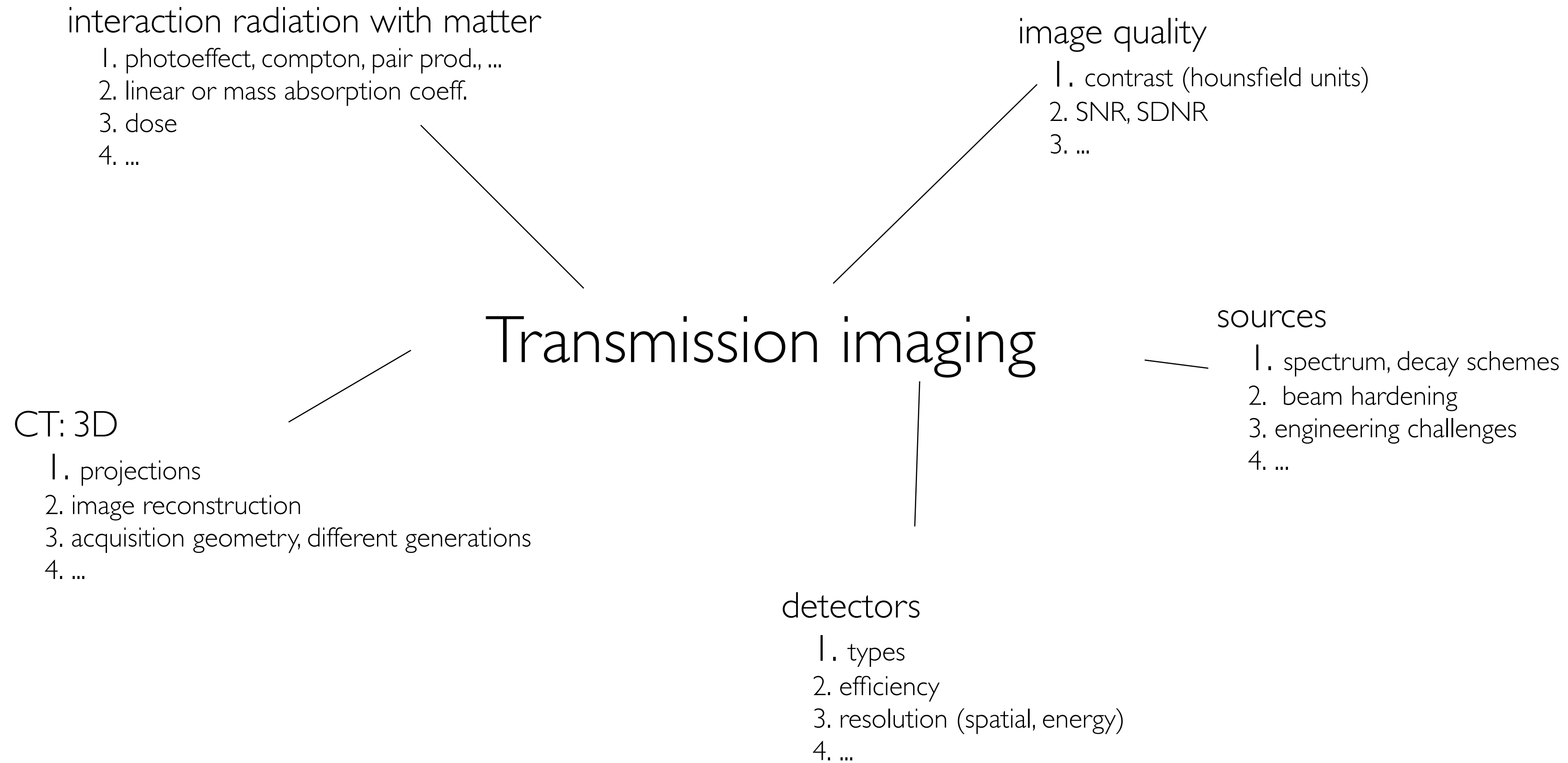
This is what you should know after this lecture + own work:

1. Def. of ionising radiation
2. Basic principles of transmission imaging
3. Fundamentals of interaction of radiation with matter of interest for transmission imaging (kinds of interaction, energy dependence)
4. Attenuation of photons: primary, attenuated, absorbed photons
5. Beer's law and how to apply it
6. Definition of linear and mass attenuation coefficient
7. Definition of object contrast
8. Image contrast vs object contrast
9. Signal Difference to Noise Ratio (SDNR)
10. Influence of Compton on contrast
11. Influence of x-ray energy on contrast
12. Energy dependence of attenuation, Photo Effect, Compton and Pair Production

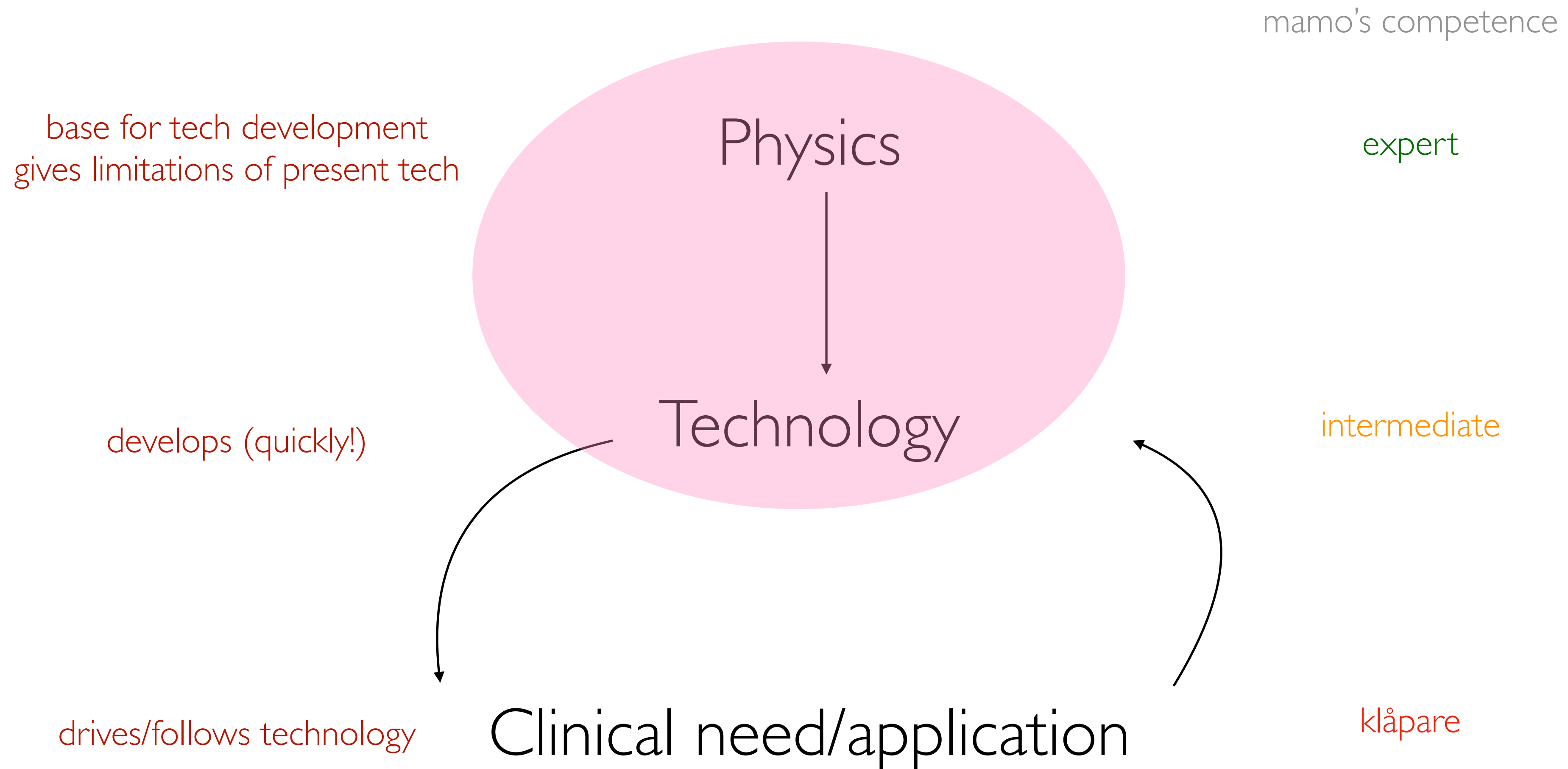
Intro: orientation chart & lecture organisation

Transmission imaging

Intro: orientation chart & lecture organisation



Philosophy of the course and motivation for it



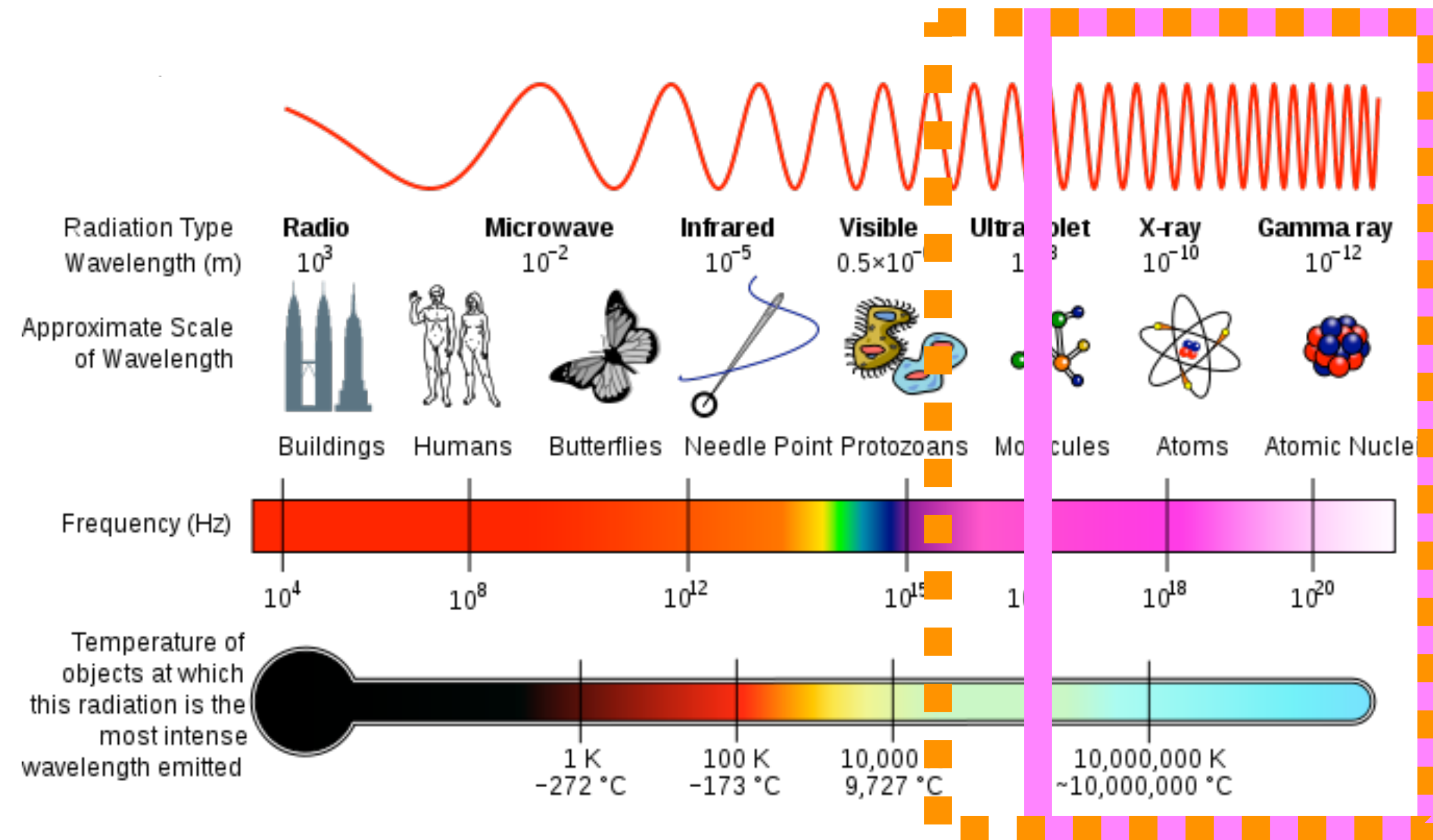
... but we will jump back and forth among these 3

Warning: the material in this lecture is at exactly the same level as you should know it in order to pass the exam (that is, I am not presenting things that are “just so you know it and have a vague idea”)

Let's get started: panorama
view of 2D transmission
imaging with ionising radiation

ionising radiation?

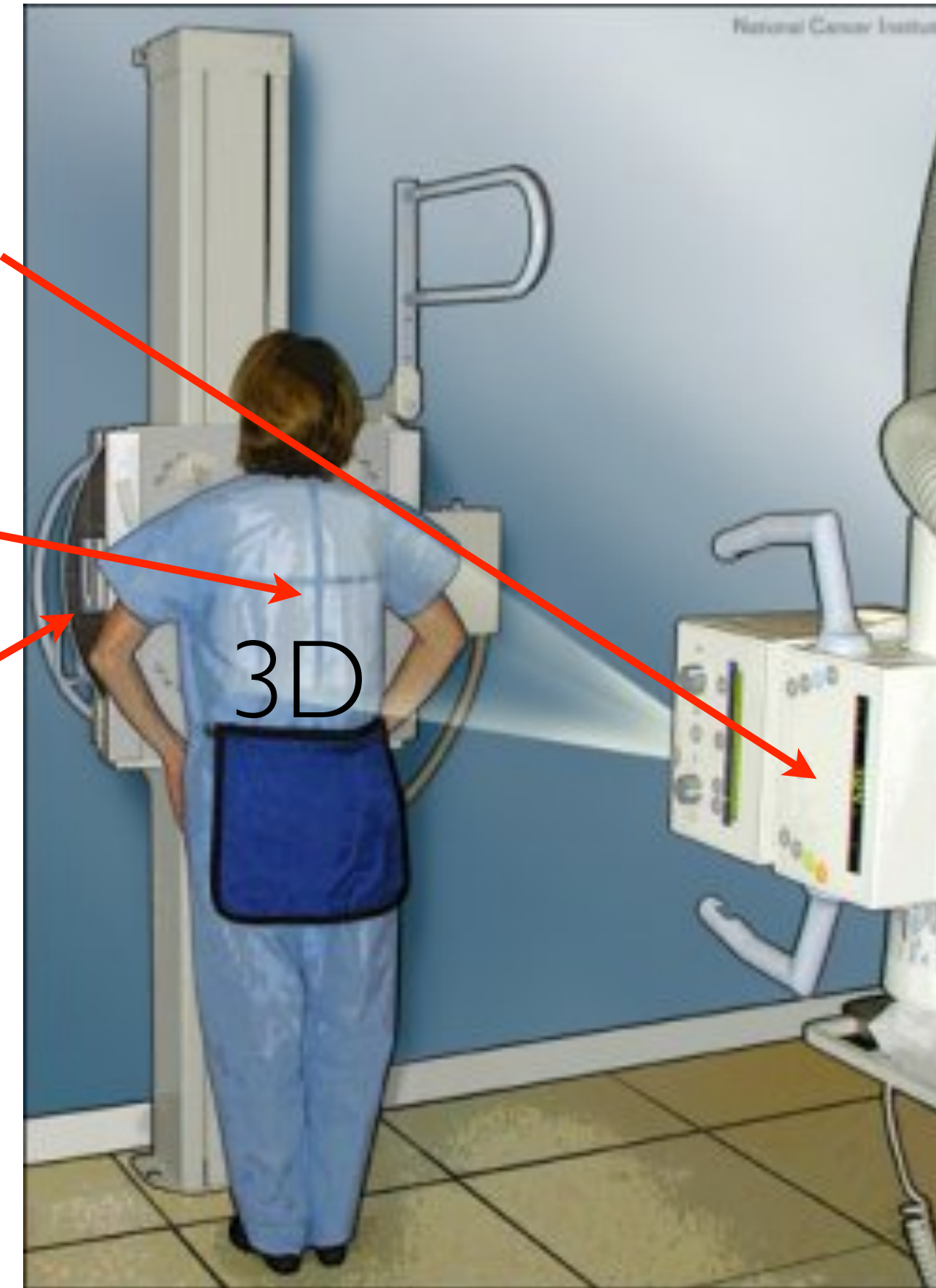
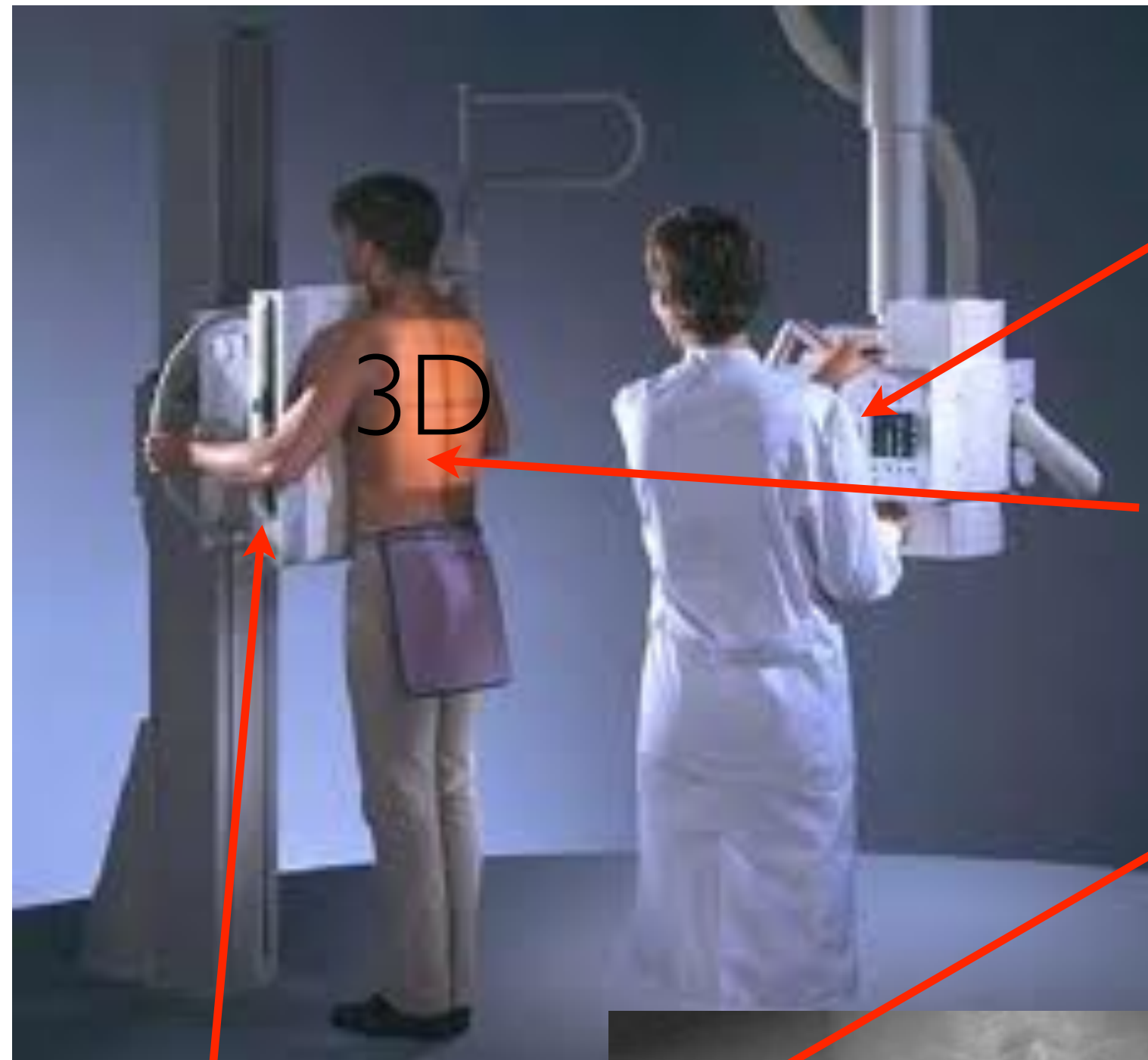
EM-strålning med tillräckligt hög energi för att jonisera atomer



> 100 eV

Let's jump to clinical
practice

Transmission imaging fundamental elements



source

object

3D

3D

detector

2D

property of the
object measured:
transparency to x-ray

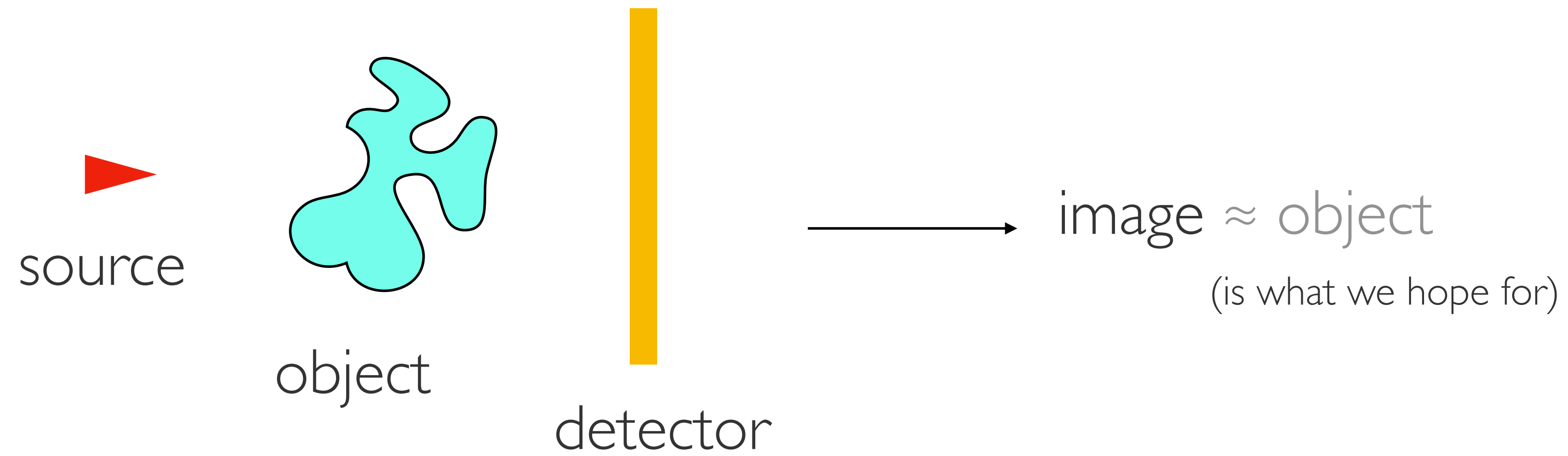
What is absolutely necessary for obtaining the kind of x-ray of the chest shown in the picture?:



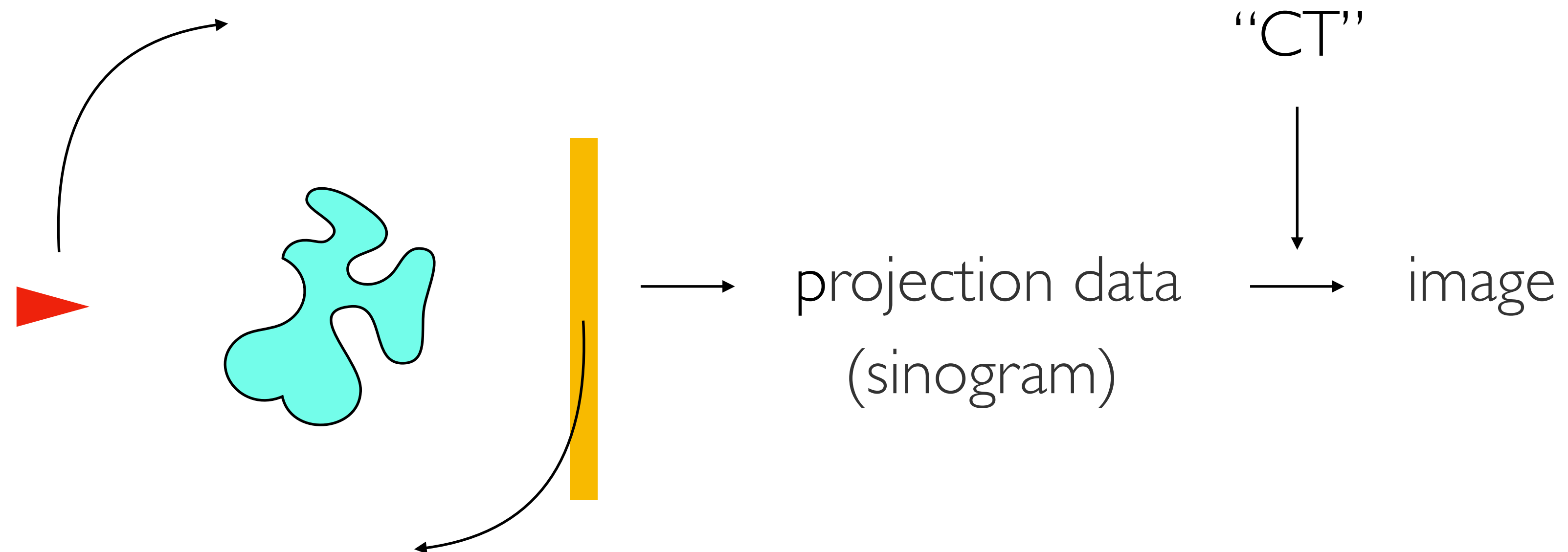
1. All of the transmitted photons must be detected by the detector
2. Bones and soft tissues must transmit x-ray in a different way
3. Some of the photons must be absorbed in the object
4. The patient must be alive

Still clinical, but simplified
(hint that we'll be heading
towards Physics soon!)

2D X-ray



3D: CT (or X-ray CT)

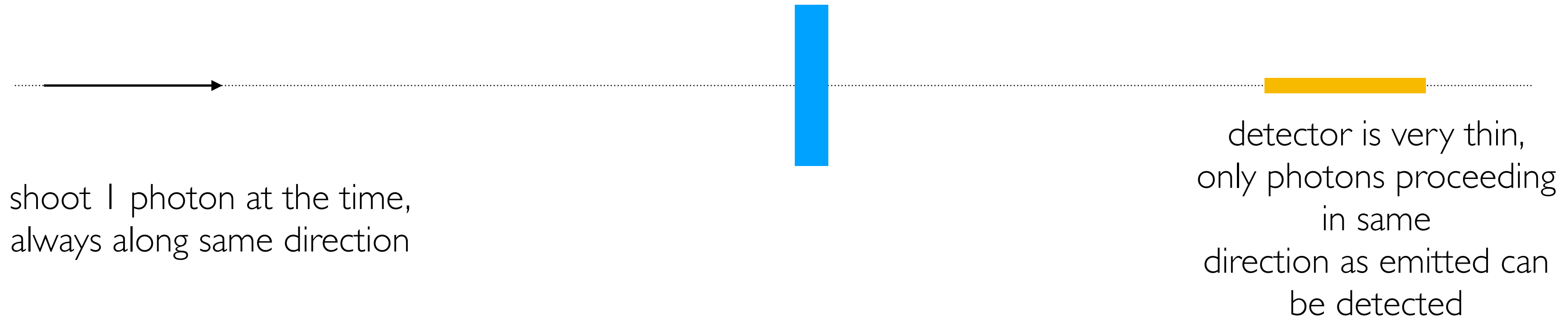


Physics!

So now we're going to look at very simple cases but we will dig very deep into details. The goal is to understand how these details are going to be important for the clinical setting

Let's consider a simple case:

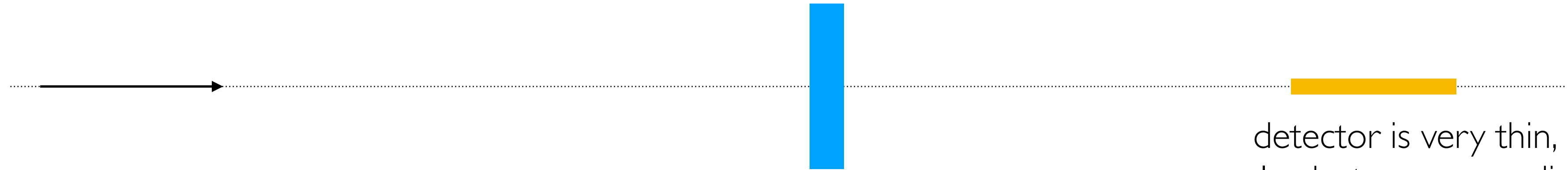
object: homogeneous material, thickness t



(Keep clinical application in mind: will the detector signal change if I remove or change in any way the object? How? Why? Can I change something in the object without changing the detector output?)

Let's consider a simple case:

object: homogeneous material, thickness t



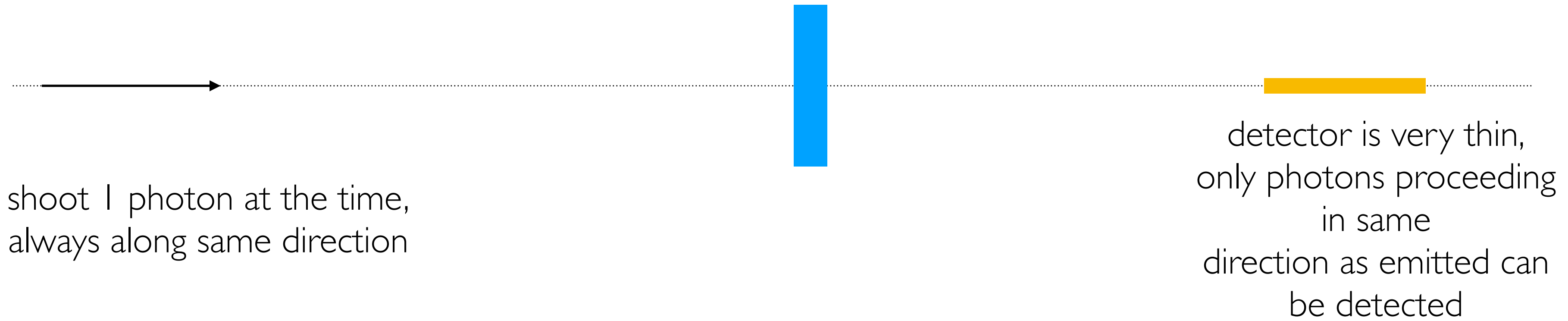
shoot 1 photon at the time,
always along same direction

detector is very thin,
only photons proceeding
in same
direction as emitted can
be detected

What can happen?

Let's consider a simple case:

object: homogeneous material, thickness t



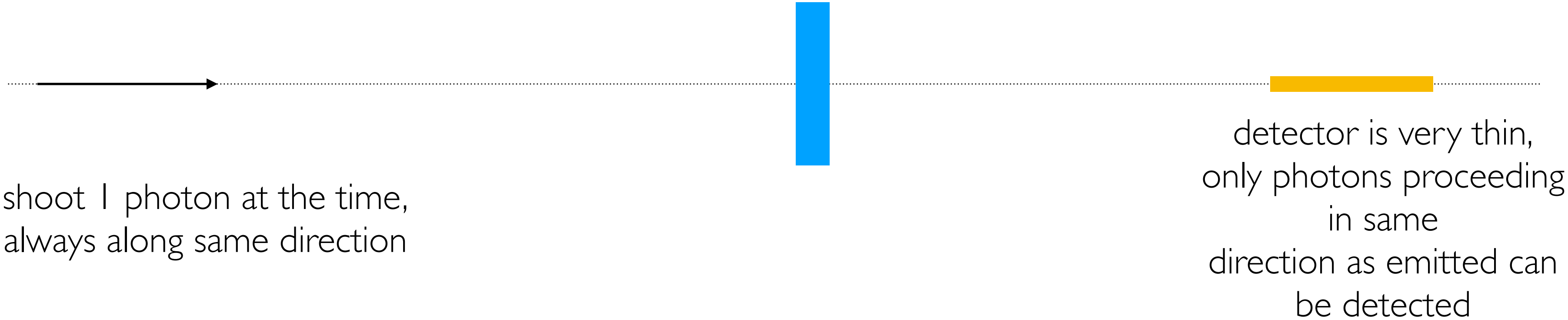
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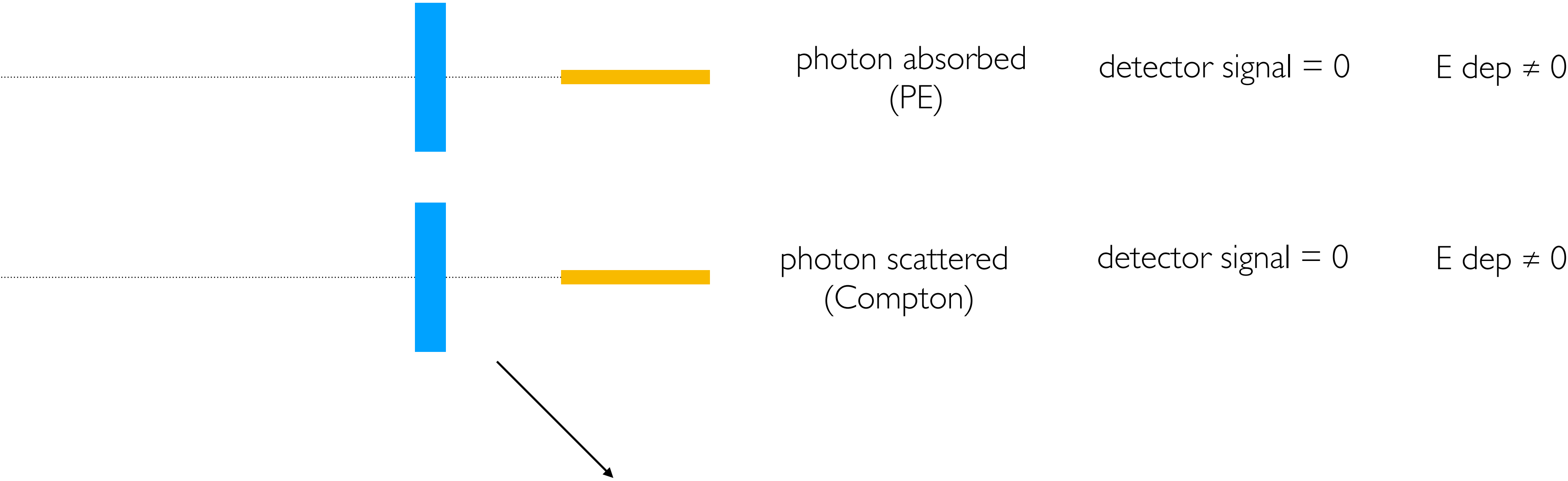


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object: homogeneous material, thickness t

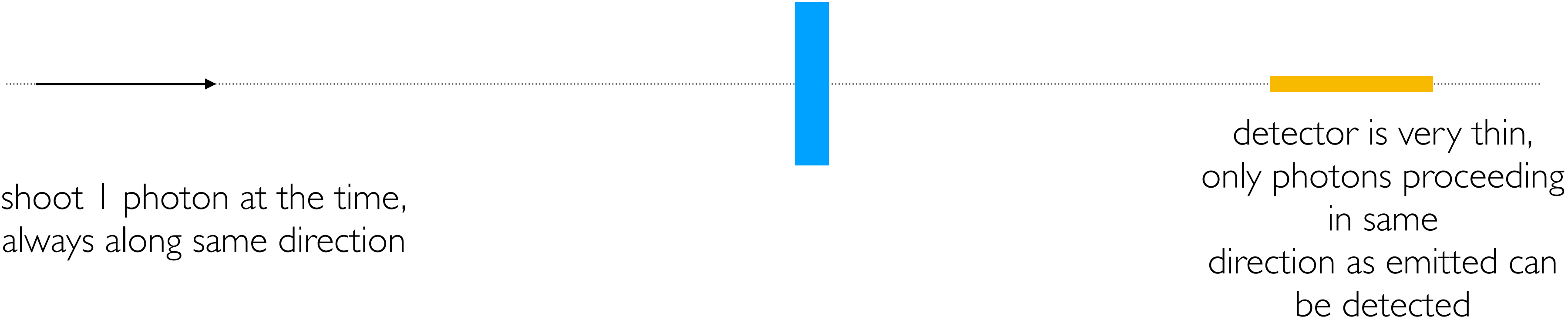


What can happen?

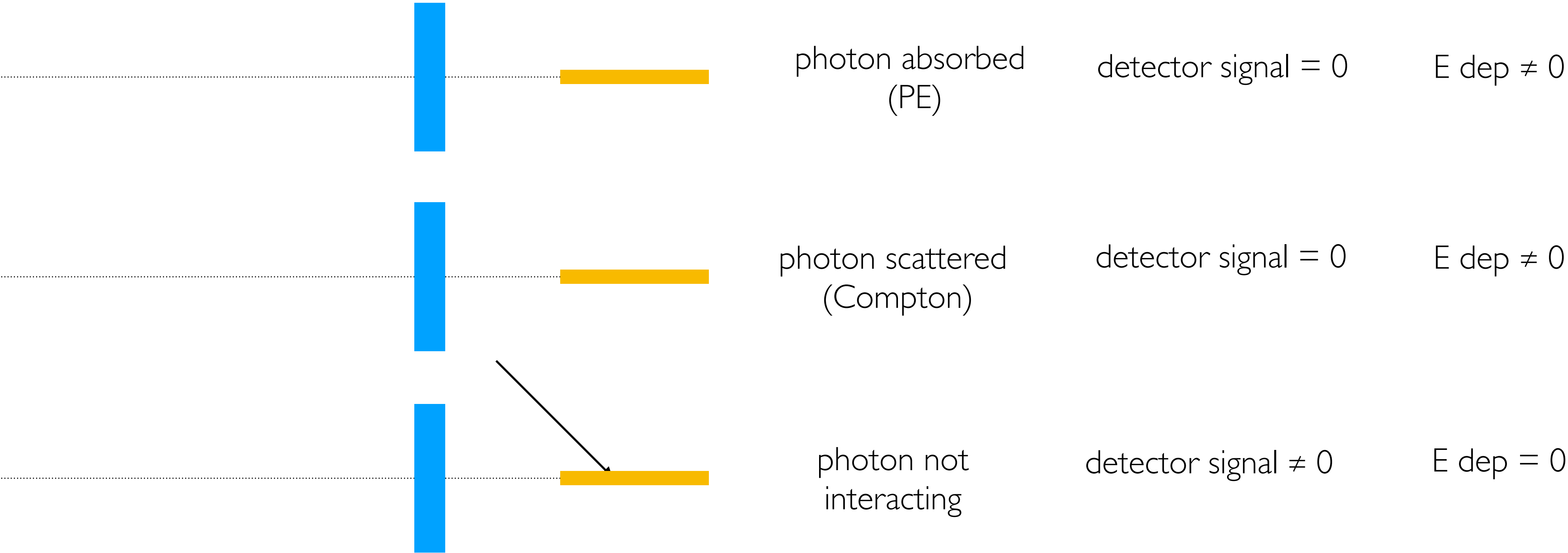


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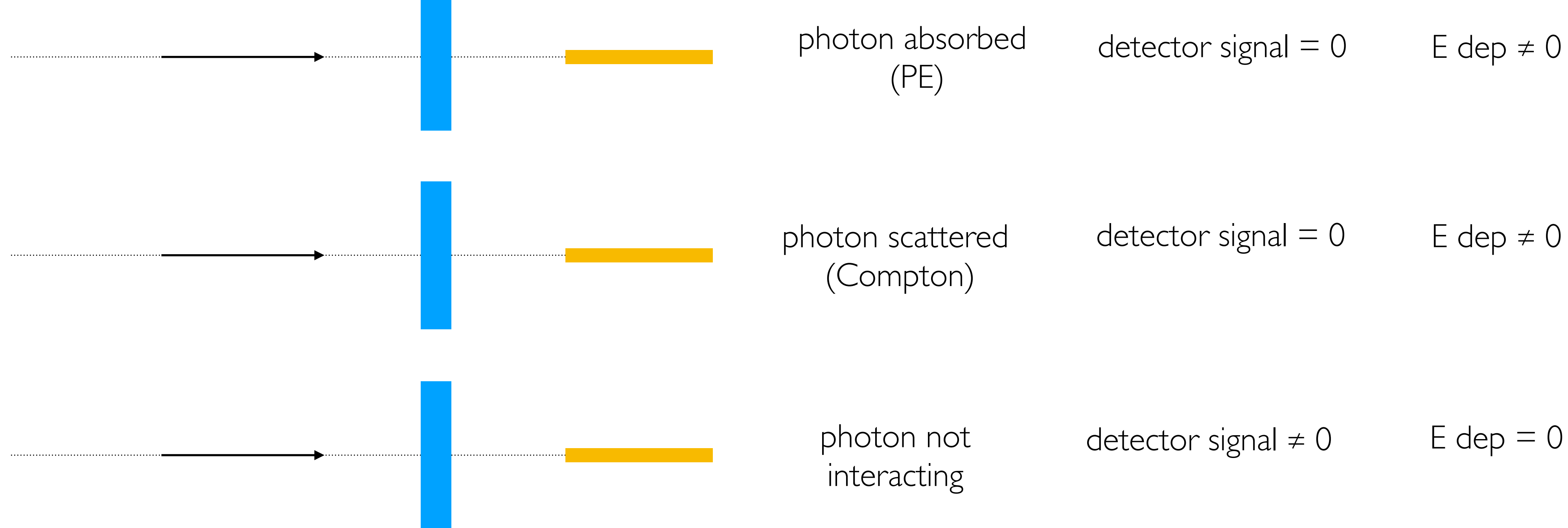
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What can happen?



Definitions ahead

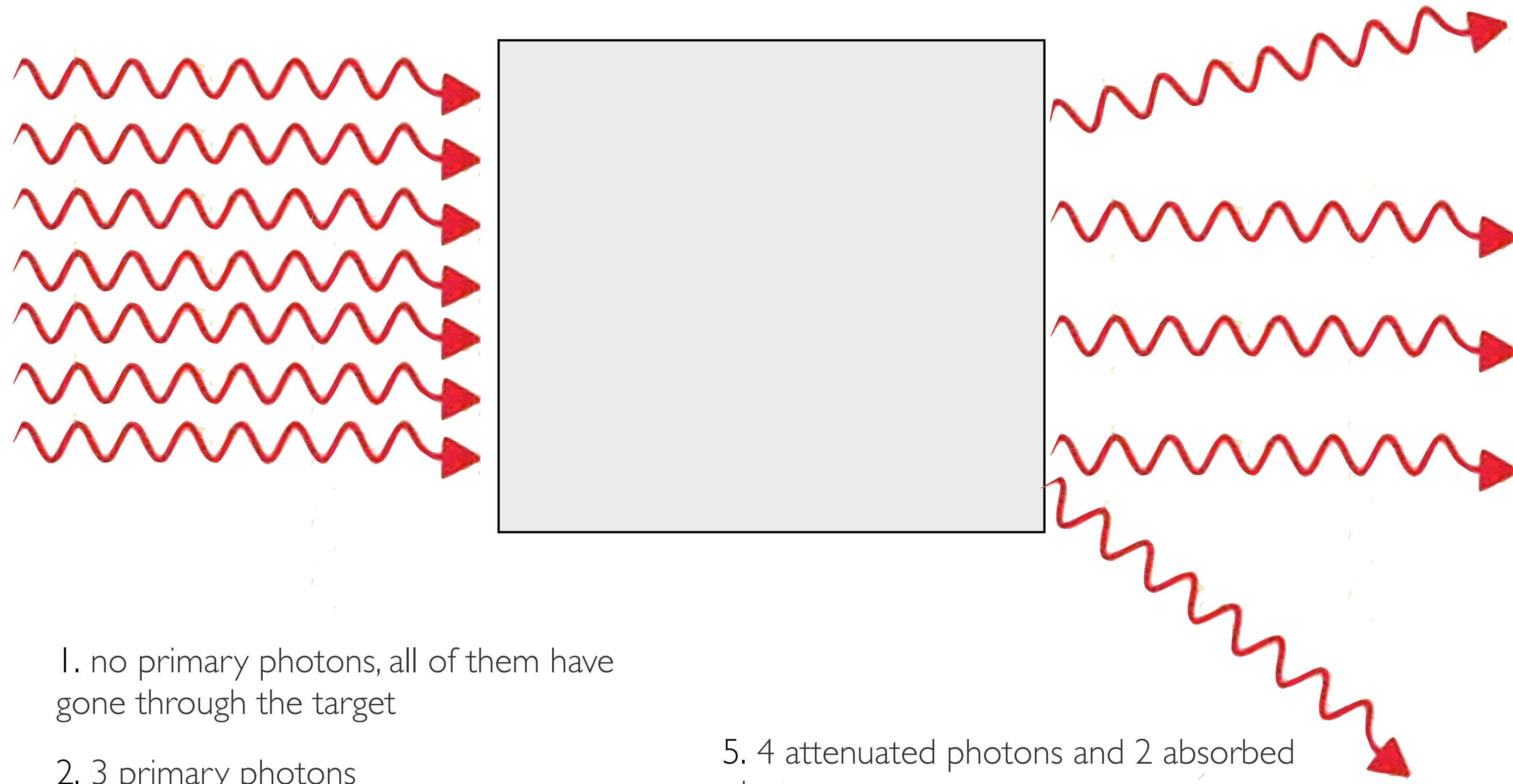


Def 1: Photons interacting in the object, that is photons depositing energy in the object are called *attenuated photons*

Def 2: Photons not interacting in the object, are called *primary photons*

Question: Could I equivalently define attenuated photons as photons not depositing energy in the detector? Hint: to answer the question, try to design at least a case in which scattered photons end up interacting in the detector

In the situation in the figure there are:



1. no primary photons, all of them have gone through the target

2. 3 primary photons

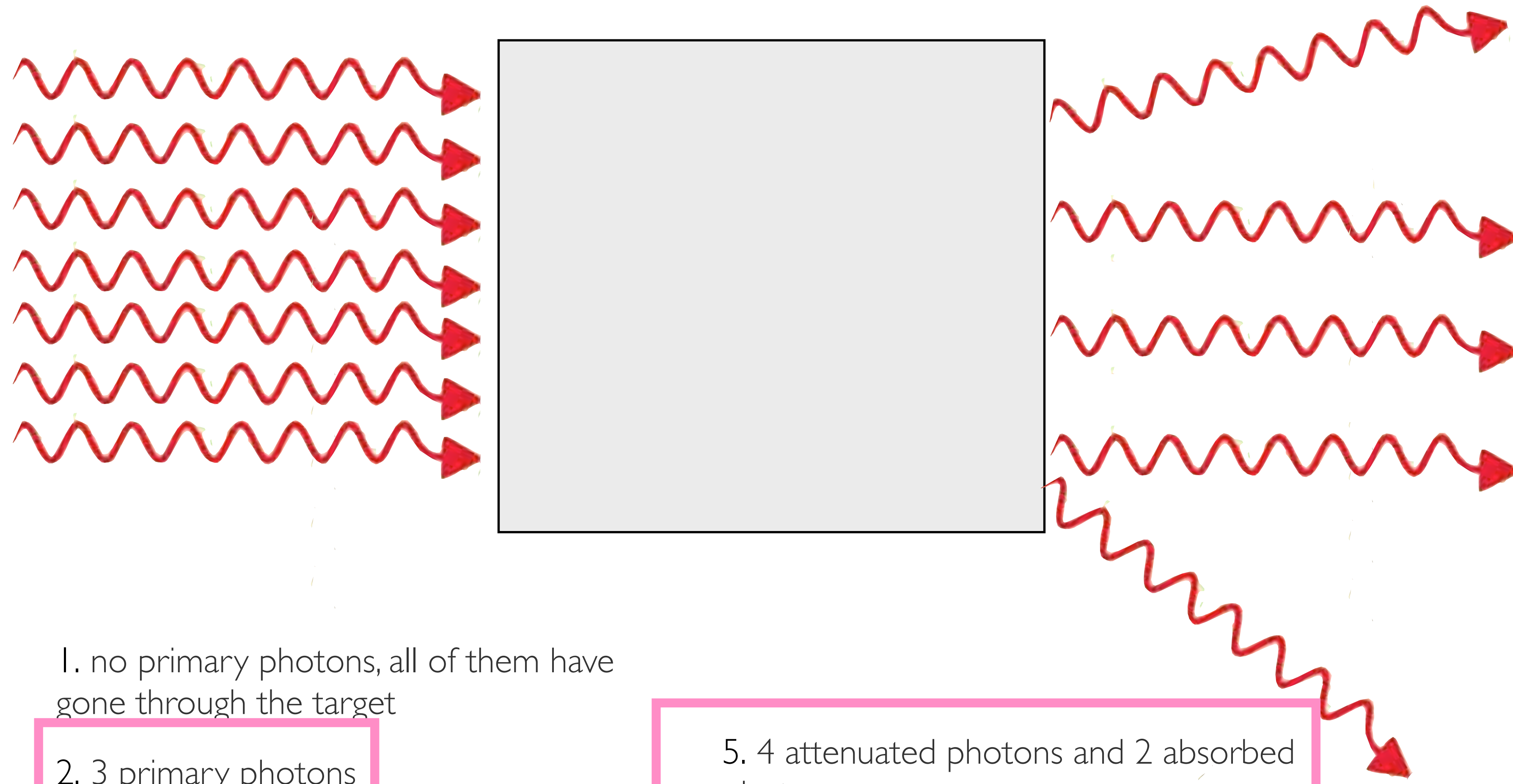
3. 5 primary photons

4. 2 attenuated photons and 2 absorbed photons

5. 4 attenuated photons and 2 absorbed photons

6. more information is needed to decide

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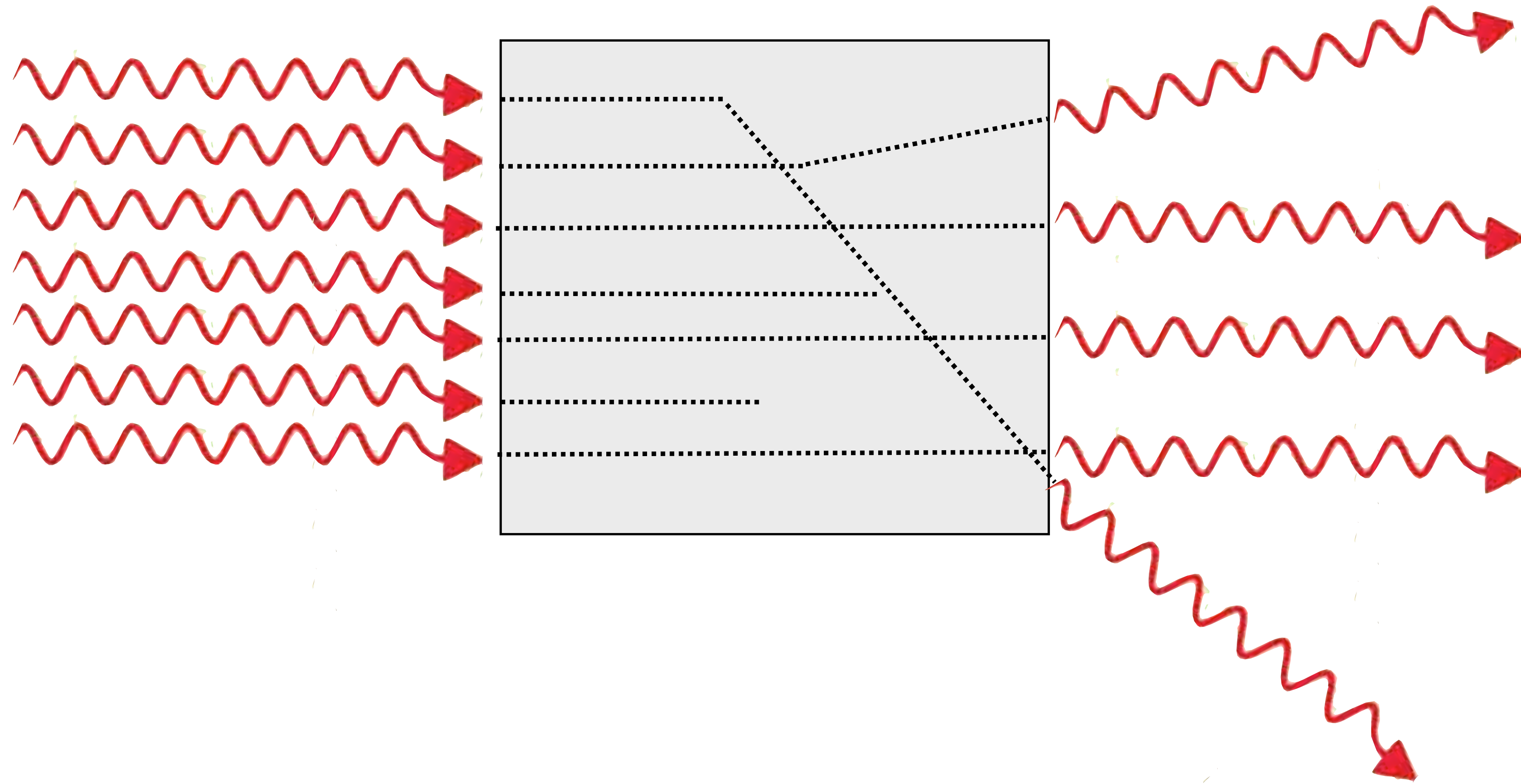
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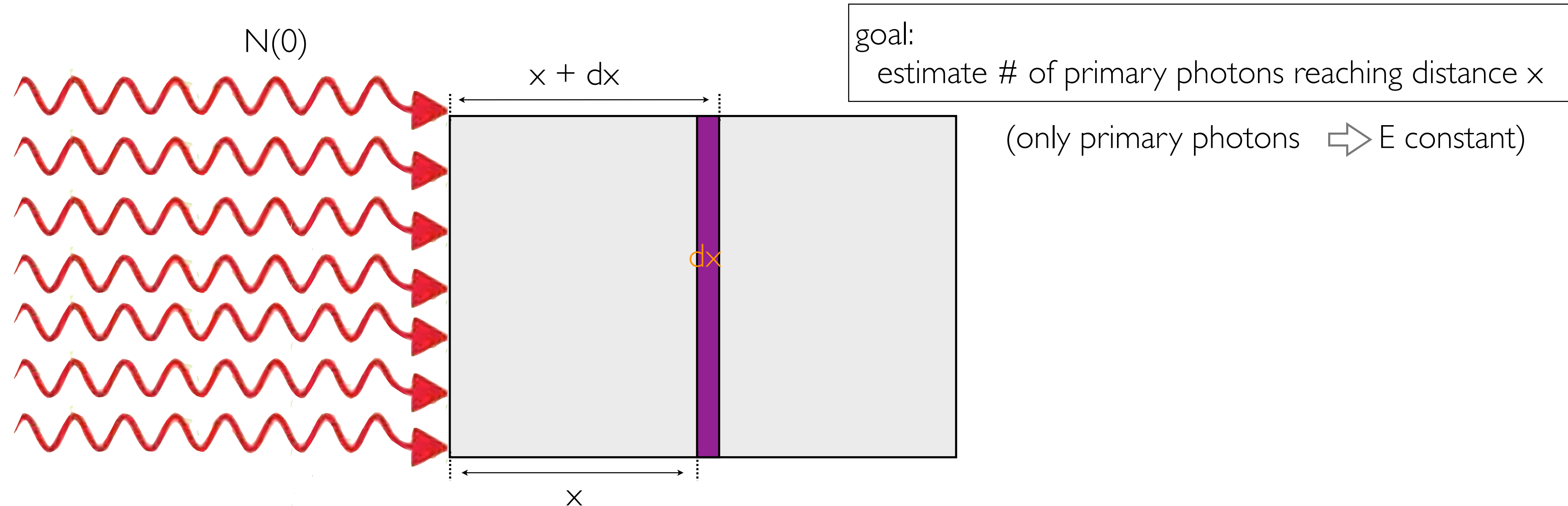
5. 4 attenuated photons and 2 absorbed photons

6. more information is needed to decide

Let us sort out the answer to the previous question



Beer's law



let σ be the probability of interaction with 1 electron in the material

let D be the density of electrons in the material

there will be Ddx electrons available for scattering in a slice of thickness dx (per unit area of the material) $\Rightarrow$ probability of interaction in that slice is $D\sigma dx$

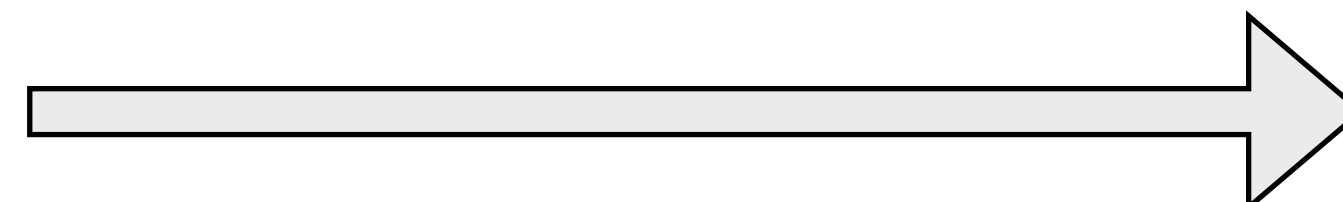
let $P(x)$ be the probability of having travelled **without** interaction to x

$\Rightarrow$ the probability of interaction in the slab between x & $x+dx$ will be:

$$P(x) D\sigma dx$$

$\Rightarrow$ the probability of travelling to $x+dx$ **without** interaction will be:

$$P(x+dx) = P(x)(1 - D\sigma dx)$$



$$P(x) = e^{-D\sigma x}$$

Q: We have derived the equation:

$$P(x+dx) = P(x)(1 - D\sigma dx)$$

$P(x)$ is found by observing that:

1. $\frac{P(x+dx) - P(x)}{dx} = -P(x)D\sigma$
2. the derivative of $P(x)$ is equal to itself times a constant
3. the probability of interaction for primary photons is independent of the distance travelled
4. none of the above, $P(x)$ is experimentally determined

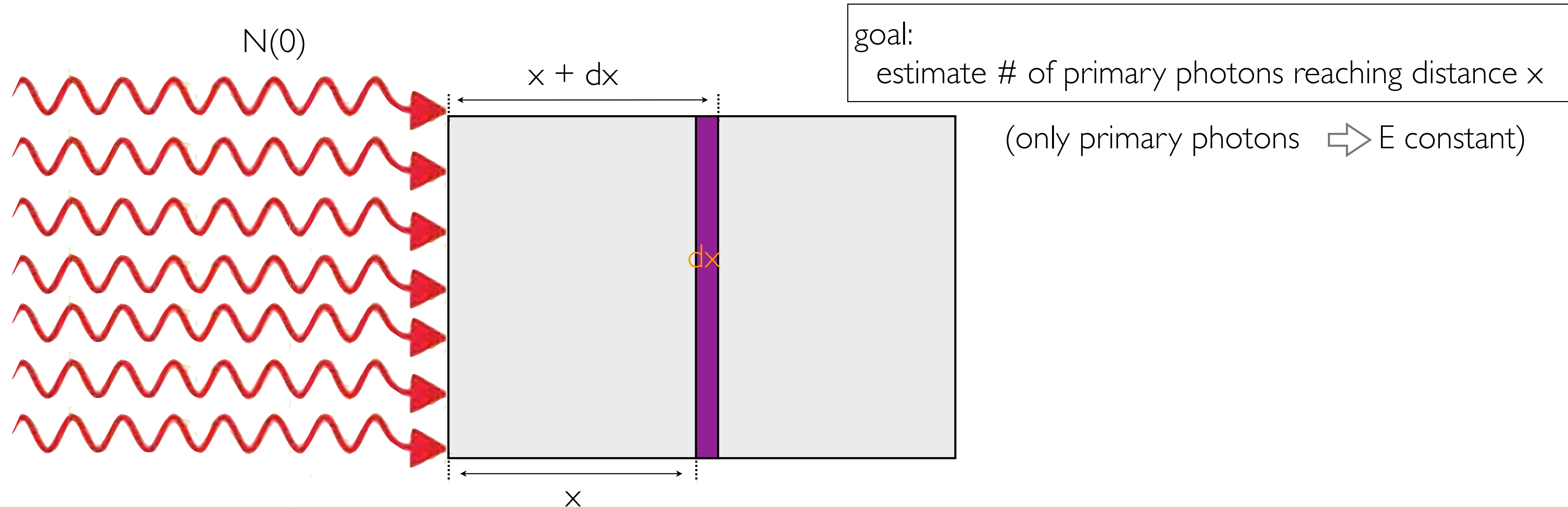
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Beer's law



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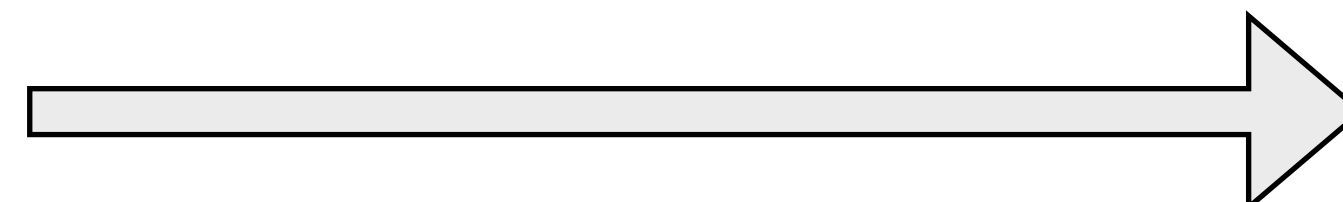
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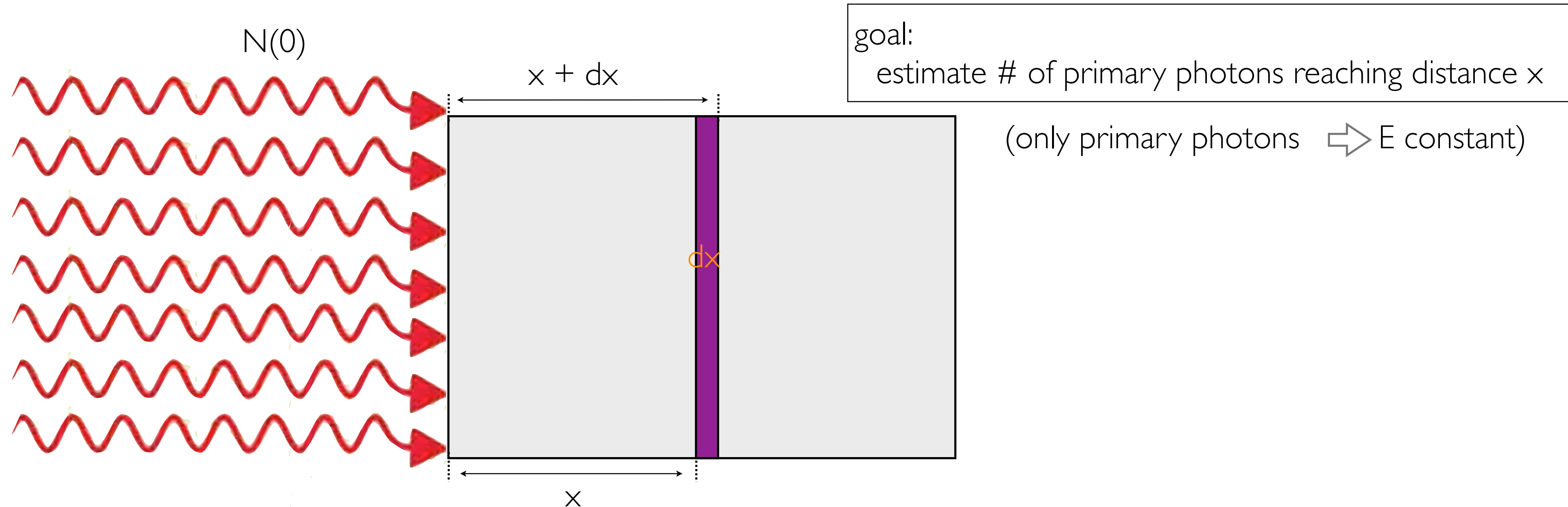


Beer's law

$$N(x) = N(0)e^{-\mu x}$$

$$P(x) = e^{-D\sigma x}$$

Beer's law



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linear attenuation coefficient

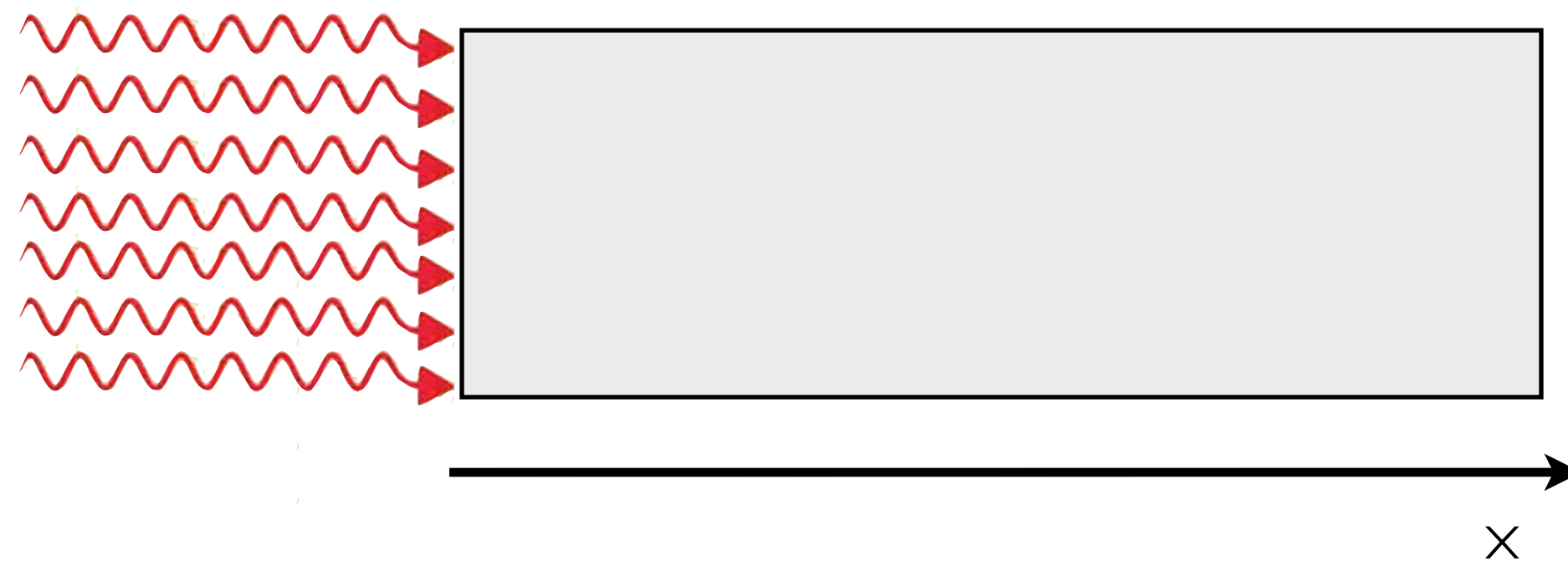
Beer's law

$$N(x) = N(0)e^{-\mu x}$$

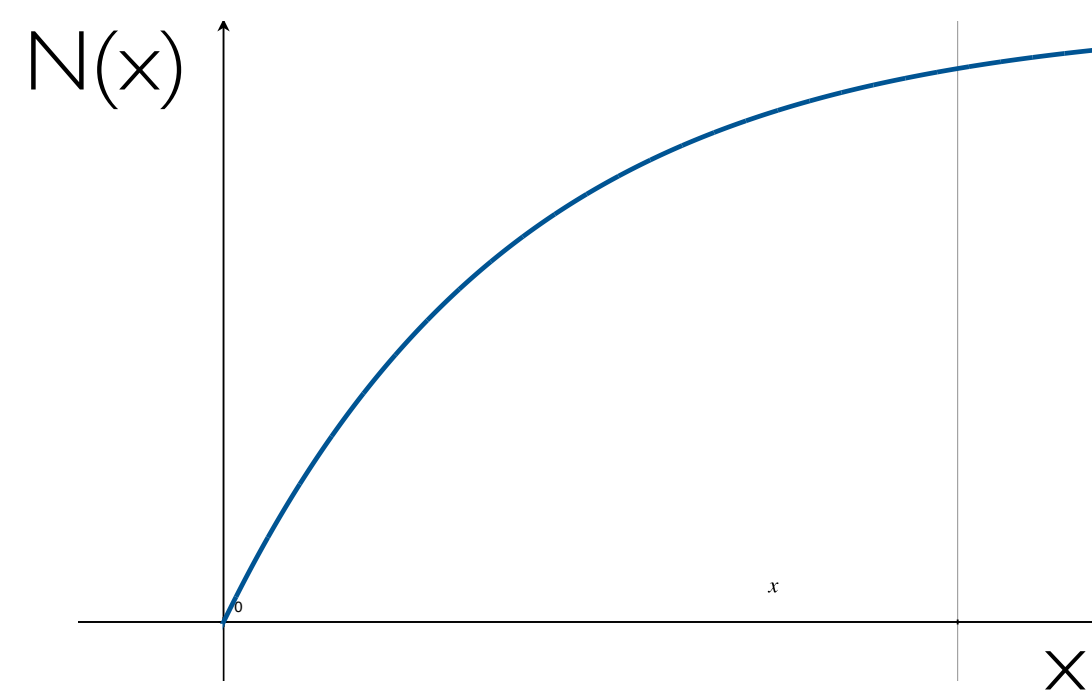
$$P(x) = e^{-D\sigma x}$$

Q:

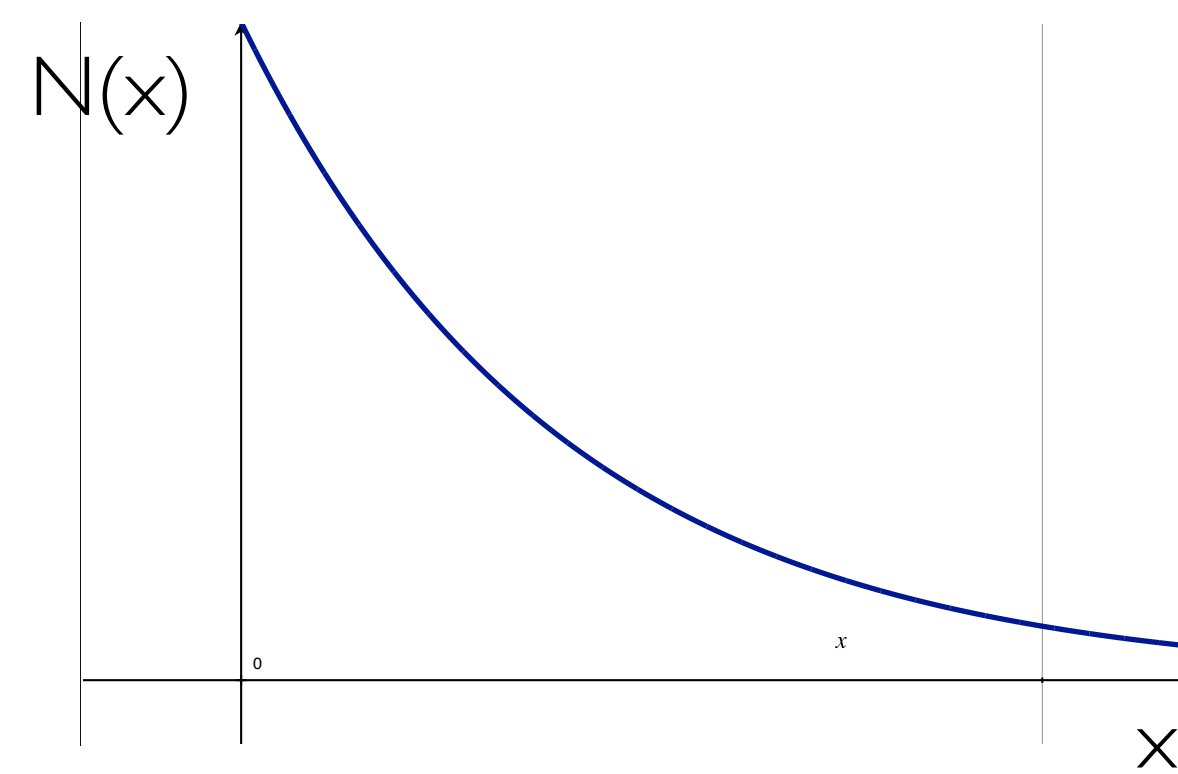
$N(0)$



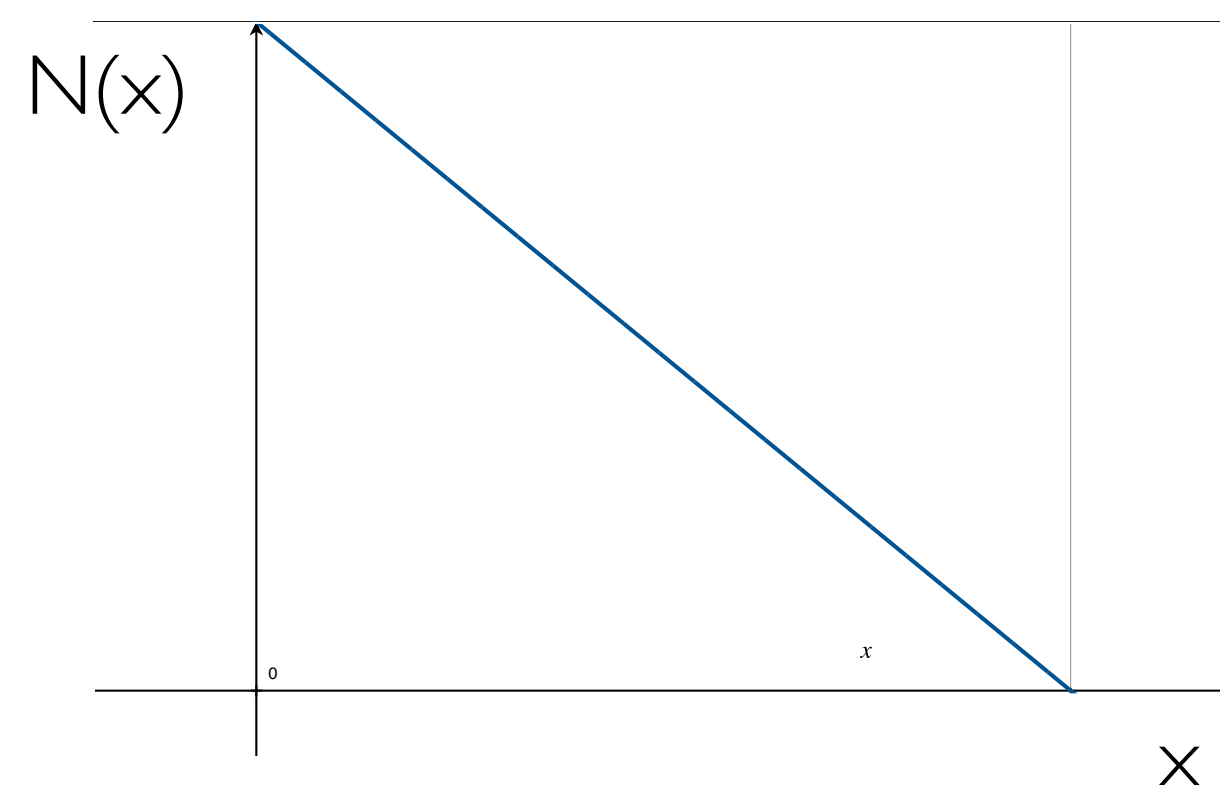
$N(x)$ looks like:



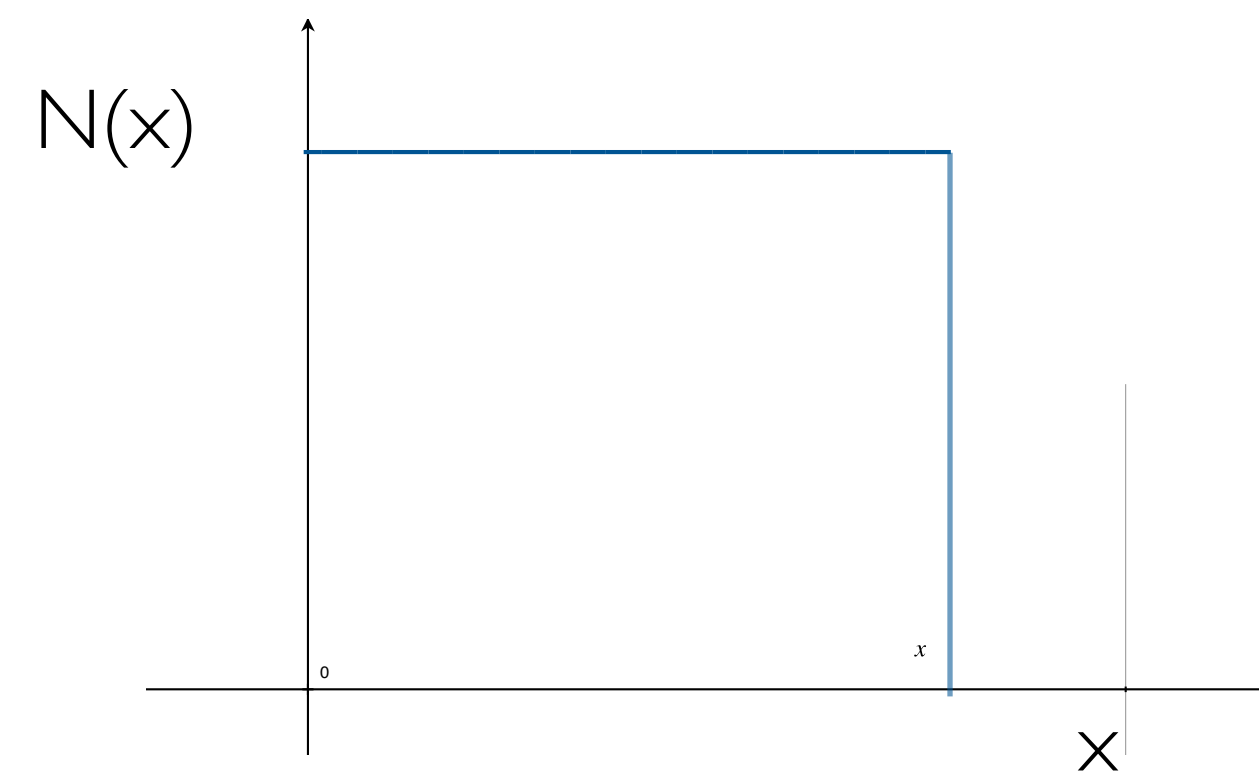
1



2



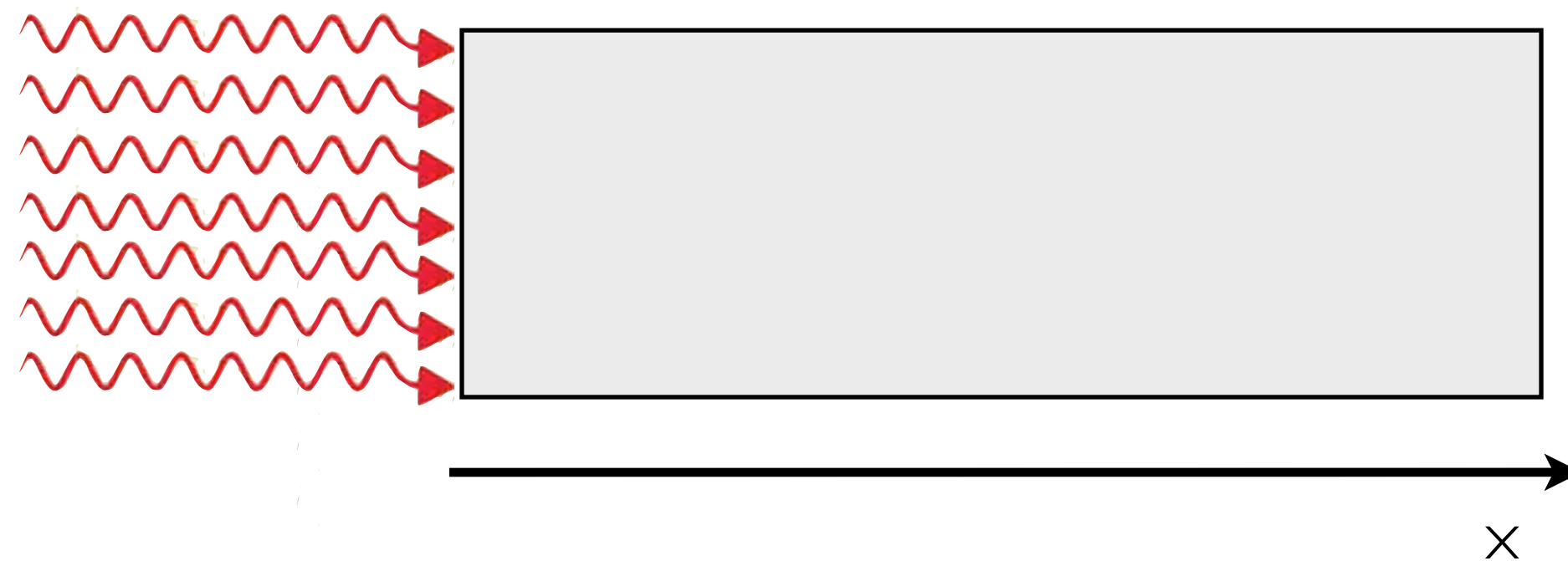
3



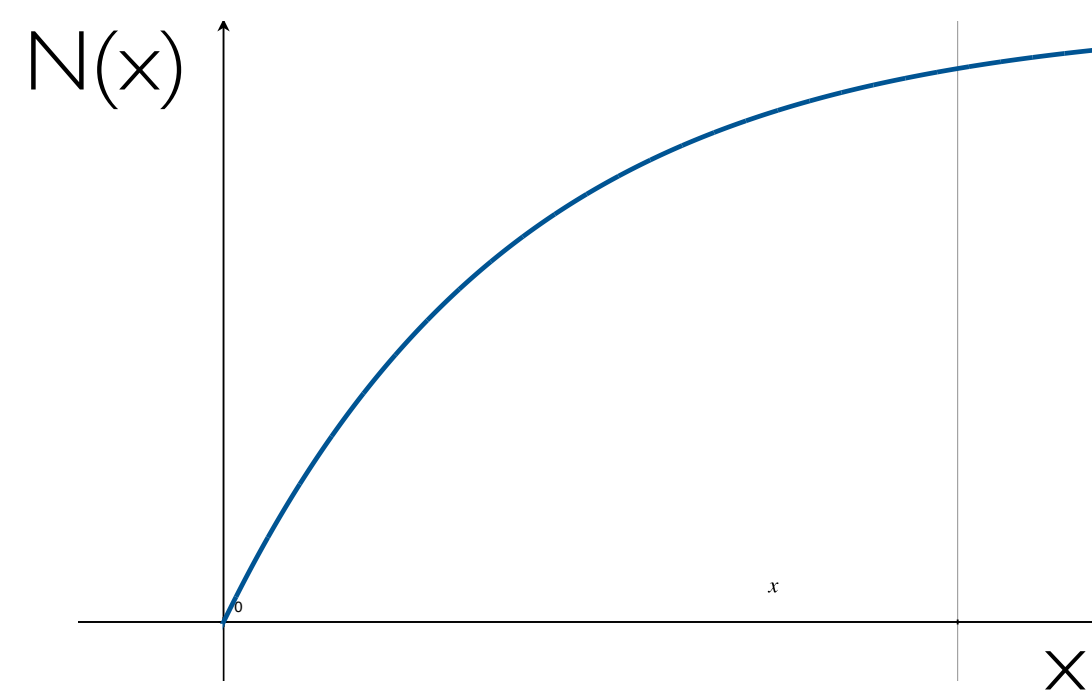
4

Q:

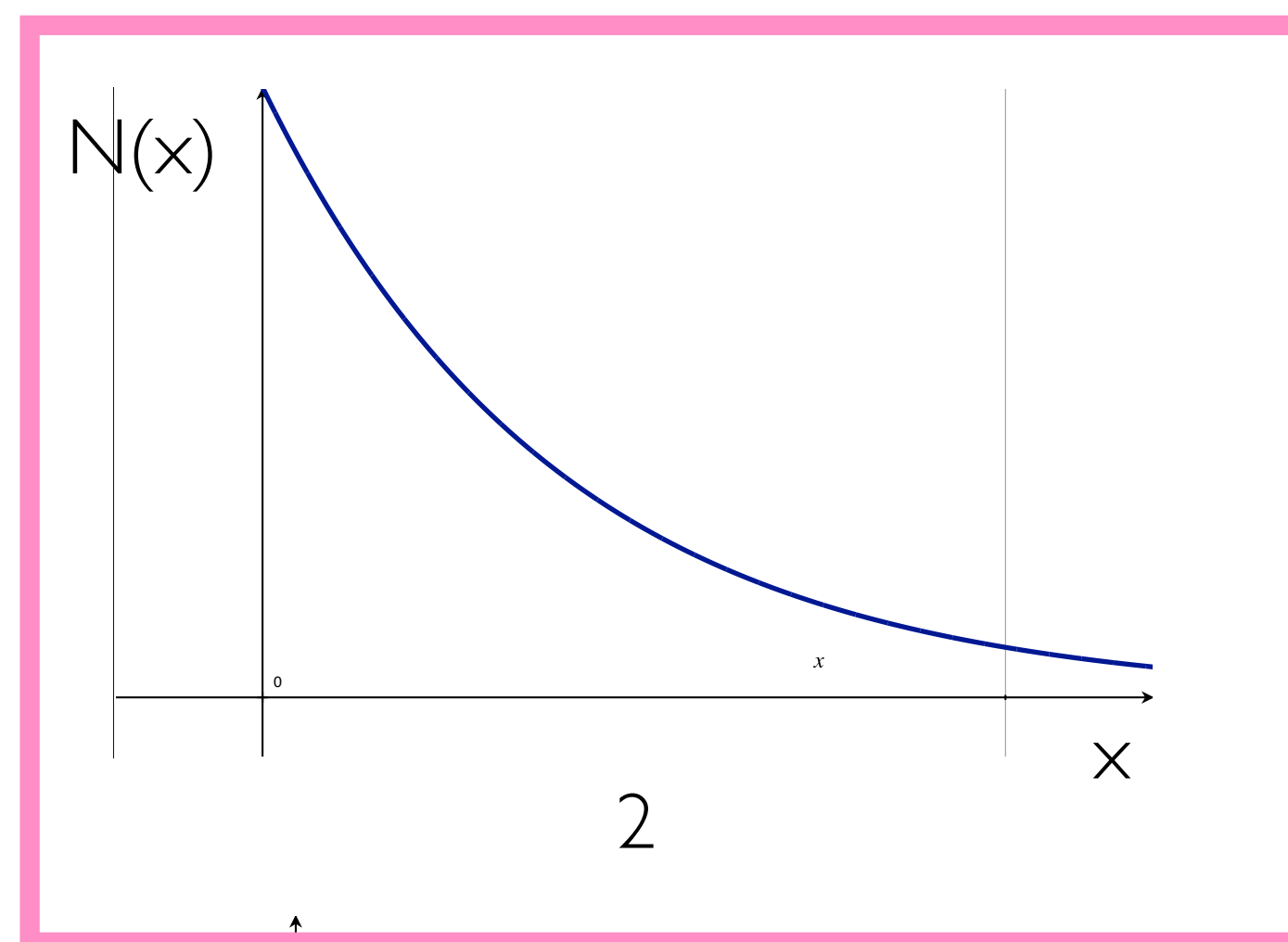
$N(0)$



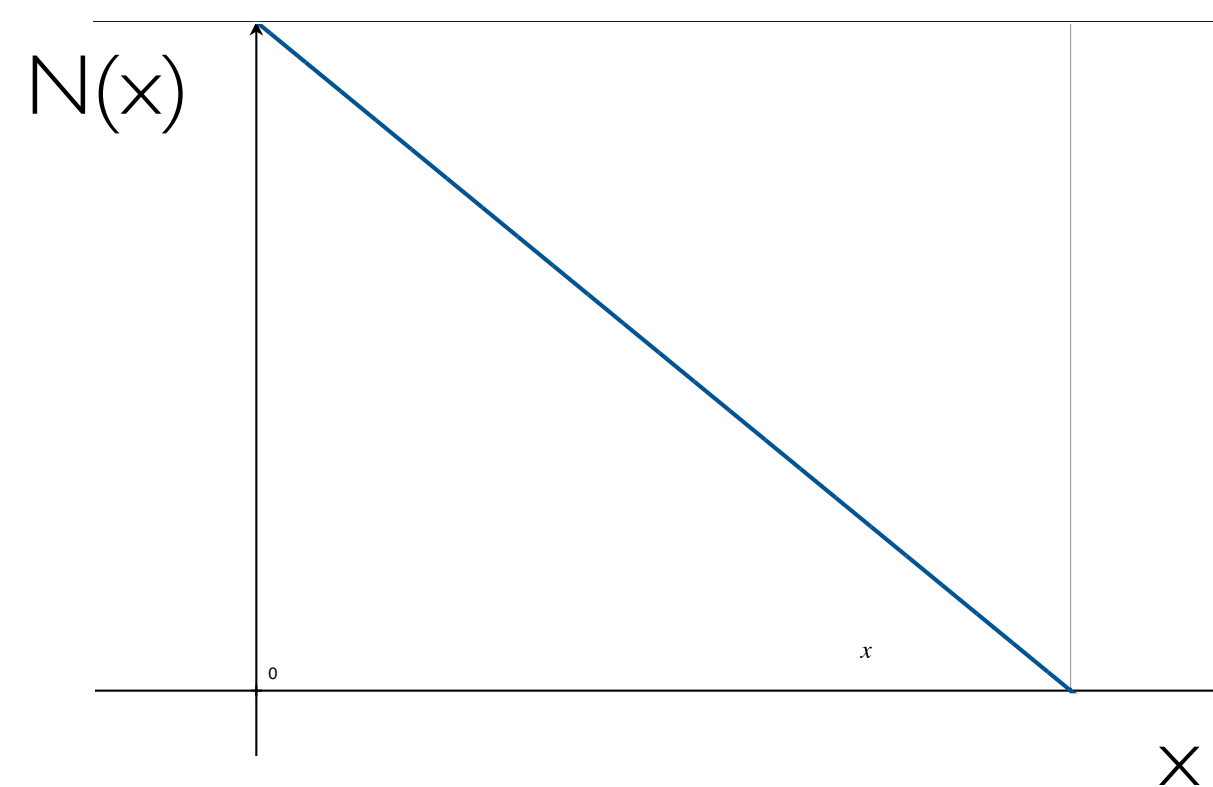
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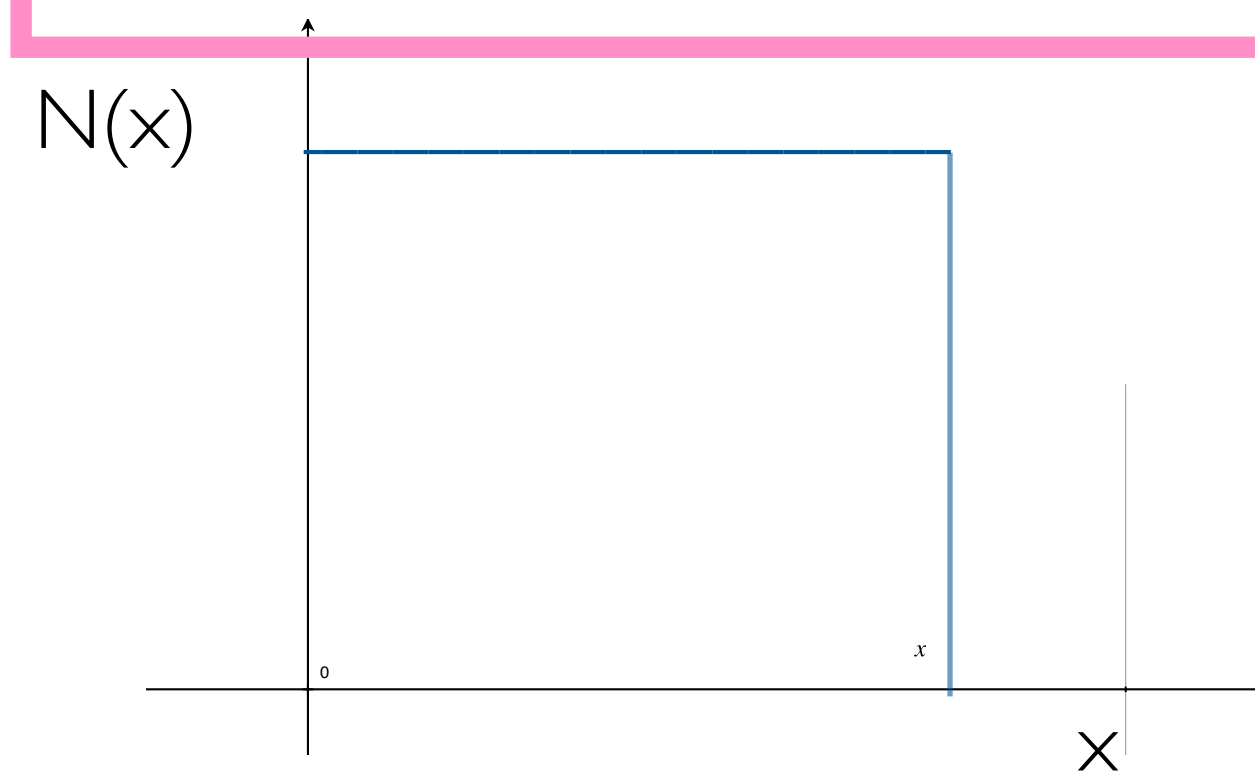
1



2



3



4

Q: How many photons are attenuated in a material with linear attenuation coefficient μ and thickness d , if N_0 photons impinge on it?

1. N_0

2. $N_0/2$

3. $N_0 e^{-\mu d}$

4. $N_0 - N_0 e^{-\mu d}$

5. $N_0 e^{-2\mu d}$

6. not sufficient data

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Definitions ahead

The fraction of incoming photons of energy E , going through a target of thickness t and linear attenuation coefficient μ , without interaction (or, equivalently the fraction of primary photons through a target):

$$\frac{N}{N_0} = e^{-\mu(E)t}$$

is called *attenuation factor*

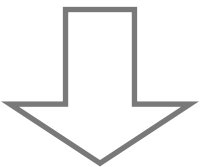
It's confusing, I know! But we're not here to change the world

The fraction of attenuated photons by a target is instead:

$$\frac{N_0 - N}{N_0} = 1 - e^{-\mu(E)t}$$

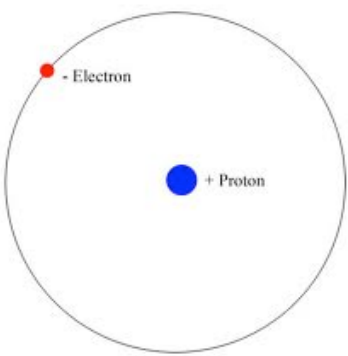
understanding at
macroscopic level
(Physics)

Compton scattering & Photoelectric effect are interactions between photons & electrons

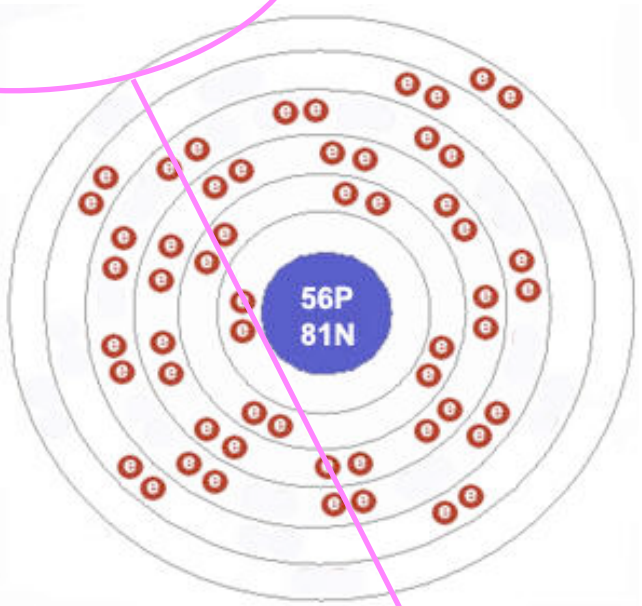
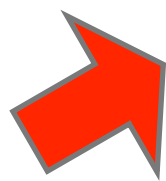


attenuation probability is proportional to electron density of the material

H
1 proton ($Z = 1$)
1 electron



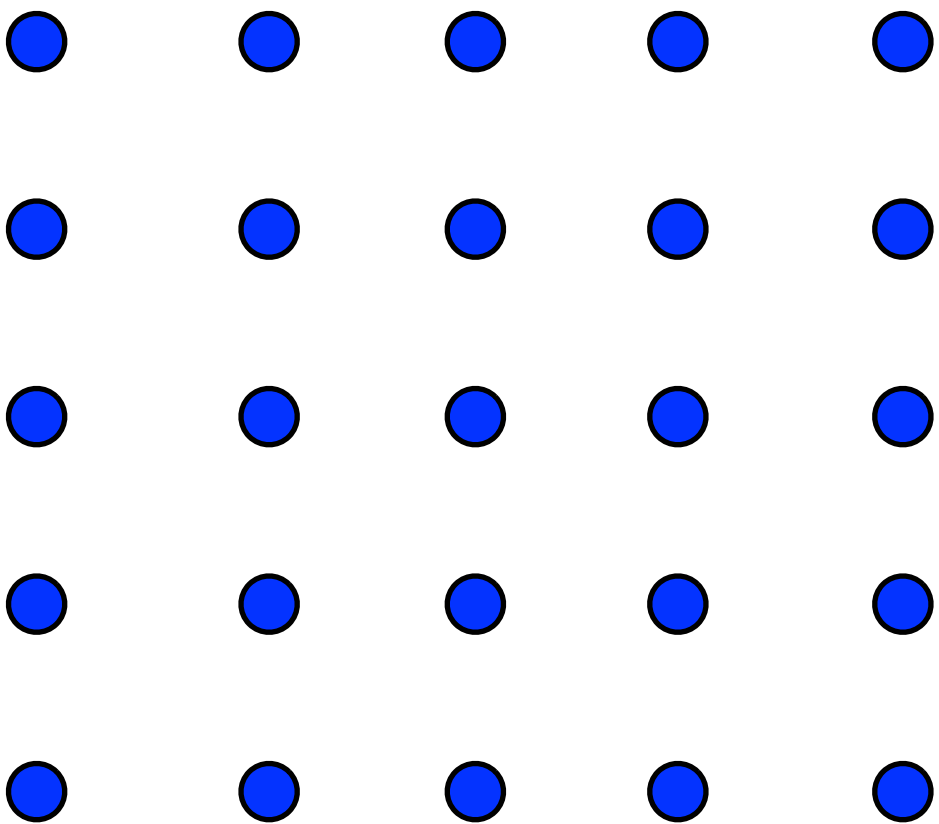
vs



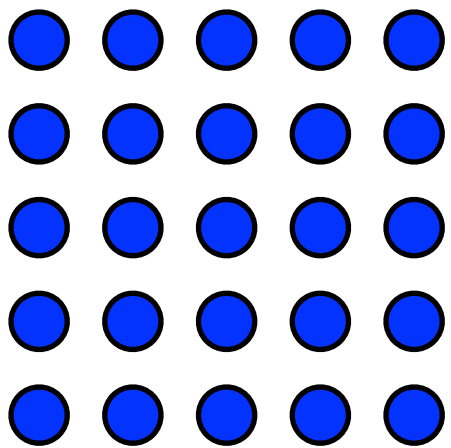
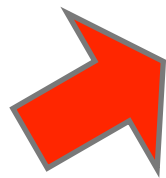
Ba

56 protons ($Z = 56$)
56 electrons
(81 neutrons)

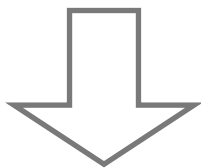
low density



vs



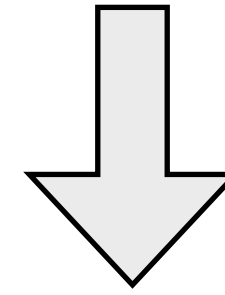
high density



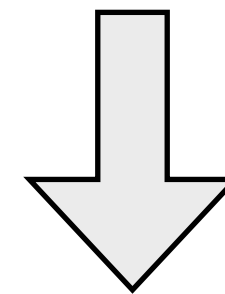
attenuation probability is proportional to Z & ρ

mass attenuation coefficient

linear attenuation coefficient, μ , depends on Z & ρ



same material in different phases or pressure will have different μ



more convenient to tabulate $\mu_{\rho} = \mu/\rho$

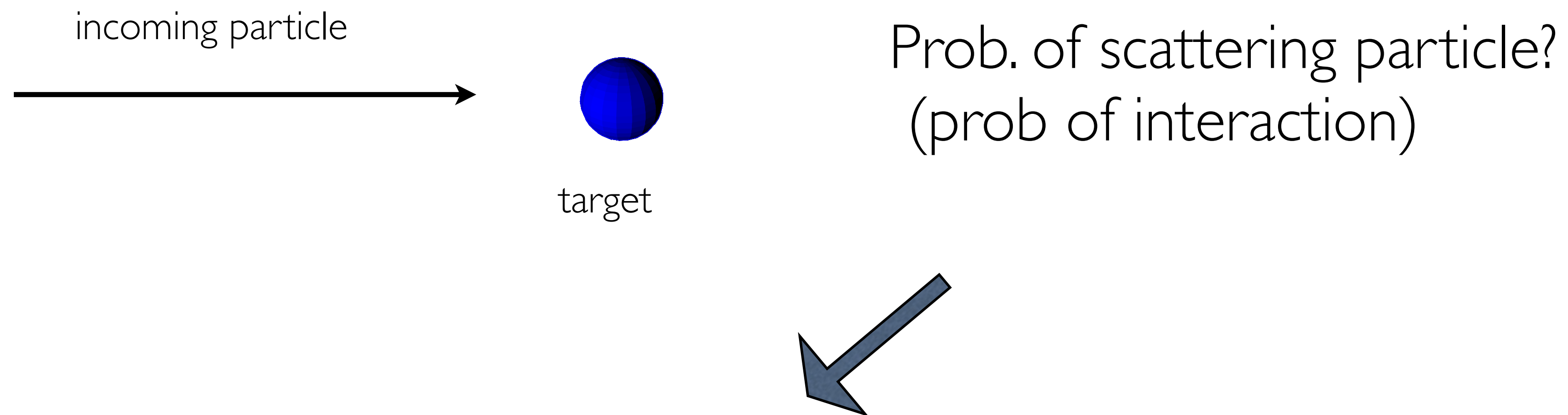
mass attenuation coefficient

<http://www.nist.gov/pml/data/xraycoef/>

(google: mass attenuation coefficient NIST)

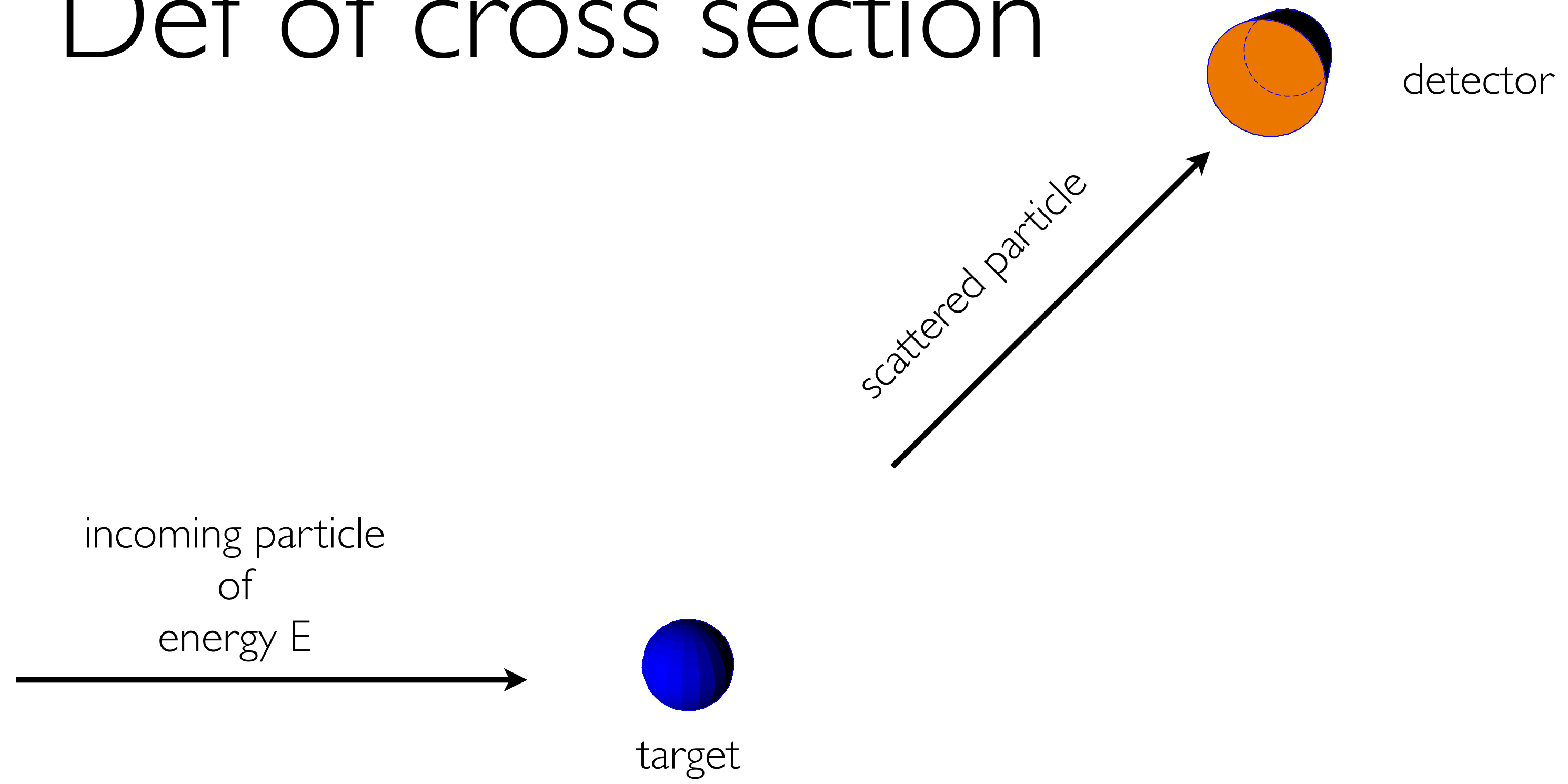
Physics:
Let's go microscopic!

cross section

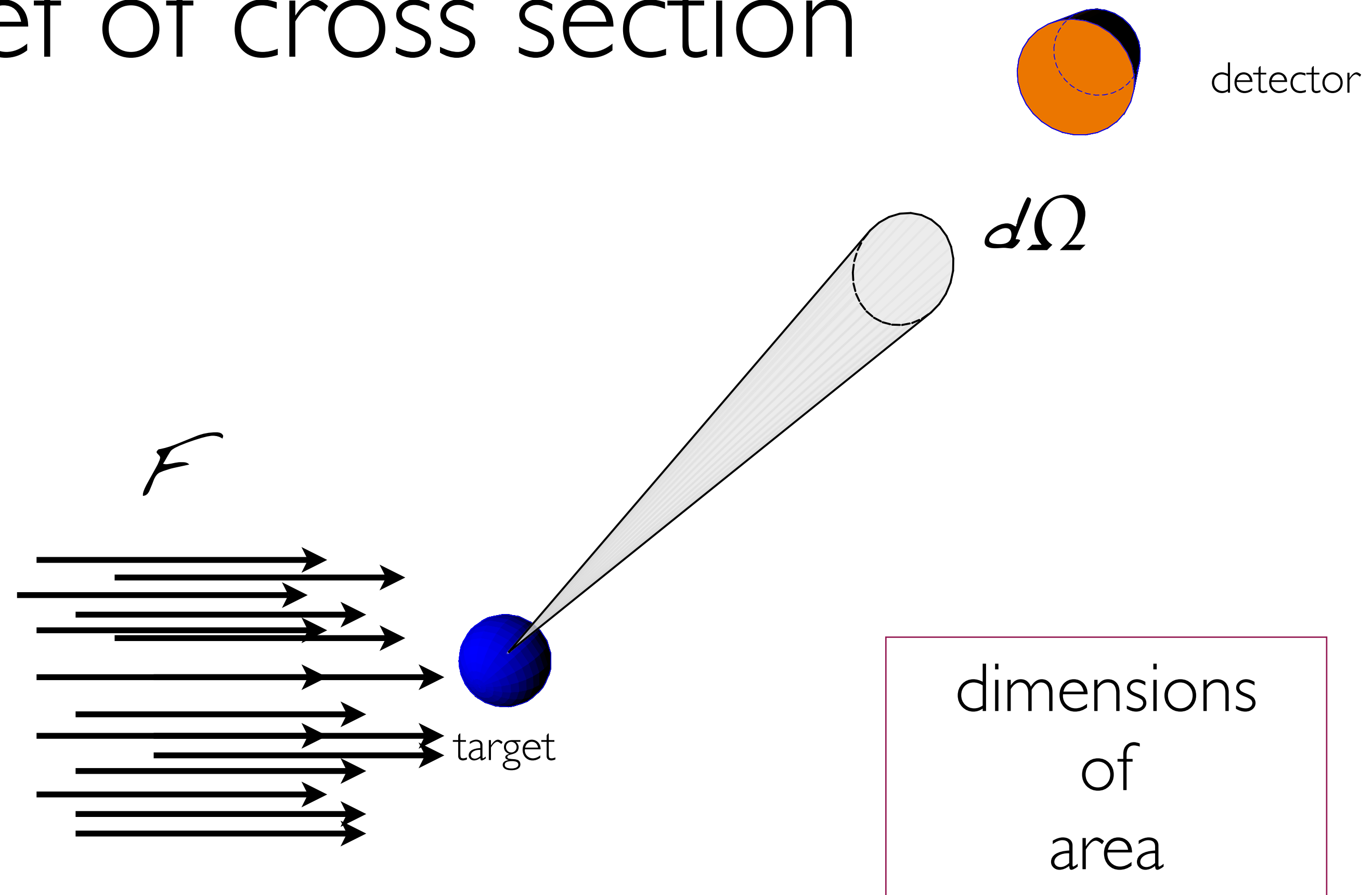


Prob. of scattering particle of energy E ?

Def of cross section



Def of cross section



Prob. of scattering particle
of energy E to angle Ω :

$$\frac{d\sigma(E, \Omega)}{d\Omega} \stackrel{\text{Def}}{=} \frac{1}{F} \frac{dN}{d\Omega}$$

Integrate over all angles
(measure at all Ω)
 $\Rightarrow$ prob. of scattering particle of
energy E :

$$\sigma(E, \Omega)$$

Mer om sambandet mellan σ , μ_ρ , μ

I g av materia av isotopen ${}^A_Z\text{X}$ innehåller $\frac{N_{\text{Avogadro}}}{A}$ atomer
(till exempel ${}^{26}_{13}\text{Al}$, ${}^{19}_{13}\text{Al}$)

och därför $\frac{N_{\text{Avogadro}}}{A}Z$ elektroner

Note on mass and linear attenuation coefficient

$$\sigma = \text{area / electron} \quad [L^2]$$

$$N_A \frac{m}{m_{\text{molar}} V} Z \sigma = \text{area / atom} \quad [L^2]$$

$$N_A \frac{\rho}{m_{\text{molar}}} Z \sigma = \text{area / volume} \quad [L^{-1}]$$

Note on mass and linear attenuation coefficient

$$\sigma = \text{area} / \text{electron} \quad [L^2]$$

$$Z\sigma = \text{area} / \text{atom} \quad [L^2]$$

linear attenuation
coefficient
 μ

$$N_A \frac{\rho}{m_{molar}} Z\sigma = \text{area} / \text{volume} \quad [L^{-1}]$$

mass attenuation
coefficient
 μ_ρ

$$N_A \frac{1}{m_{molar}} Z\sigma = \text{area} / \text{mass} \quad \left[\frac{L^2}{M} \right]$$

Tabulated!

Google: NIST mass attenuation coefficient
(till government shutdown är last titta på pdf I CANVAS)

Let's see if we understand
how to use this for clinical
applications!

Q: In order to better image veins a water solution of Ba is injected to patients before an x-ray examination. What is the reason for this?

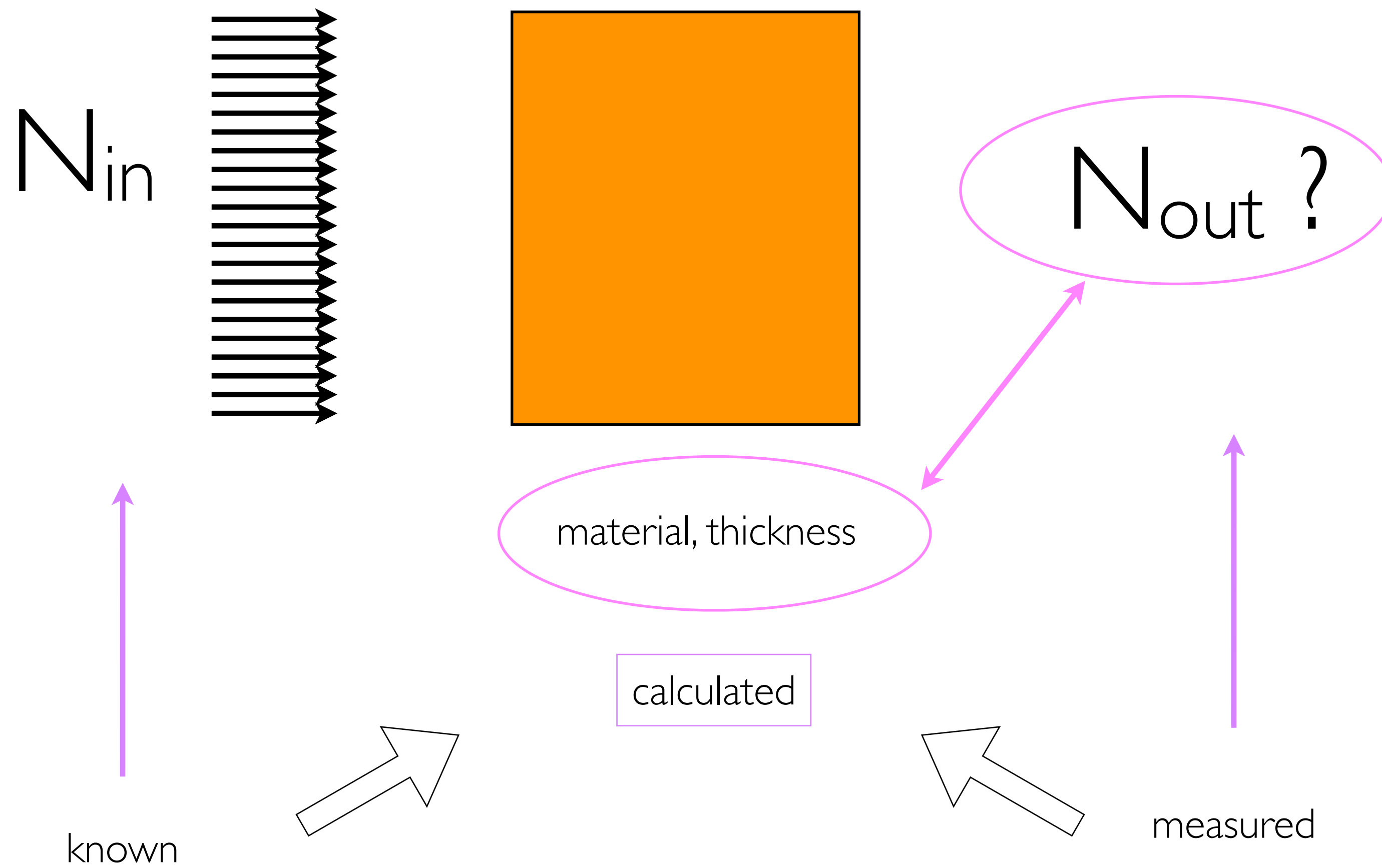
1. Ba has high Z , this will result in a much higher x-ray absorption than in surrounding tissue
2. Ba in standard condition has around 3,5 times the density of blood => the higher density will produce much higher absorption
3. both 1 and 2

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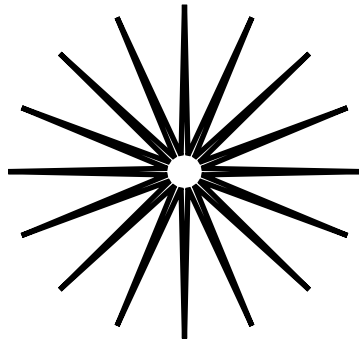
Physics:
back to a macroscopic

Transmission basic idea



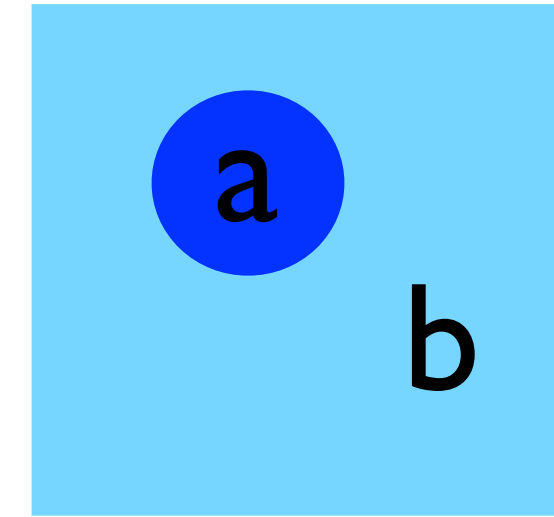
toy model

S



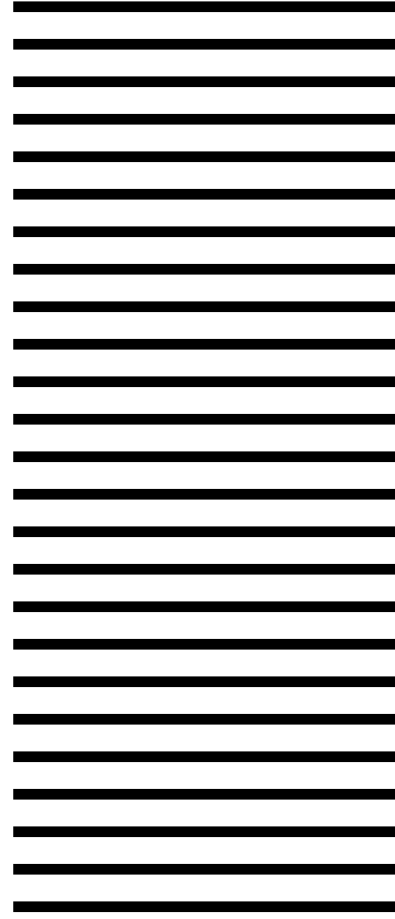
O

D

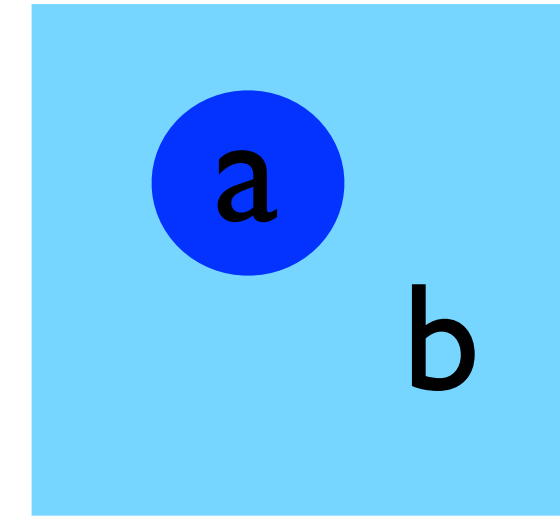


toy model

S



O



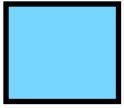
D



toy model



in order to hope to get an image:

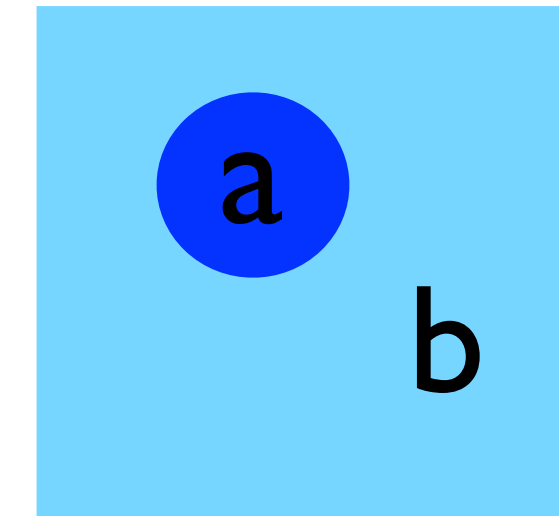
1.  and  must attenuate x-ray differently
2. D must translate x-rays in something visible or measurable
3. a lot of other things you'll know by the end of this week

toy model

S



O



D



• N_a

• N_b

object contrast defined as:

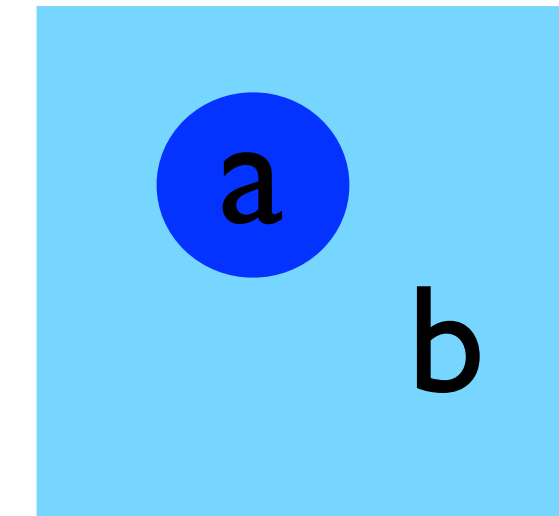
$$\frac{N_b - N_a}{N_b}$$

toy model

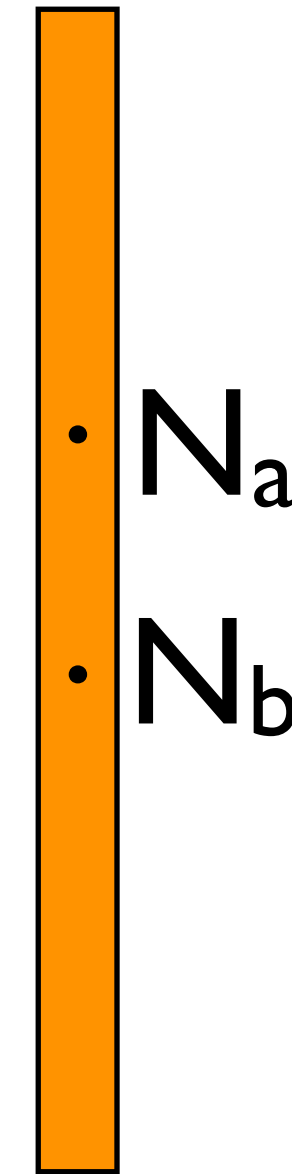
S



O



D



object contrast defined as:

$$\frac{N_b - N_a}{N_b}$$

toy model



object contrast depends on:

1. energy of incoming photons
2. thickness of b
3. thickness of a
4. difference in linear attenuation coefficient between a and b

recall: $\mu \equiv N_A \frac{\rho}{m_{\text{molar}}} Z \sigma(E)$

toy model



object contrast depends on:

1. energy of incoming photons
- ~~2. thickness of b~~
3. thickness of a
4. difference in linear attenuation coefficient between a and b

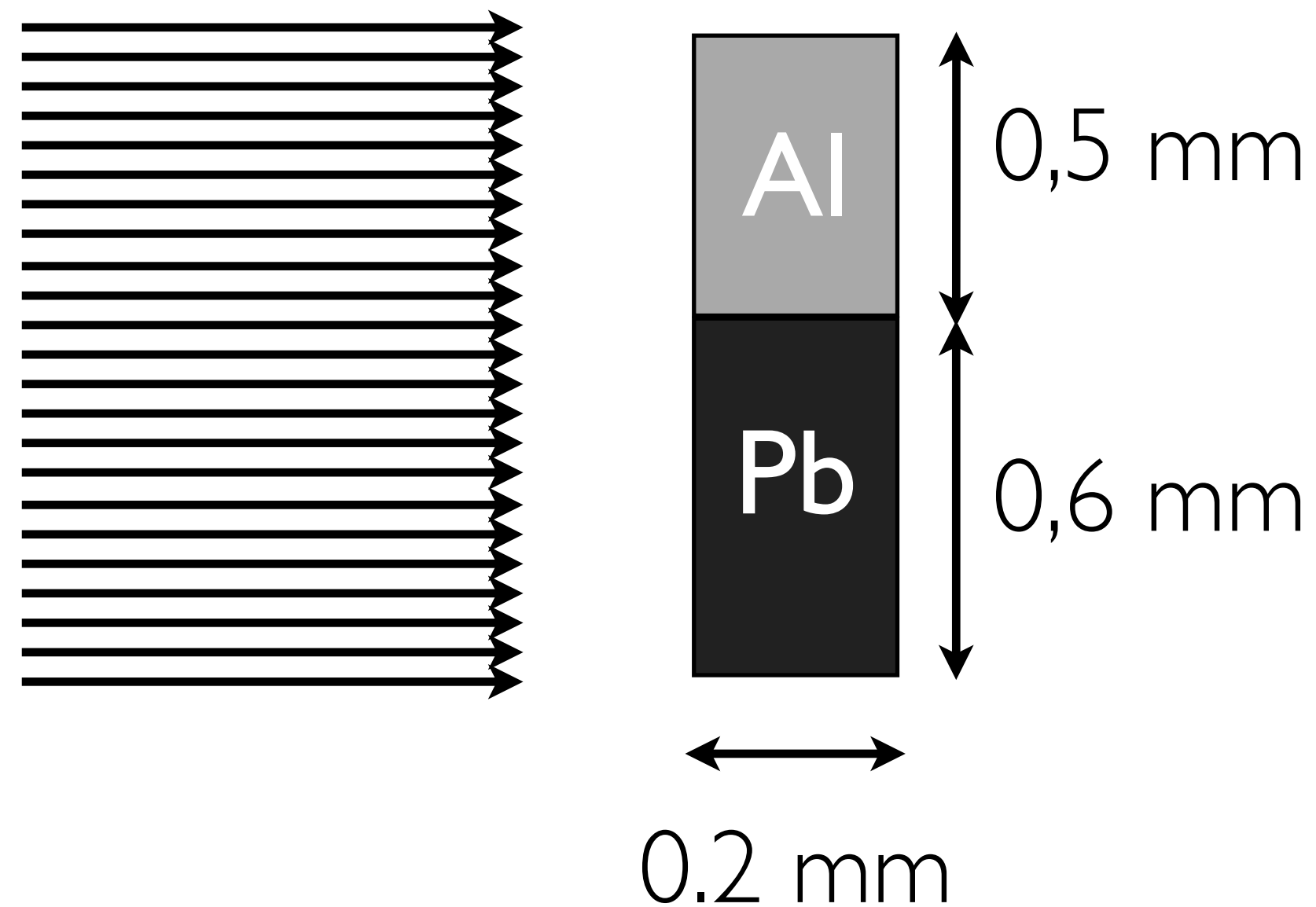
recall: $\mu \equiv N_A \frac{\rho}{m_{\text{molar}}} Z \sigma(E)$

object contrast:



object contrast: numerisk uppgift

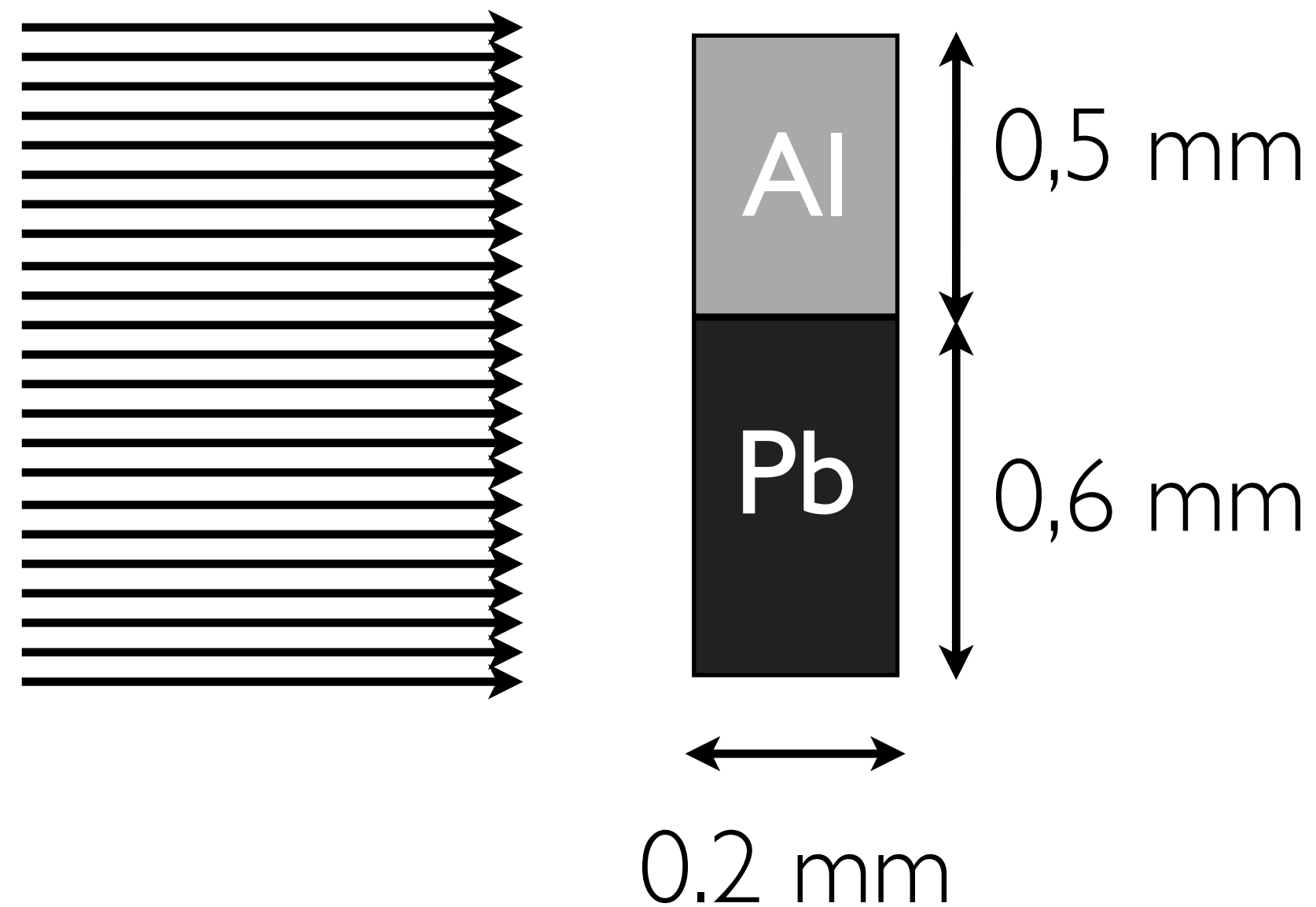
$E = 100 \text{ keV}$



$$= \frac{N_0 e^{-\mu_{Al}(100 \text{ keV})t_{Al}} - N_0 e^{-\mu_{Pb}(100 \text{ keV})t_{Pb}}}{N_0 e^{-\mu_{Al}(100 \text{ keV})t_{Al}}} = 1 - e^{-(\mu_{Pb}(100 \text{ keV}) - \mu_{Al}(100 \text{ keV}))t_{Al}}$$

object contrast: numerisk uppgift

$E = 100 \text{ keV}$



$$\text{Object contrast} = \frac{N \text{ through Al} - N \text{ through Pb}}{N \text{ through Al}} = \frac{N_0 e^{-\mu_{Al}(100 \text{ keV})t_{Al}} - N_0 e^{-\mu_{Pb}(100 \text{ keV})t_{Pb}}}{N_0 e^{-\mu_{Al}(100 \text{ keV})t_{Al}}} = 1 - e^{-(\mu_{Pb}(100 \text{ keV}) - \mu_{Al}(100 \text{ keV}))t_{Al}}$$

tables given by Hubbell in the International Journal of Applied Radiation and Isotopes **33**, 1269 (1982).

Note on NIST X-ray Attenuation Databases

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Elemental Media

or

Compounds & Mixtures

NIST Standard Reference Database 126

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edges are indicated by the shell designation.

Z	Element	Z	Element	Z	Element	Z	Element
1	H Hydrogen	24	Cr Chromium	47	Ag Silver	70	Yb Ytterbium
2	He Helium	25	Mn Manganese	48	Cd Cadmium	71	Lu Lutetium
3	Li Lithium	26	Fe Iron	49	In Indium	72	Hf Hafnium
4	Be Beryllium	27	Co Cobalt	50	Sn Tin	73	Ta Tantalum
5	B Boron	28	Ni Nickel	51	Sb Antimony	74	W Tungsten
6	C Carbon, Graphite	29	Cu Copper	52	Te Tellurium	75	Re Rhenium
7	N Nitrogen	30	Zn Zinc	53	I Iodine	76	Os Osmium
8	O Oxygen	31	Ga Gallium	54	Xe Xenon	77	Ir Iridium
9	F Fluorine	32	Ge Germanium	55	Cs Cesium	78	Pt Platinum
10	Ne Neon	33	As Arsenic	56	Ba Barium	79	Au Gold
11	Na Sodium	34	Se Selenium	57	La Lanthanum	80	Hg Mercury
12	Mg Magnesium	35	Br Bromine	58	Ce Cerium	81	Tl Thallium
13	Al Aluminum	36	Kr Krypton	59	Pr Praseodymium	82	Pb Lead
14	Si Silicon	37	Rb Rubidium	60	Nd Neodymium	83	Bi Bismuth
15	P Phosphorus	38	Sr Strontium	61	Pm Promethium	84	Po Polonium
16	S Sulfur	39	Y Yttrium	62	Sm Samarium	85	At Astatine
17	Cl Chlorine	40	Zr Zirconium	63	Eu Europium	86	Rn Radon
18	Ar Argon	41	Nb Niobium	64	Gd Gadolinium	87	Fr Francium
19	K Potassium	42	Mo Molybdenum	65	Tb Terbium	88	Ra Radium
20	Ca Calcium	43	Tc Technetium	66	Dy Dysprosium	89	Ac Actinium
21	Sc Scandium	44	Ru Ruthenium	67	Ho Holmium	90	Th Thorium
22	Ti Titanium	45	Rh Rhodium	68	Er Erbium	91	Pa Protactinium
23	V Vanadium	46	Pd Palladium	69	Tm Thulium	92	U Uranium

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Al

	Energy (MeV)	μ/ρ (cm ² /g)	μ_{en}/ρ (cm ² /g)
	1.00000E-03	1.185E+03	1.183E+03
	1.50000E-03	4.022E+02	4.001E+02
	1.55960E-03	3.621E+02	3.600E+02
K	1.55960E-03	3.957E+03	3.829E+03
	2.00000E-03	2.263E+03	2.204E+03
	3.00000E-03	7.880E+02	7.732E+02
	4.00000E-03	3.605E+02	3.545E+02
	5.00000E-03	1.934E+02	1.902E+02
	6.00000E-03	1.153E+02	1.133E+02
	8.00000E-03	5.033E+01	4.918E+01
	1.00000E-02	2.623E+01	2.543E+01
	1.50000E-02	7.955E+00	7.487E+00
	2.00000E-02	3.441E+00	3.094E+00
	3.00000E-02	1.128E+00	8.778E-01
	4.00000E-02	5.685E-01	3.601E-01
	5.00000E-02	3.681E-01	1.840E-01
	6.00000E-02	2.778E-01	1.099E-01
	8.00000E-02	2.018E-01	5.511E-02
	1.00000E-01	1.704E-01	3.794E-02
	1.50000E-01	1.378E-01	2.827E-02
	2.00000E-01	1.223E-01	2.745E-02
	3.00000E-01	1.042E-01	2.816E-02
	4.00000E-01	9.276E-02	2.862E-02
	5.00000E-01	8.445E-02	2.868E-02
	6.00000E-01	7.802E-02	2.851E-02
	8.00000E-01	6.841E-02	2.778E-02
	1.00000E+00	6.146E-02	2.686E-02
	1.25000E+00	5.496E-02	2.565E-02
	1.50000E+00	5.006E-02	2.451E-02

	Energy (MeV)	μ/ρ (cm ² /g)	μ_{en}/ρ (cm ² /g)
	1.00000E-03	1.185E+03	1.183E+03
	1.50000E-03	4.022E+02	4.001E+02
	1.55960E-03	3.621E+02	3.600E+02
K	1.55960E-03	3.957E+03	3.829E+03
	2.00000E-03	2.263E+03	2.204E+03
	3.00000E-03	7.880E+02	7.732E+02
	4.00000E-03	3.605E+02	3.545E+02
	5.00000E-03	1.934E+02	1.902E+02
	6.00000E-03	1.153E+02	1.133E+02
	8.00000E-03	5.033E+01	4.918E+01
	1.00000E-02	2.623E+01	2.543E+01
	1.50000E-02	7.955E+00	7.487E+00
	2.00000E-02	3.441E+00	3.094E+00
	3.00000E-02	1.128E+00	8.778E-01
	4.00000E-02	5.685E-01	3.601E-01
	5.00000E-02	3.681E-01	1.840E-01
	6.00000E-02	2.778E-01	1.099E-01
	8.00000E-02	2.018E-01	5.511E-02
	1.00000E-01	1.704E-01	3.794E-02
	1.50000E-01	1.378E-01	2.827E-02
	2.00000E-01	1.223E-01	2.745E-02
	3.00000E-01	1.042E-01	2.816E-02
	4.00000E-01	9.276E-02	2.862E-02
	5.00000E-01	8.445E-02	2.868E-02
	6.00000E-01	7.802E-02	2.851E-02
	8.00000E-01	6.841E-02	2.778E-02
	1.00000E+00	6.146E-02	2.686E-02
	1.25000E+00	5.496E-02	2.565E-02
	1.50000E+00	5.006E-02	2.451E-02
	2.00000E+00	4.324E-02	2.266E-02
	3.00000E+00	3.541E-02	2.024E-02
	4.00000E+00	3.106E-02	1.882E-02
	5.00000E+00	2.836E-02	1.795E-02
	6.00000E+00	2.655E-02	1.739E-02
	8.00000E+00	2.437E-02	1.678E-02
	1.00000E+01	2.318E-02	1.650E-02
	1.50000E+01	2.195E-02	1.631E-02
	2.00000E+01	2.168E-02	1.633E-02

Al

	Energy (MeV)	μ/ρ (cm ² /g)	μ_{en}/ρ (cm ² /g)
K	1.00000E-03	1.185E+03	1.183E+03
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	1.55960E-03	3.957E+03	3.829E+03
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	3.00000E-03	7.880E+02	7.732E+02
	4.00000E-03	3.605E+02	3.545E+02
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	1.00000E-02	2.623E+01	2.543E+01
	1.50000E-02	7.955E+00	7.487E+00
	2.00000E-02	3.441E+00	3.094E+00
	3.00000E-02	1.128E+00	8.778E-01
	4.00000E-02	5.685E-01	3.601E-01
	5.00000E-02	3.681E-01	1.840E-01
	6.00000E-02	2.778E-01	1.099E-01
	8.00000E-02	2.018E-01	5.511E-02
	1.00000E-01	1.704E-01	3.794E-02
	1.50000E-01	1.378E-01	2.827E-02
100 keV	2.00000E-01	1.223E-01	2.745E-02
	3.00000E-01	1.042E-01	2.816E-02
	4.00000E-01	9.276E-02	2.862E-02
	5.00000E-01	8.445E-02	2.868E-02
	6.00000E-01	7.802E-02	2.851E-02
	8.00000E-01	6.841E-02	2.778E-02
	1.00000E+00	6.146E-02	2.686E-02
	1.25000E+00	5.496E-02	2.565E-02
	1.50000E+00	5.006E-02	2.451E-02

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	1.55960E-03	3.957E+03	3.829E+03
	2.00000E-03	2.263E+03	2.204E+03
	3.00000E-03	7.880E+02	7.732E+02
	4.00000E-03	3.605E+02	3.545E+02
	5.00000E-03	1.934E+02	1.902E+02
	6.00000E-03	1.153E+02	1.133E+02
	8.00000E-03	5.033E+01	4.918E+01
	1.00000E-02	2.623E+01	2.543E+01
	1.50000E-02	7.955E+00	7.487E+00
	2.00000E-02	3.441E+00	3.094E+00
	3.00000E-02	1.128E+00	8.778E-01
	4.00000E-02	5.685E-01	3.601E-01
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	8.00000E-01	6.841E-02	2.778E-02
	1.00000E+00	6.146E-02	2.686E-02
	1.25000E+00	5.496E-02	2.565E-02
	1.50000E+00	5.006E-02	2.451E-02
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	3.00000E+00	3.541E-02	2.024E-02
	4.00000E+00	3.106E-02	1.882E-02
	5.00000E+00	2.836E-02	1.795E-02
	6.00000E+00	2.655E-02	1.739E-02
	8.00000E+00	2.437E-02	1.678E-02
	1.00000E+01	2.318E-02	1.650E-02
	1.50000E+01	2.195E-02	1.631E-02
	2.00000E+01	2.168E-02	1.633E-02

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Z	Element	Z/A	I (eV)	Density (g/cm ³)
1 H	Hydrogen	0.99212	19.2	8.375E-05
2 He	Helium	0.49968	41.8	1.663E-04
3 Li	Lithium	0.43221	40.0	5.340E-01
4 Be	Beryllium	0.44384	63.7	1.848E+00
5 B	Boron	0.46245	76.0	2.370E+00
6 C	Carbon, Graphite	0.49954	78.0	1.700E+00
7 N	Nitrogen	0.49976	82.0	1.165E-03
8 O	Oxygen	0.50002	95.0	1.332E-03
9 F	Fluorine	0.47372	115.0	1.580E-03
10 Ne	Neon	0.49555	137.0	8.385E-04
11 Na	Sodium	0.47847	149.0	9.710E-01
12 Mg	Magnesium	0.49373	156.0	1.740E+00
13 Al	Aluminum	0.48181	166.0	2.699E+00
14 Si	Silicon	0.49848	173.0	2.330E+00
15 P	Phosphorus	0.48428	173.0	2.200E+00
16 S	Sulfur	0.49897	180.0	2.000E+00
17 Cl	Chlorine	0.47951	174.0	2.995E-03
18 Ar	Argon	0.45059	188.0	1.662E-03
19 K	Potassium	0.48595	190.0	8.620E-01
20 Ca	Calcium	0.49903	191.0	1.550E+00

Z	Element	Z/A	I (eV)	Density (g/cm ³)
1 H	Hydrogen	0.99212	19.2	8.375E-05
2 He	Helium	0.49968	41.8	1.663E-04
3 Li	Lithium	0.43221	40.0	5.340E-01
4 Be	Beryllium	0.44384	63.7	1.848E+00
5 B	Boron	0.46245	76.0	2.370E+00
6 C	Carbon, Graphite	0.49954	78.0	1.700E+00
7 N	Nitrogen	0.49976	82.0	1.165E-03
8 O	Oxygen	0.50002	95.0	1.332E-03
9 F	Fluorine	0.47372	115.0	1.580E-03
10 Ne	Neon	0.49555	137.0	8.385E-04
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13 Al	Aluminum	0.48181	166.0	2.699E+00
14 Si	Silicon	0.49848	173.0	2.330E+00
15 P	Phosphorus	0.48428	173.0	2.200E+00
16 S	Sulfur	0.49897	180.0	2.000E+00
17 Cl	Chlorine	0.47951	174.0	2.995E-03
18 Ar	Argon	0.45059	188.0	1.662E-03
19 K	Potassium	0.48595	190.0	8.620E-01
20 Ca	Calcium	0.49903	191.0	1.550E+00

Pb

HTML table format

	Energy (MeV)	μ/ρ (cm ² /g)	μ_{en}/ρ (cm ² /g)
	1.00000E-03	5.210E+03	5.197E+03
	1.50000E-03	2.356E+03	2.344E+03
	2.00000E-03	1.285E+03	1.274E+03
	2.48400E-03	8.006E+02	7.895E+02
M5	2.48400E-03	1.397E+03	1.366E+03
	2.53429E-03	1.726E+03	1.682E+03
	2.58560E-03	1.944E+03	1.895E+03
M4	2.58560E-03	2.458E+03	2.390E+03
	3.00000E-03	1.965E+03	1.913E+03
	3.06640E-03	1.857E+03	1.808E+03
M3	3.06640E-03	2.146E+03	2.090E+03
	3.30130E-03	1.796E+03	1.748E+03
	3.55420E-03	1.496E+03	1.459E+03
M2	3.55420E-03	1.585E+03	1.546E+03
	3.69948E-03	1.442E+03	1.405E+03
	3.85070E-03	1.311E+03	1.279E+03
M1	3.85070E-03	1.368E+03	1.335E+03
	4.00000E-03	1.251E+03	1.221E+03
	5.00000E-03	7.304E+02	7.124E+02
	6.00000E-03	4.672E+02	4.546E+02
	8.00000E-03	2.287E+02	2.207E+02
	1.00000E-02	1.306E+02	1.247E+02
	1.30352E-02	6.701E+01	6.270E+01
L3	1.30352E-02	1.621E+02	1.291E+02
	1.50000E-02	1.116E+02	9.100E+01
	1.52000E-02	1.078E+02	8.807E+01
L2	1.52000E-02	1.485E+02	1.131E+02
	1.55269E-02	1.416E+02	1.083E+02
	1.58608E-02	1.344E+02	1.032E+02
L1	1.58608E-02	1.548E+02	1.180E+02
	2.00000E-02	8.636E+01	6.899E+01
	3.00000E-02	3.032E+01	2.536E+01
	4.00000E-02	1.436E+01	1.211E+01
	5.00000E-02	8.041E+00	6.740E+00
	6.00000E-02	5.021E+00	4.149E+00
	8.00000E-02	2.419E+00	1.916E+00
	8.80045E-02	1.910E+00	1.482E+00
K	8.80045E-02	7.683E+00	2.160E+00
	1.00000E-01	5.549E+00	1.976E+00
	1.50000E-01	2.014E+00	1.056E+00
	2.00000E-01	9.985E-01	5.870E-01
	3.00000E-01	4.031E-01	2.455E-01
	4.00000E-01	2.323E-01	1.370E-01
	5.00000E-01	1.614E-01	9.128E-02
	6.00000E-01	1.248E-01	6.819E-02
	8.00000E-01	8.870E-02	4.644E-02
	1.00000E+00	7.102E-02	3.654E-02
	1.25000E+00	5.876E-02	2.988E-02

ASCII format

	Energy (MeV)	μ/ρ (cm ² /g)	μ_{en}/ρ (cm ² /g)
	1.00000E-03	5.210E+03	5.197E+03
	1.50000E-03	2.356E+03	2.344E+03
	2.00000E-03	1.285E+03	1.274E+03
	2.48400E-03	8.006E+02	7.895E+02
M5	2.48400E-03	1.397E+03	1.366E+03
	2.53429E-03	1.726E+03	1.682E+03
	2.58560E-03	1.944E+03	1.895E+03
M4	2.58560E-03	2.458E+03	2.390E+03
	3.00000E-03	1.965E+03	1.913E+03
	3.06640E-03	1.857E+03	1.808E+03
M3	3.06640E-03	2.146E+03	2.090E+03
	3.30130E-03	1.796E+03	1.748E+03
	3.55420E-03	1.496E+03	1.459E+03
M2	3.55420E-03	1.585E+03	1.546E+03
	3.69948E-03	1.442E+03	1.405E+03
	3.85070E-03	1.311E+03	1.279E+03
M1	3.85070E-03	1.368E+03	1.335E+03
	4.00000E-03	1.251E+03	1.221E+03
	5.00000E-03	7.304E+02	7.124E+02
	6.00000E-03	4.672E+02	4.546E+02
	8.00000E-03	2.287E+02	2.207E+02
	1.00000E-02	1.306E+02	1.247E+02
	1.30352E-02	6.701E+01	6.270E+01
L3	1.30352E-02	1.621E+02	1.291E+02
	1.50000E-02	1.116E+02	9.100E+01
	1.52000E-02	1.078E+02	8.807E+01
L2	1.52000E-02	1.485E+02	1.131E+02
	1.55269E-02	1.416E+02	1.083E+02
	1.58608E-02	1.344E+02	1.032E+02
L1	1.58608E-02	1.548E+02	1.180E+02
	2.00000E-02	8.636E+01	6.899E+01
	3.00000E-02	3.032E+01	2.536E+01
	4.00000E-02	1.436E+01	1.211E+01
	5.00000E-02	8.041E+00	6.740E+00
	6.00000E-02	5.021E+00	4.149E+00
	8.00000E-02	2.419E+00	1.916E+00
	8.80045E-02	1.910E+00	1.482E+00
K	8.80045E-02	7.683E+00	2.160E+00
	1.00000E-01	5.549E+00	1.976E+00
	1.50000E-01	2.014E+00	1.056E+00
	2.00000E-01	9.985E-01	5.870E-01
	3.00000E-01	4.031E-01	2.455E-01
	4.00000E-01	2.323E-01	1.370E-01
	5.00000E-01	1.614E-01	9.128E-02
	6.00000E-01	1.248E-01	6.819E-02
	8.00000E-01	8.870E-02	4.644E-02
	1.00000E+00	7.102E-02	3.654E-02
	1.25000E+00	5.876E-02	2.988E-02
	1.50000E+00	5.222E-02	2.640E-02
	2.00000E+00	4.606E-02	2.360E-02
	3.00000E+00	4.234E-02	2.322E-02
	4.00000E+00	4.197E-02	2.449E-02
	5.00000E+00	4.272E-02	2.600E-02
	6.00000E+00	4.391E-02	2.744E-02
	8.00000E+00	4.675E-02	2.989E-02
	1.00000E+01	4.972E-02	3.181E-02
	1.50000E+01	5.658E-02	3.478E-02
	2.00000E+01	6.206E-02	3.595E-02

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$
 $\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$C =$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$C = \left| - \right.$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$$

$$C = 1 - e^{-(\mu_{Pb} - \mu_{Al})x}$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$C = 1 - e^{-(\mu_{Pb} - \mu_{Al})x}$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$x = x_{Al} = x_{Pb}$$

$$C = 1 - e^{-(\mu_{Pb} - \mu_{Al})x}$$

Med numeriska värden:

$$C = 1 - e^{-(5,549 * 11,35 - 0,1704 * 2,699)0,02}$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$x = x_{Al} = x_{Pb}$$

$$C = 1 - e^{-(\mu_{Pb} - \mu_{Al})x}$$

Med numeriska värden:

$$C = 1 - e^{-(5,549 * 11,35 - 0,1704 * 2,699)0,02}$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$x = x_{Al} = x_{Pb}$$

$$C = 1 - e^{-(\mu_{Pb} - \mu_{Al})x}$$

Med numeriska värden:

$$C = 1 - e^{-(5,549 * 11,35 - 0,1704 * 2,699)0,02}$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Pb} = \mu_p \rho_{Pb}$$

$$\mu_{Al} = \mu_p \rho_{Al}$$

$$x = x_{Al} = x_{Pb}$$

$$C = 1 - e^{-(\mu_{Pb} - \mu_{Al})x}$$

Med numeriska värden:

$$C = 1 - e^{-(5,549 * 11,35 - 0,1704 * 2,699)0,02}$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$x = x_{Al} = x_{Pb}$$

$$C = 1 - e^{-(\mu_{Pb} - \mu_{Al})x}$$

Med numeriska värden:

$$C = 1 - e^{-(5,549 * 11,35 - 0,1704 * 2,699)0,02}$$

$$C = \frac{e^{-\mu_{Al} x_{Al}} - e^{-\mu_{Pb} x_{Pb}}}{e^{-\mu_{Al} x_{Al}}}$$

$$\mu_{Pb} = \mu_{\rho_{Pb}} \rho_{Pb}$$

$$\mu_{Al} = \mu_{\rho_{Al}} \rho_{Al}$$

$$x = x_{Al} = x_{Pb}$$

$$C = 1 - e^{-(\mu_{Pb} - \mu_{Al})x}$$

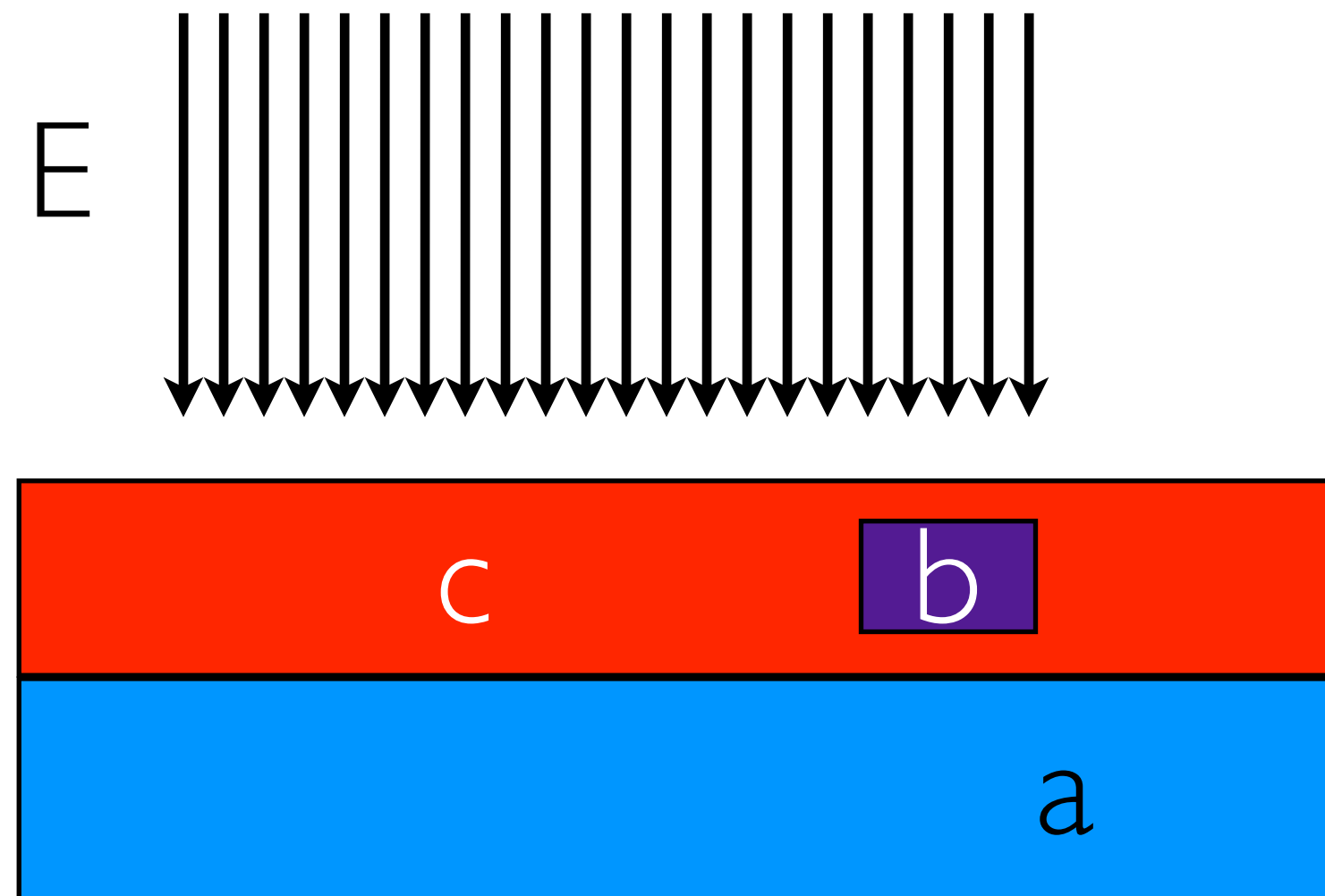
0,2 mm

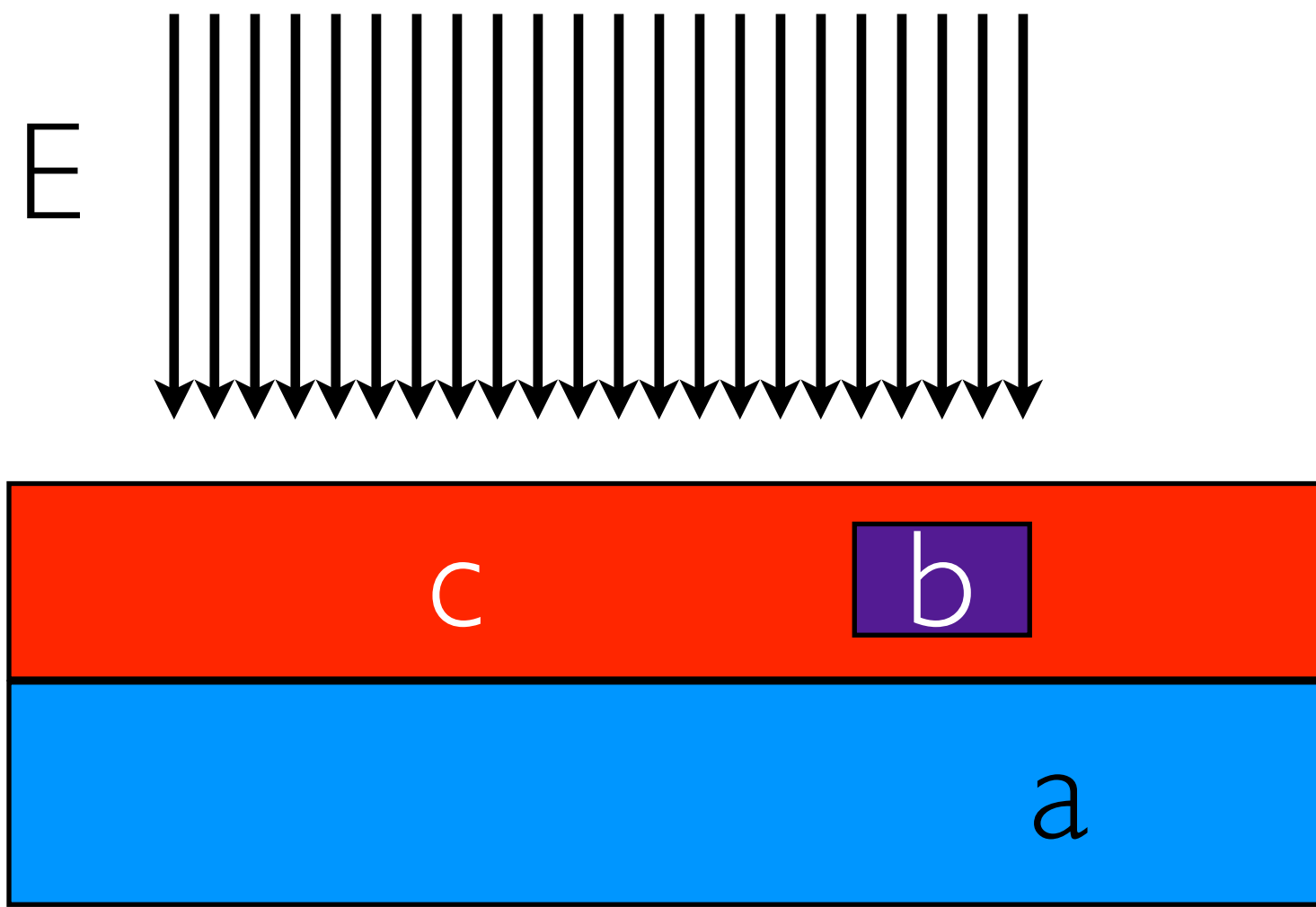
Med numeriska värden:

$$C = 1 - e^{-(5,549 * 11,35 - 0,1704 * 2,699)0,02}$$

$$C \approx 70\%$$

Assume c attenuates less than b and calculate object contrast:





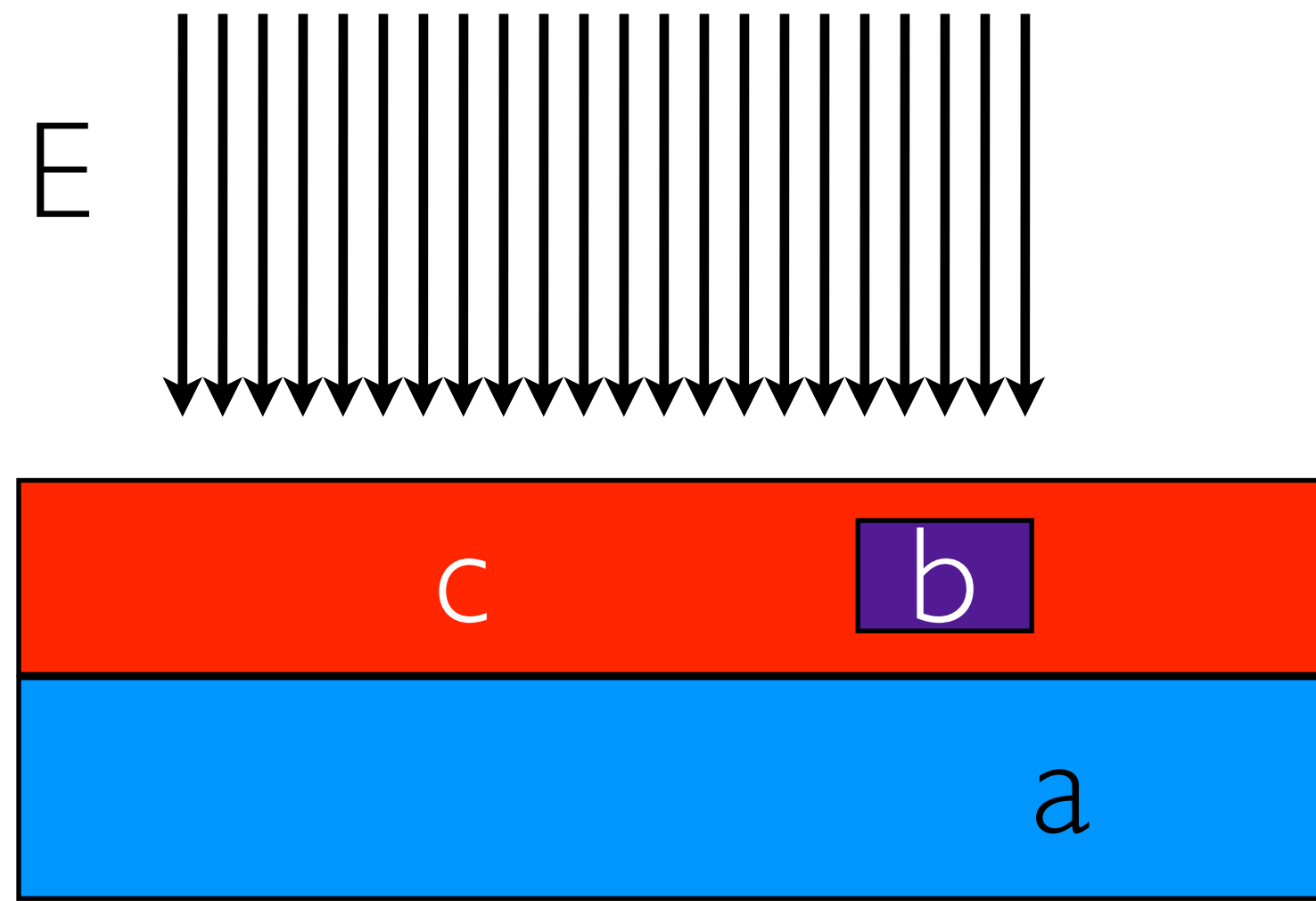
Assume c attenuates less than b and calculate object contrast:

$$\begin{array}{|c|} \hline \begin{array}{cc} c & e^{-\mu_c(E)x_c} \\ \hline e^{-\mu_a(E)x_a} & a \end{array} & - & \begin{array}{cc} c & e^{-\mu_c(E)(x_c-x_b)} \\ \hline e^{-\mu_a(E)x_a} & a \end{array} \\ \hline \end{array}$$

Diagram illustrating the subtraction of two attenuation paths. The left path shows the full thickness of layer c (red) and layer a (blue). The right path shows the same layers, but with a small purple square b (containing $e^{-\mu_b(E)x_b}$) removed from the top layer c . The resulting expression is:

$$\begin{array}{|c|} \hline c \quad e^{-\mu_c(E)x_c} \\ \hline e^{-\mu_a(E)x_a} \quad a \end{array}$$

$$C = 1 - e^{[\mu_b(E) - \mu_c(E)]x_b}$$



Assume c attenuates less than b and calculate object contrast:

$$e^{-\mu_c(E)x_b}$$



—

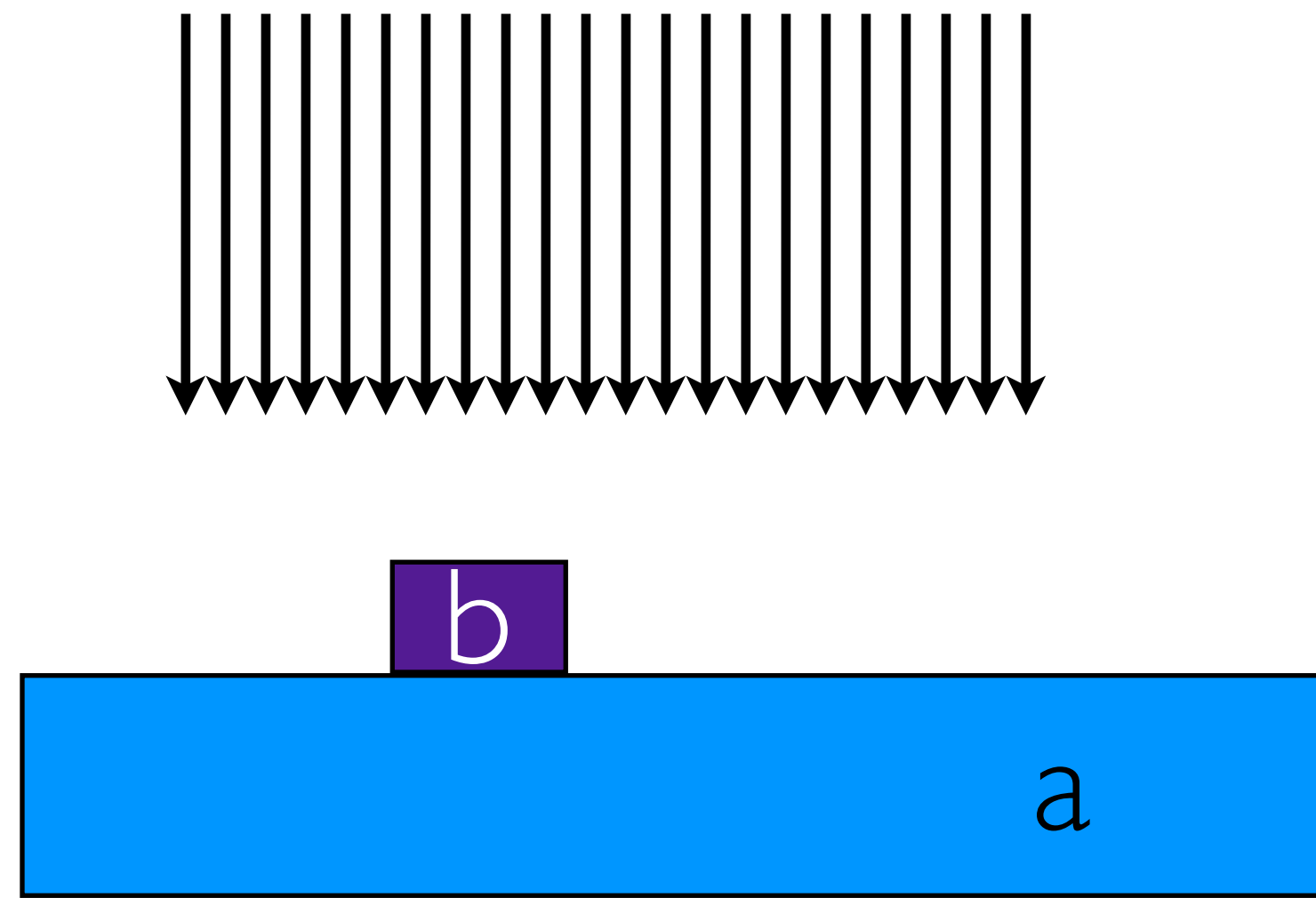


$$e^{-\mu_b(E)x_b}$$

$$e^{-\mu_c(E)x_b}$$



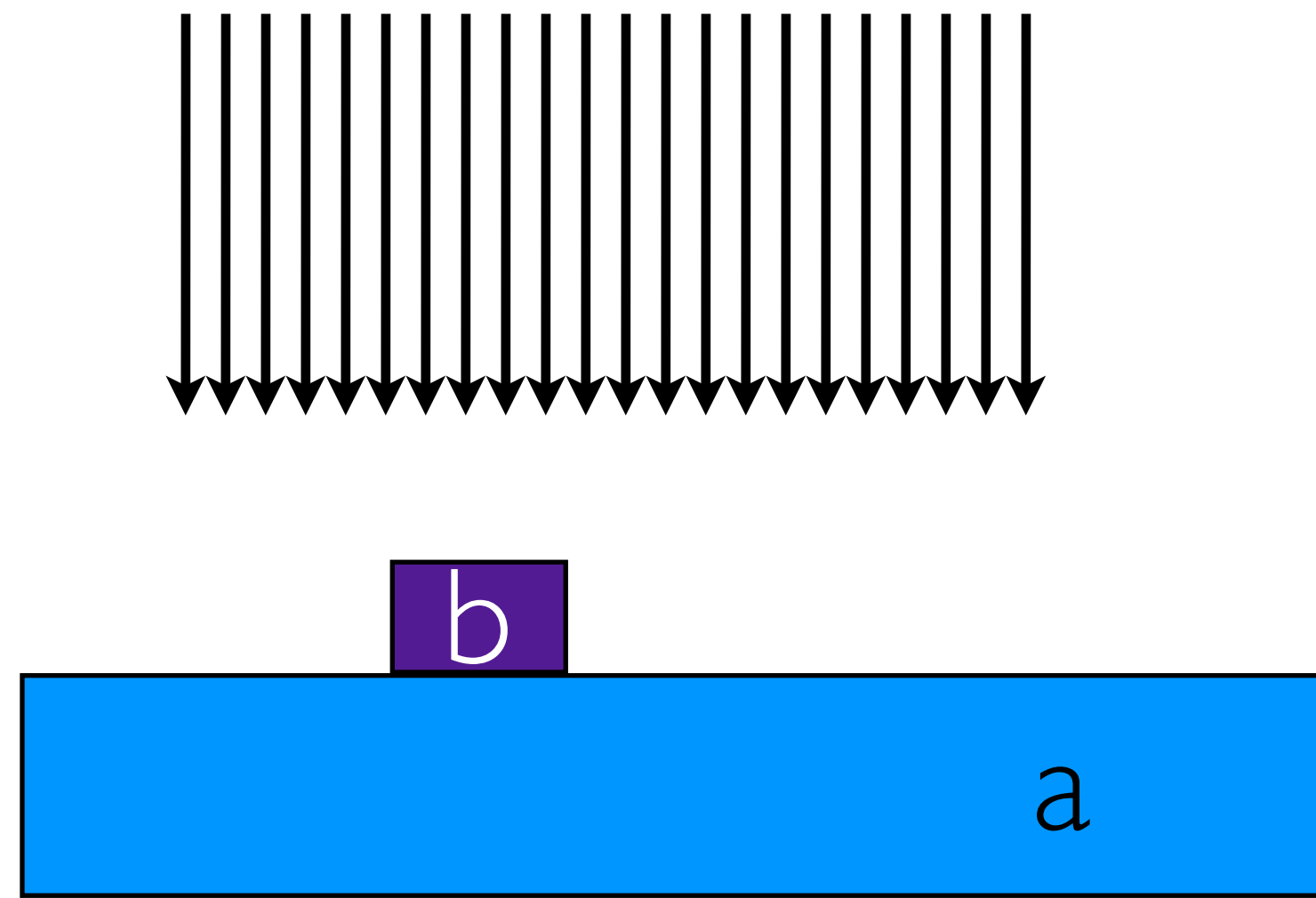
$$C = 1 - e^{[\mu_b(E) - \mu_c(E)]x_b}$$



Object b is on background a. The contrast of object b will depend on:

1. linear attenuation coefficient and thickness of both a and b
2. difference in linear attenuation coefficient between a and b and thickness of b

3. thickness and linear attenuation coefficient of a
4. thickness and linear attenuation coefficient of b

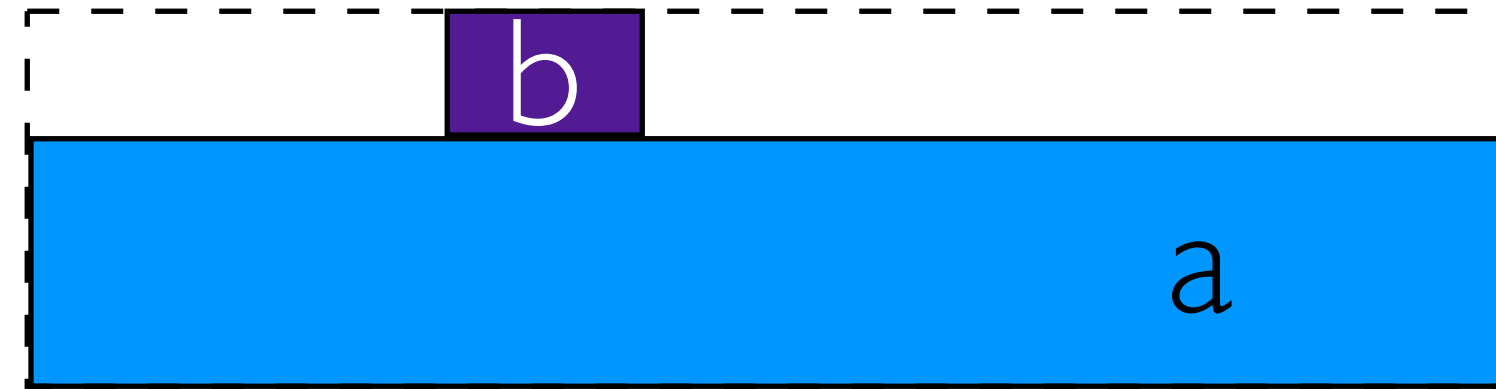
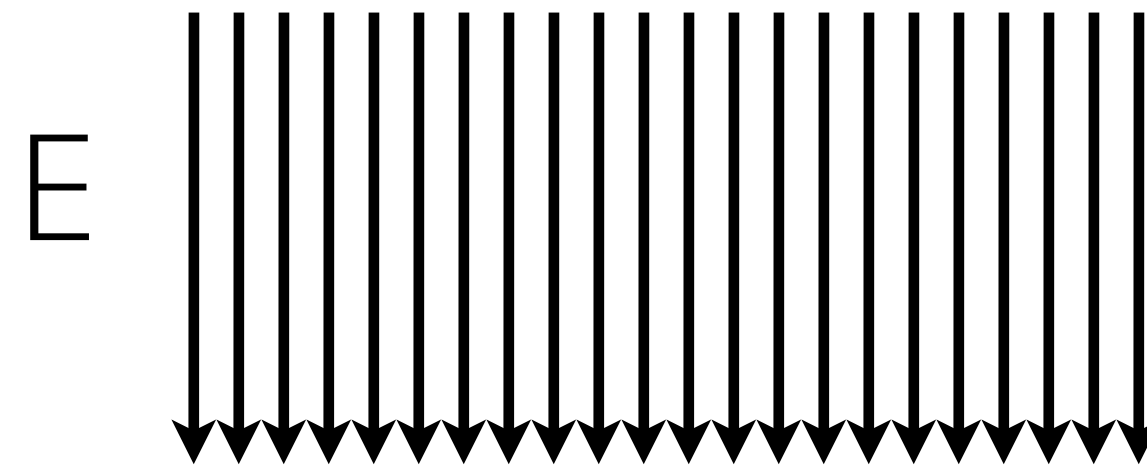


Object b is on background a. The contrast of object b will depend on:

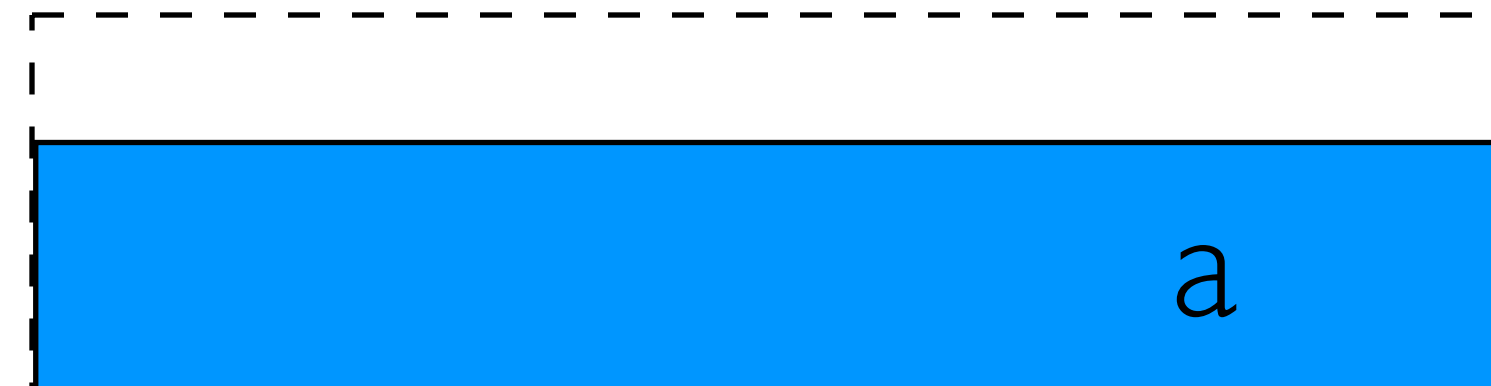
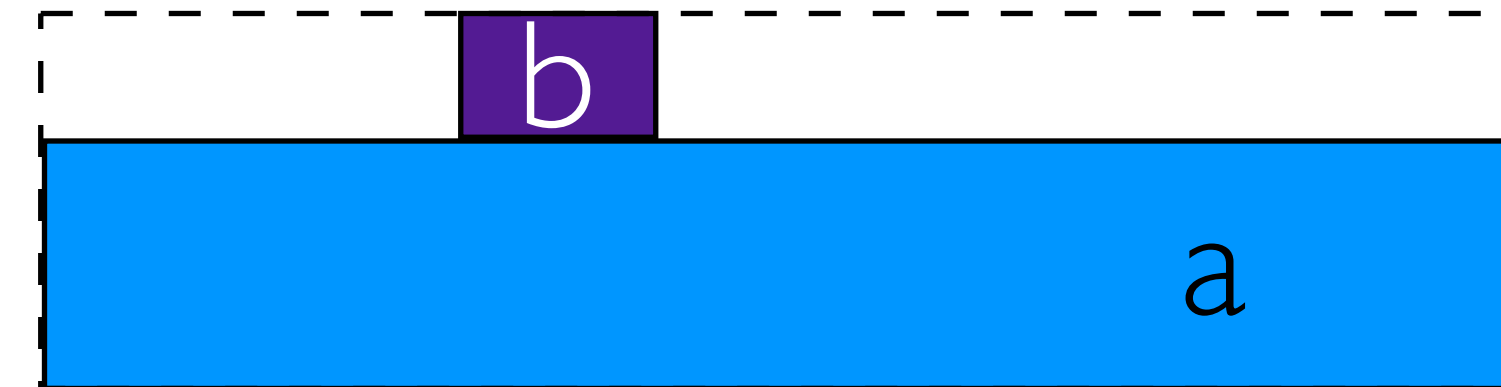
1. linear attenuation coefficient and thickness of both a and b
2. difference in linear attenuation coefficient between a and b and thickness of b

3. thickness and linear attenuation coefficient of a

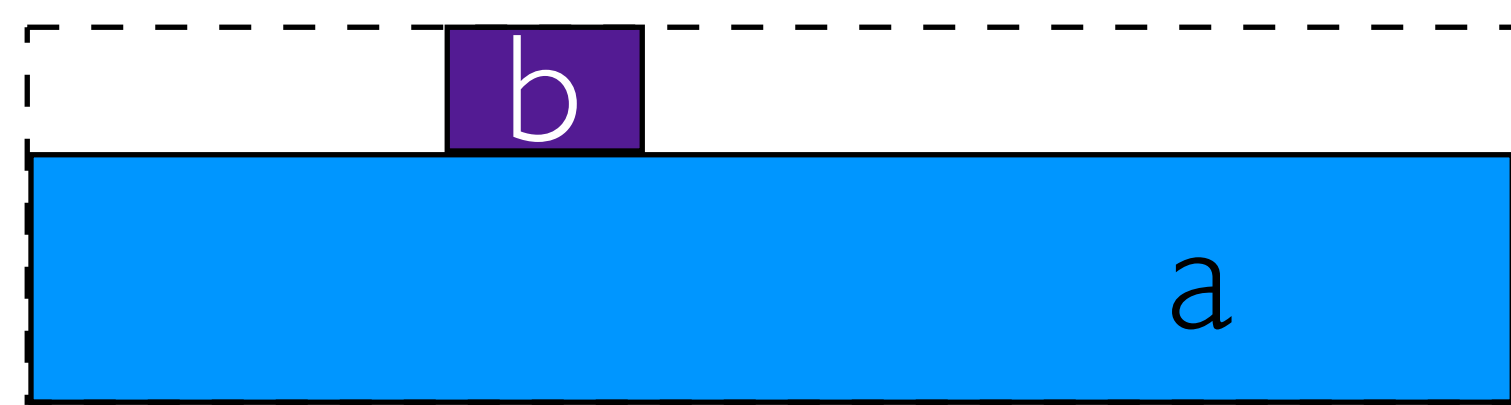
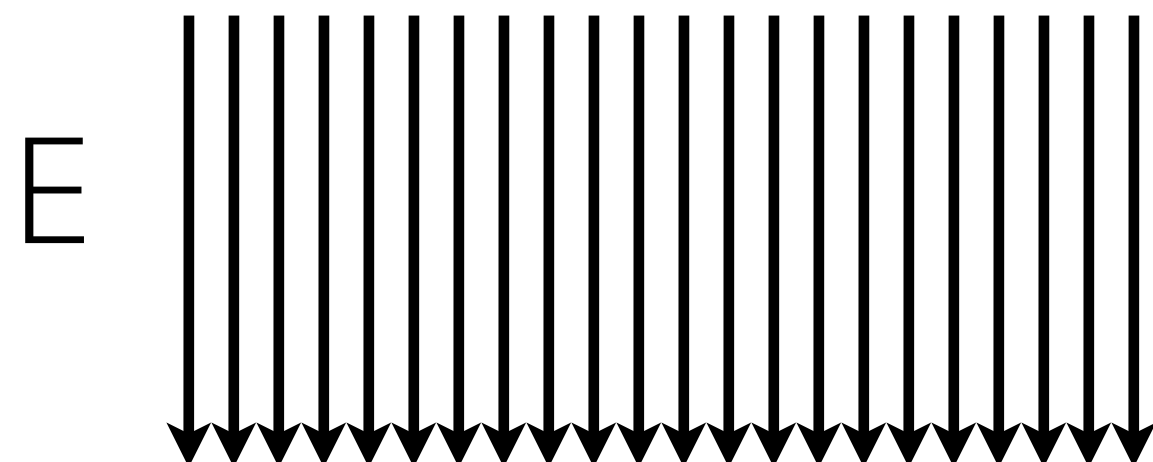
4. thickness and linear attenuation coefficient of b



-



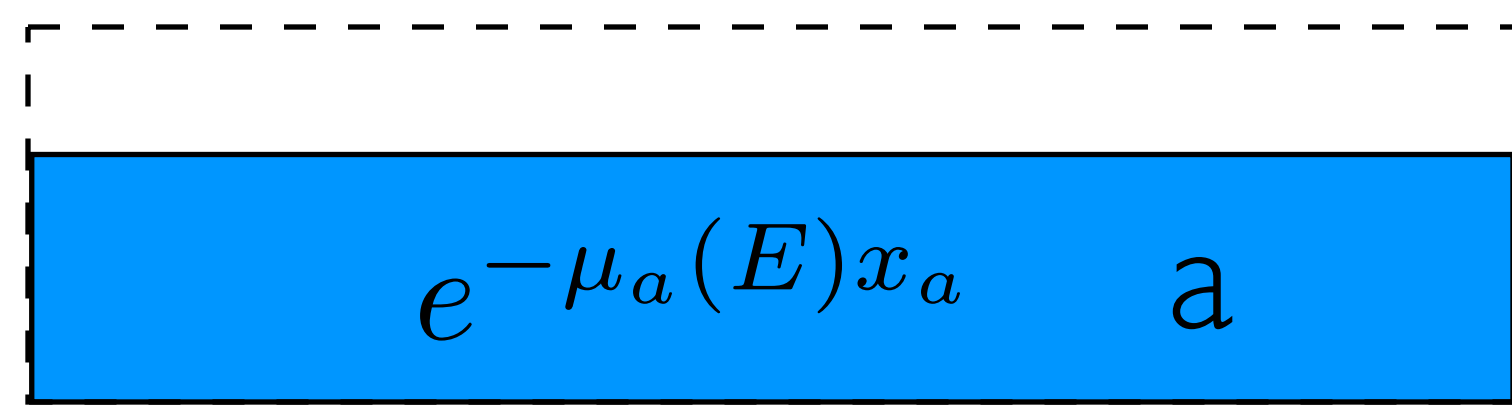
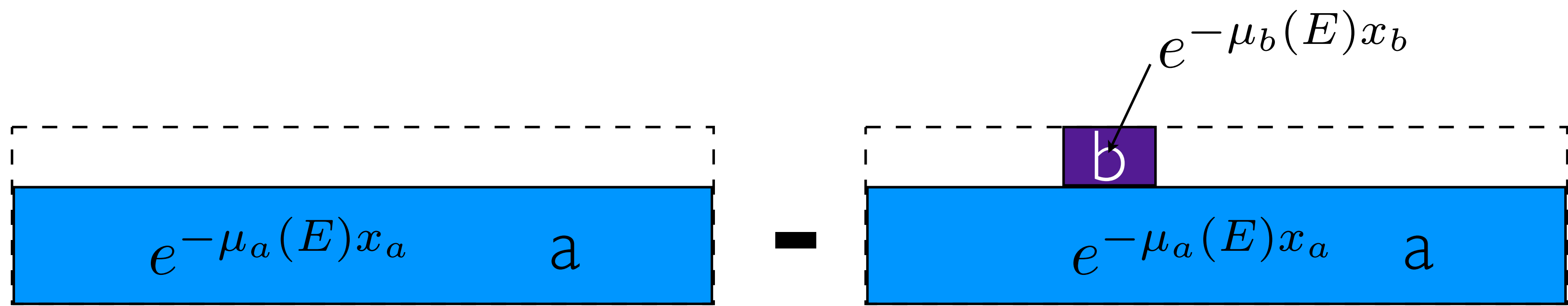
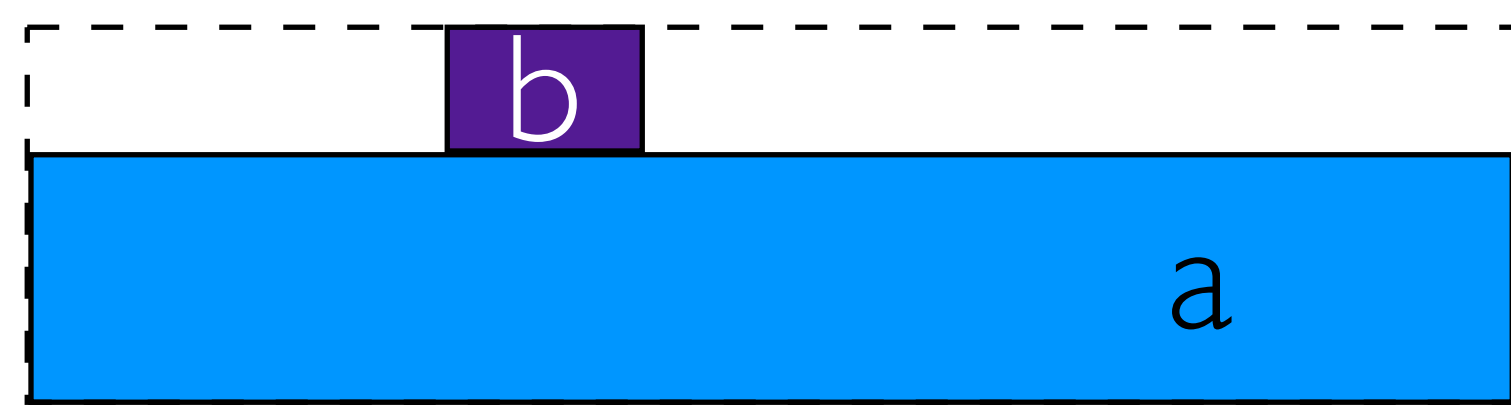
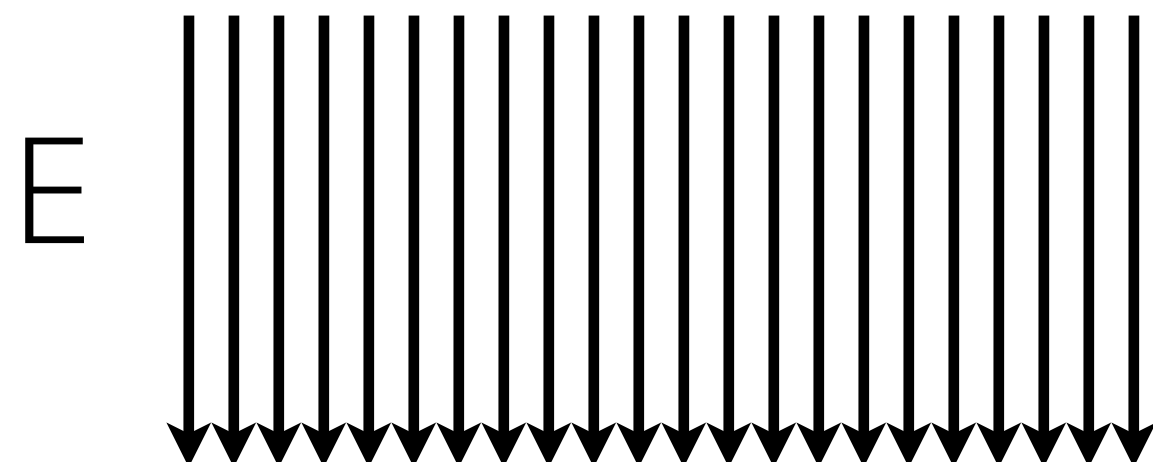
bara för att lin. att. coeff.
för luft är 0! $\Rightarrow$ lin. att. skillnad
är samma som lin. att av b



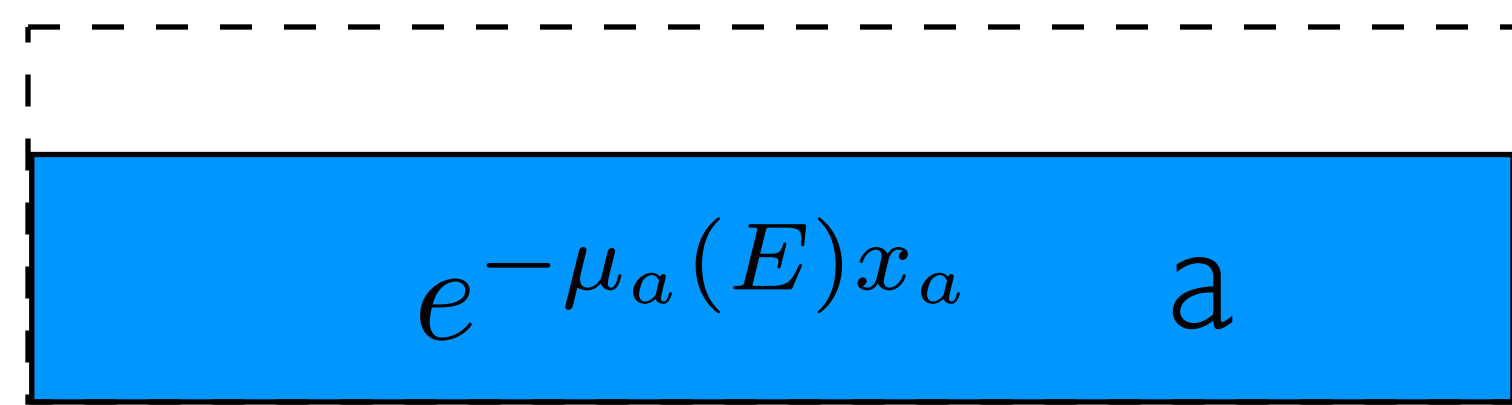
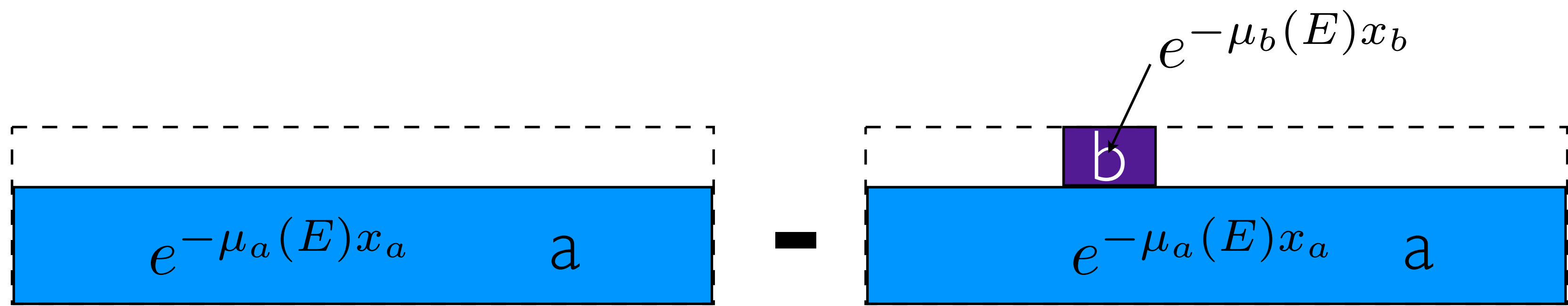
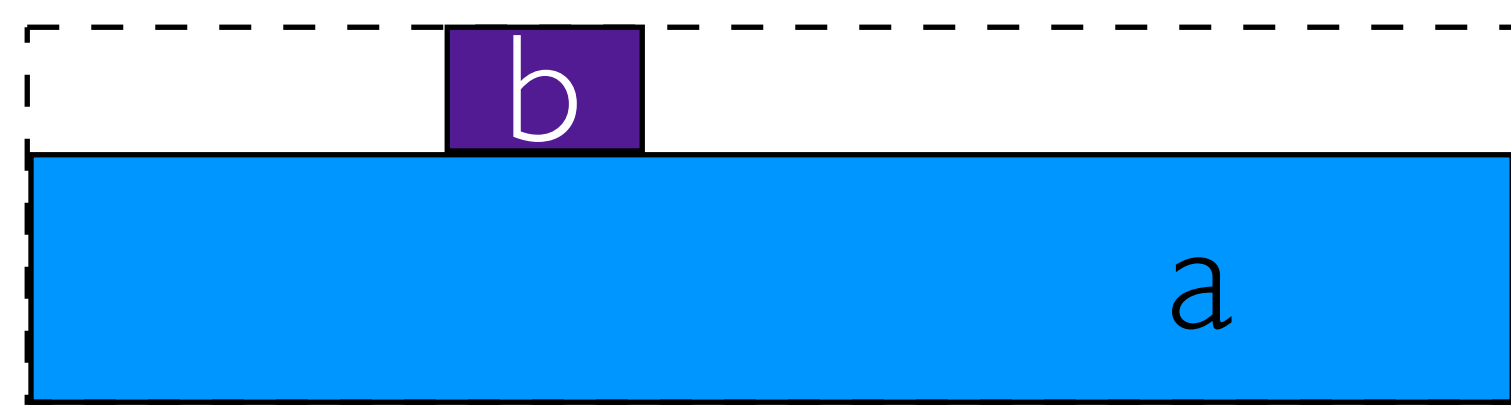
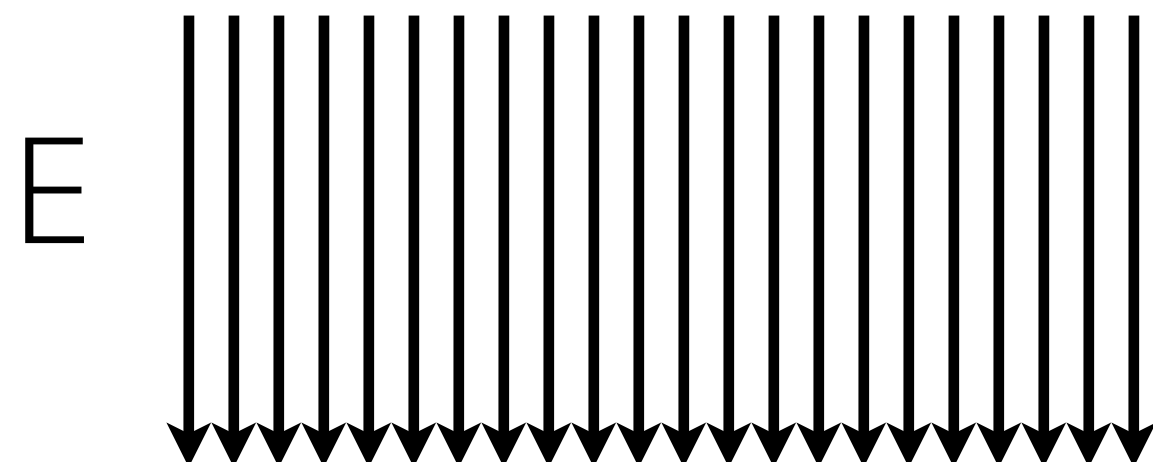
$$e^{-\mu_a(E)x_a} \quad a \quad - \quad e^{-\mu_a(E)x_a} \quad a$$

$$e^{-\mu_a(E)x_a} \quad a$$

bara för att lin. att. coeff.
 för luft är 0! $\Rightarrow$ lin. att. skillnad
 är samma som lin. att av b



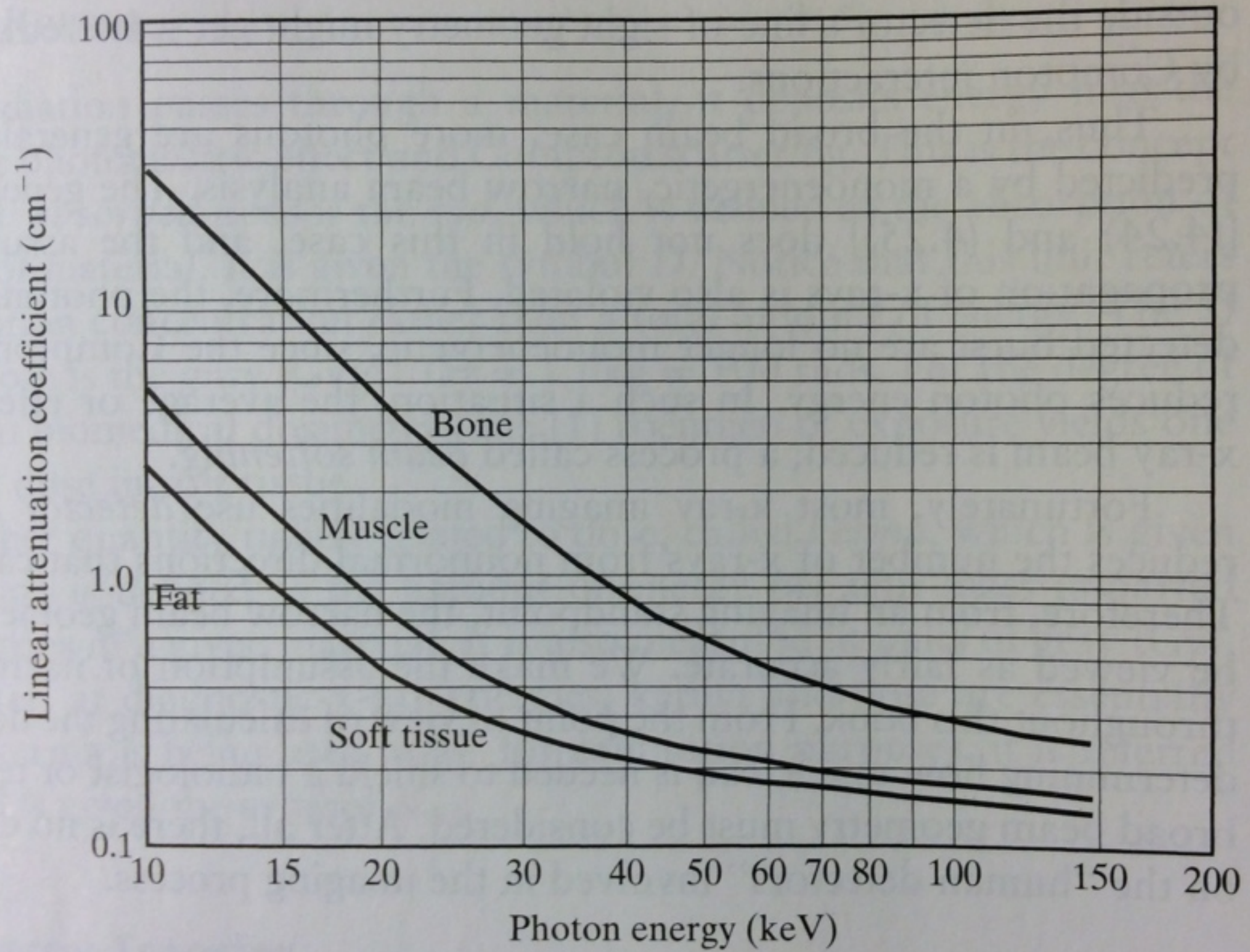
bara för att lin. att. coeff.
för luft är 0! $\Rightarrow$ lin. att. skillnad
är samma som lin. att av b

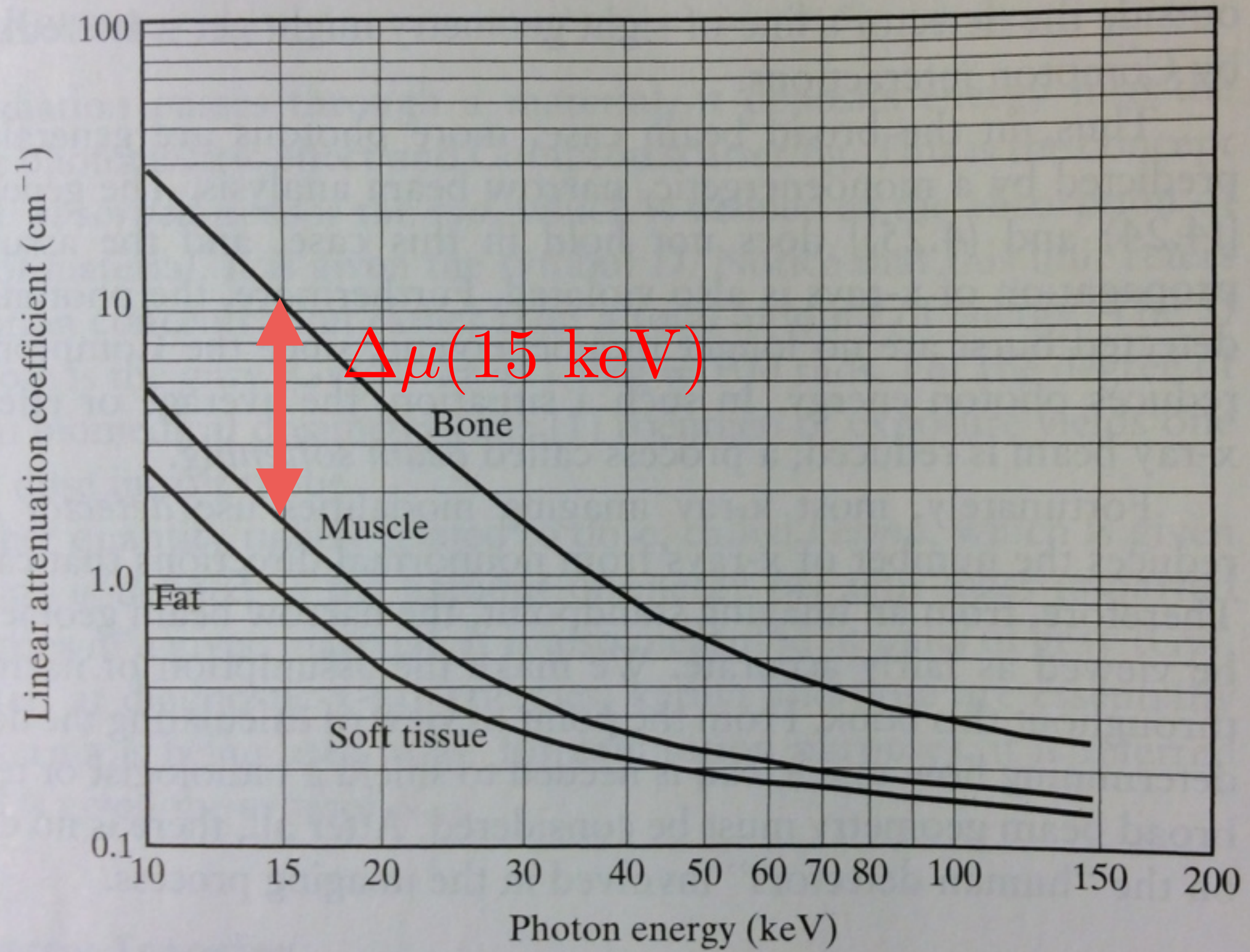


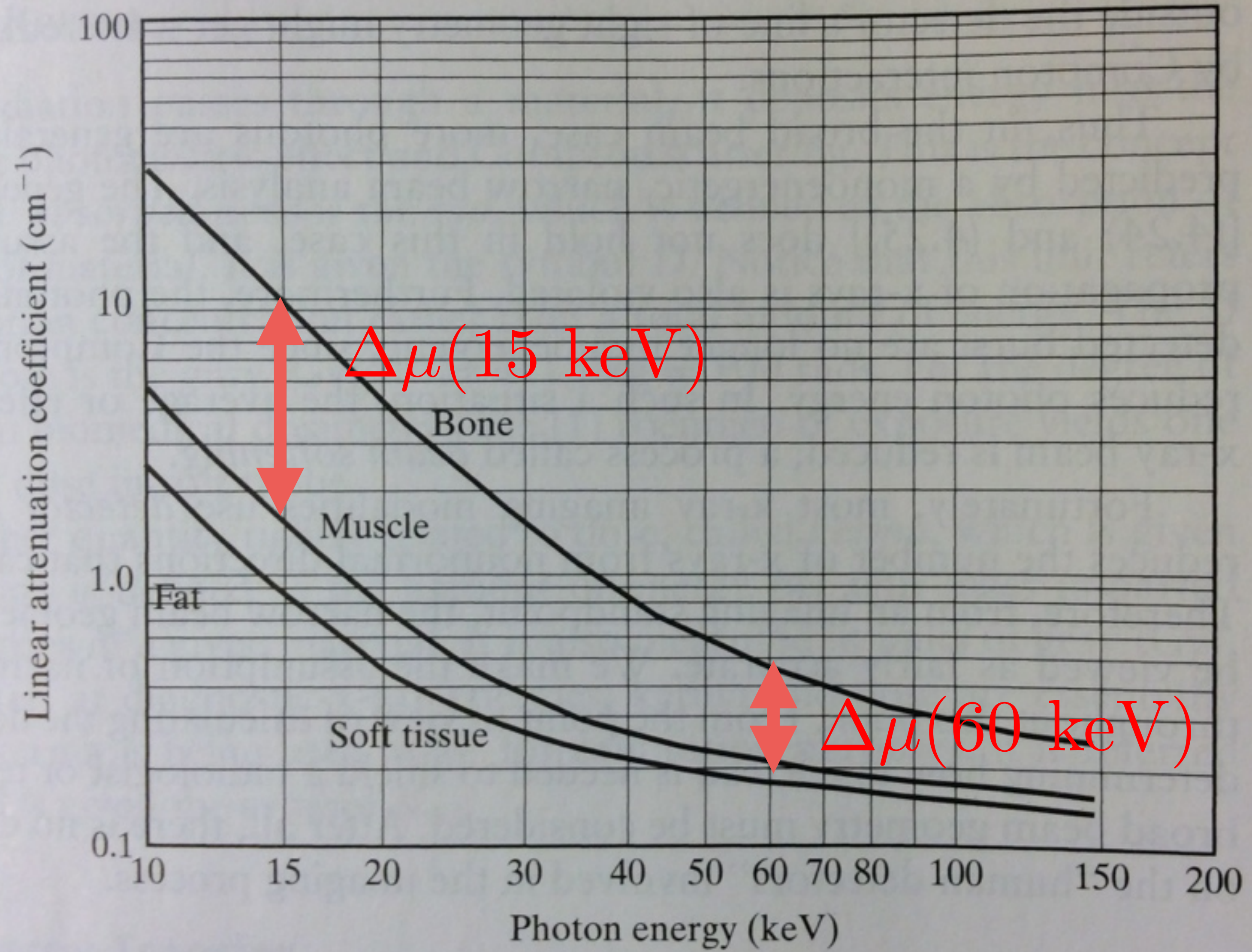
$$C = 1 - e^{-\mu_b(E)x_b}$$

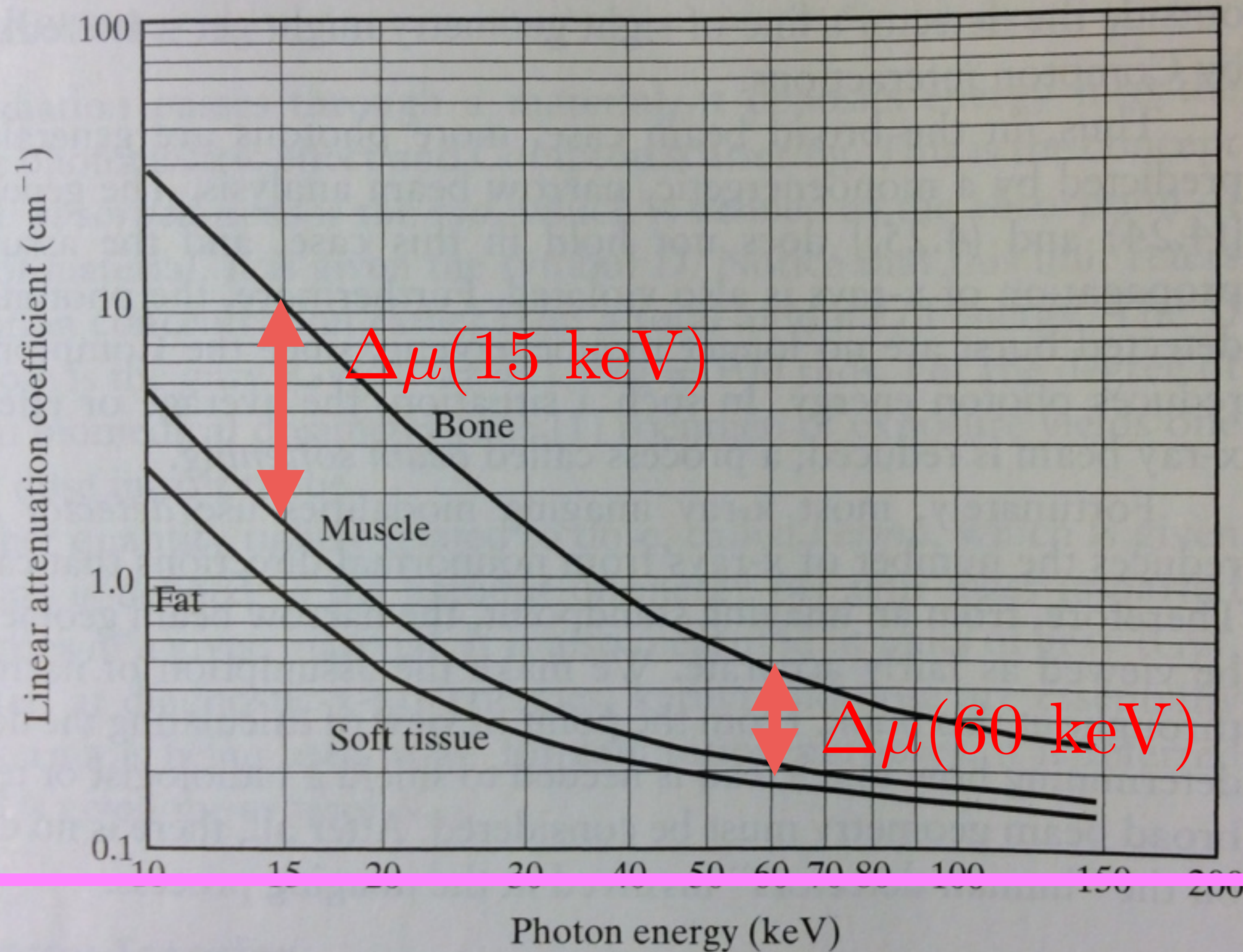
bara för att lin. att. coeff.
för luft är 0! $\Rightarrow$ lin. att. skillnad
är samma som lin. att av b

Hur påverkas transmissionsavbildning
av fotonernas energi?



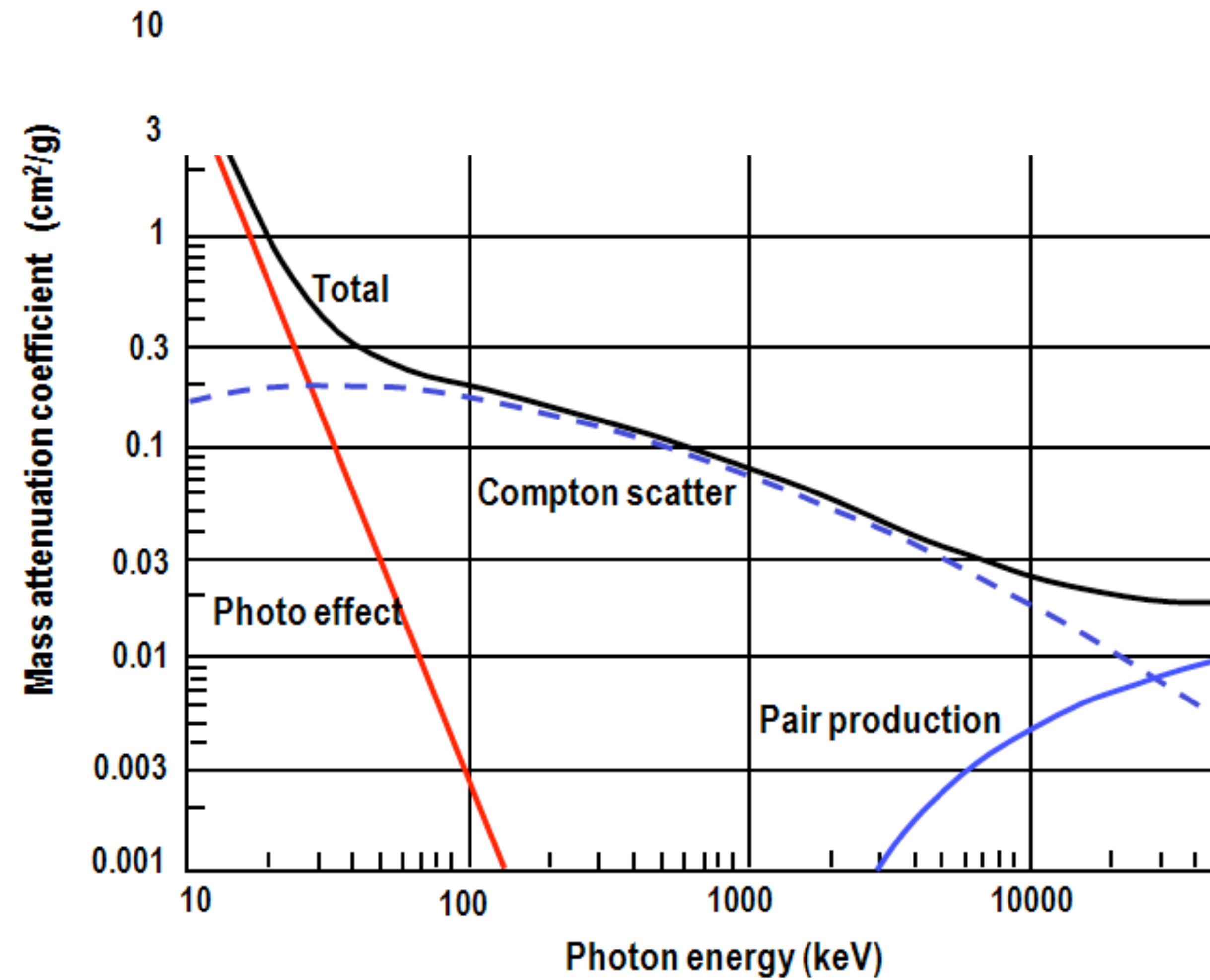




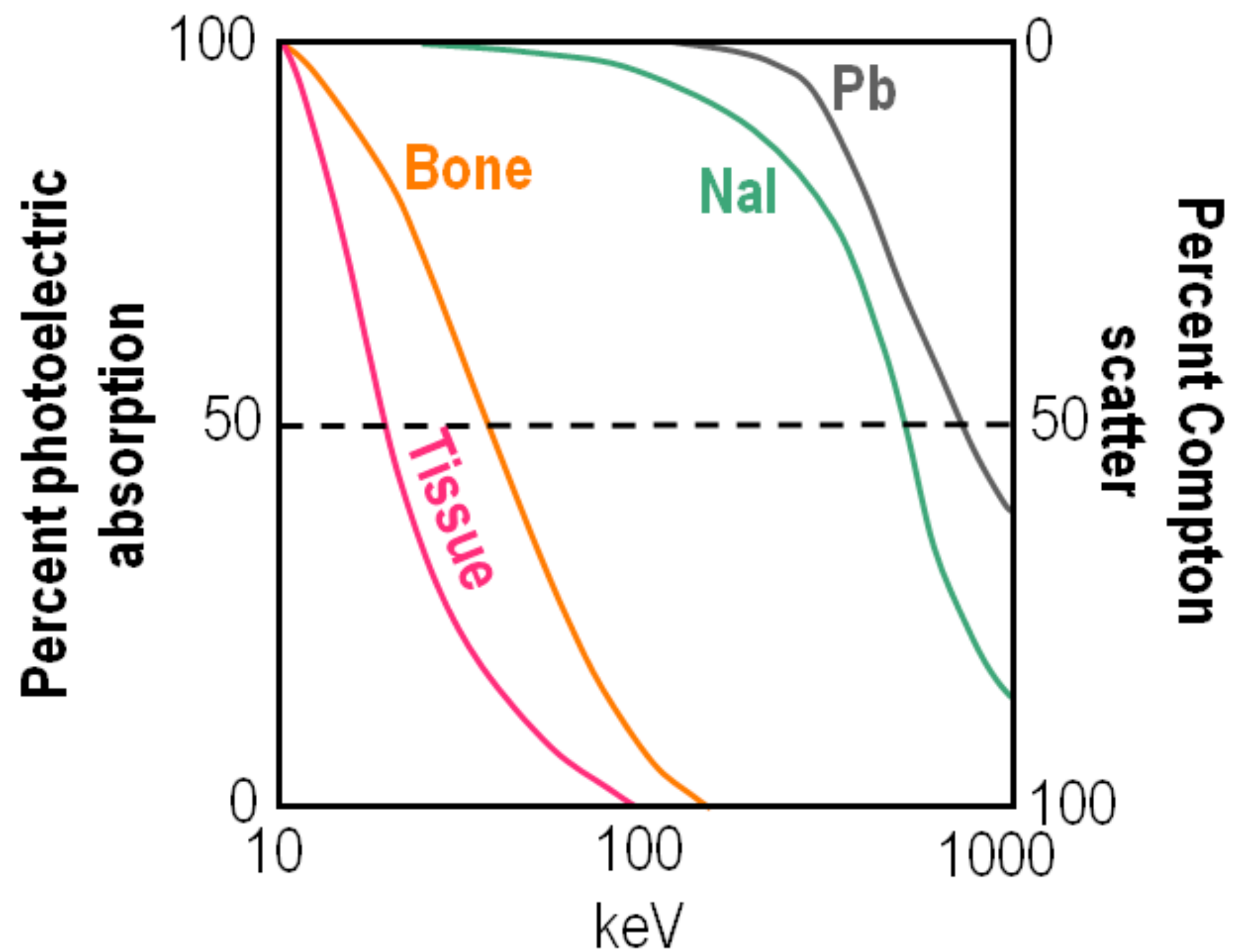


Lesson to learn:
Energi påverkar object contrast!

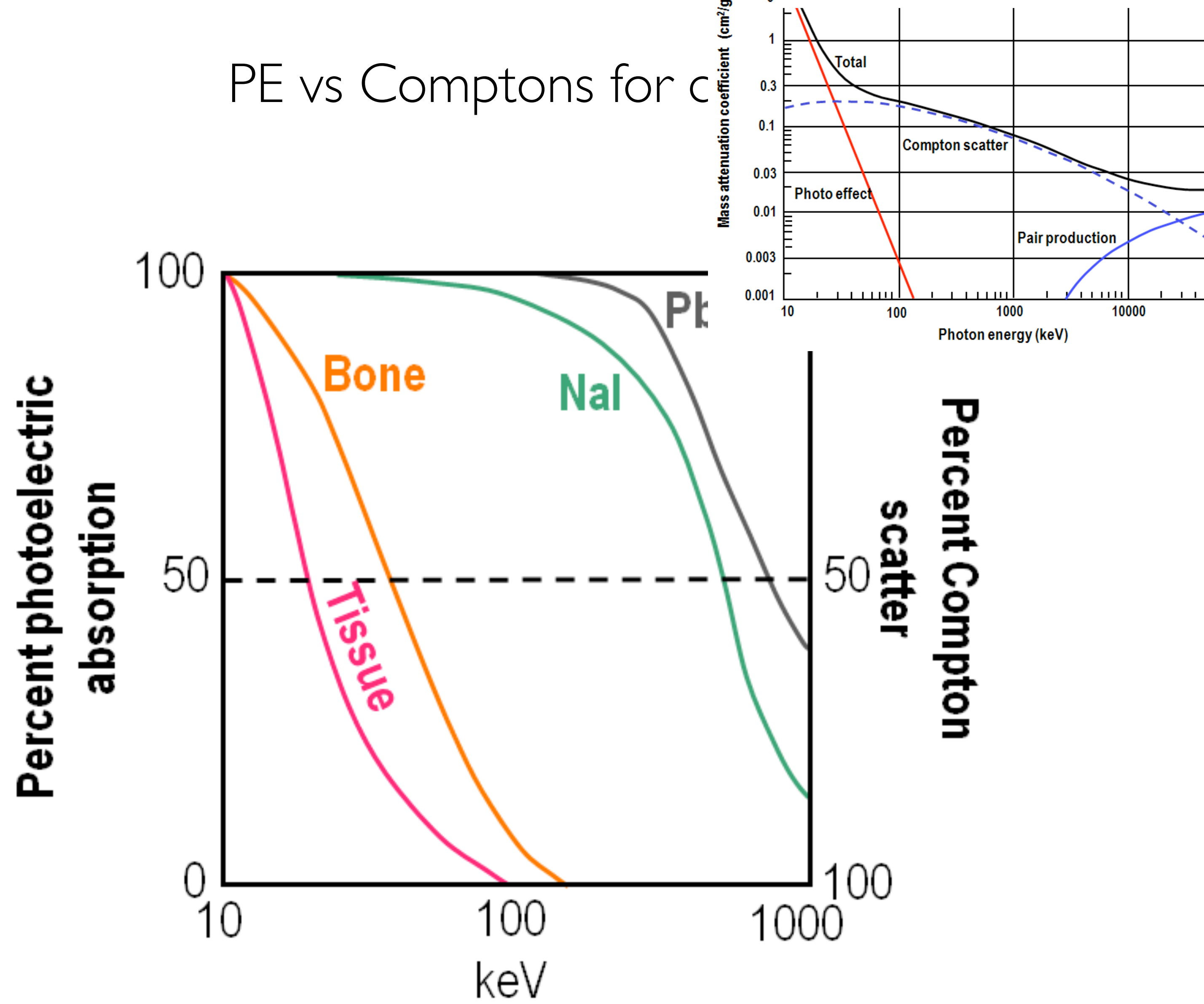
Mass attenuation coefficient: Energy dependence



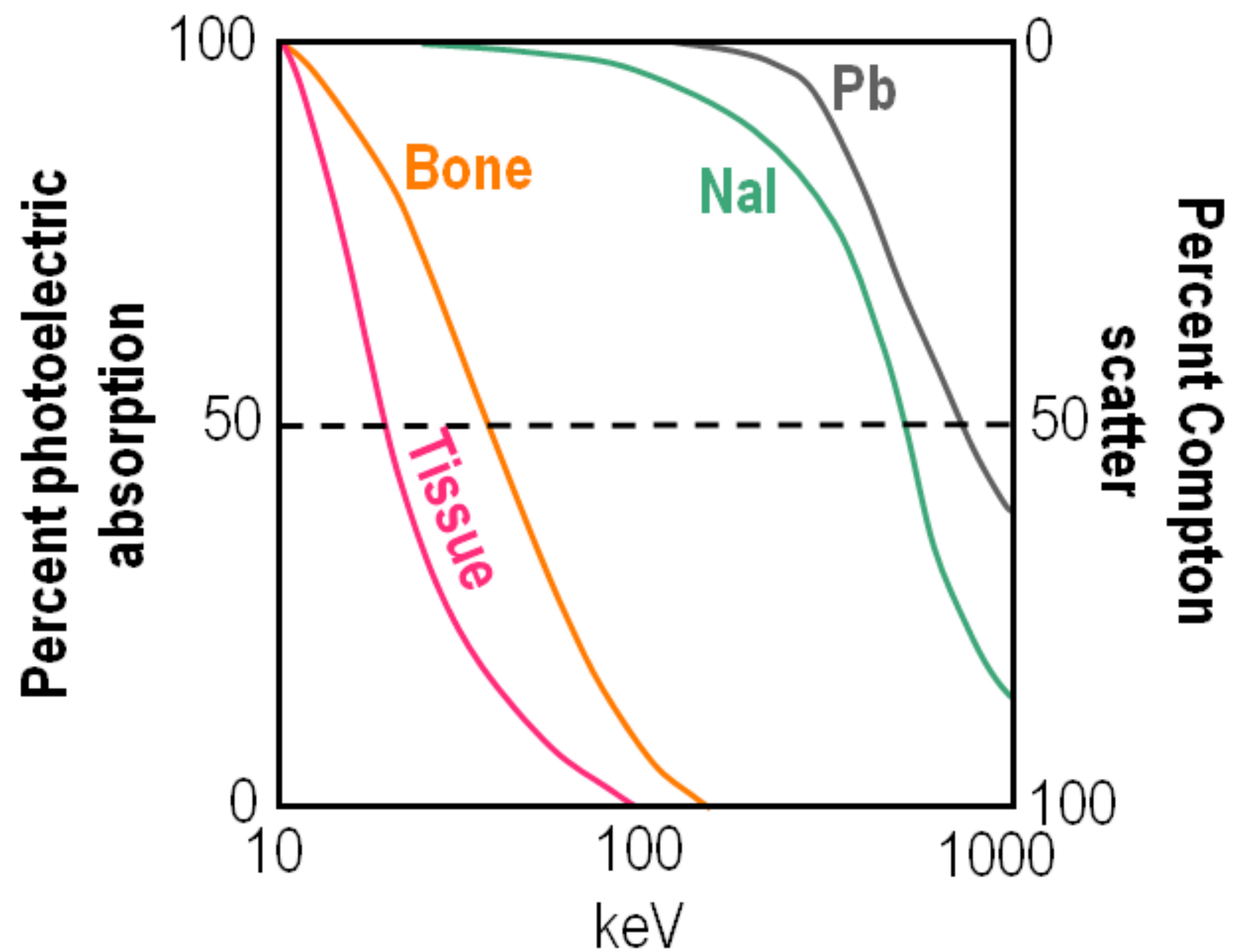
PE vs Comptons for different materials

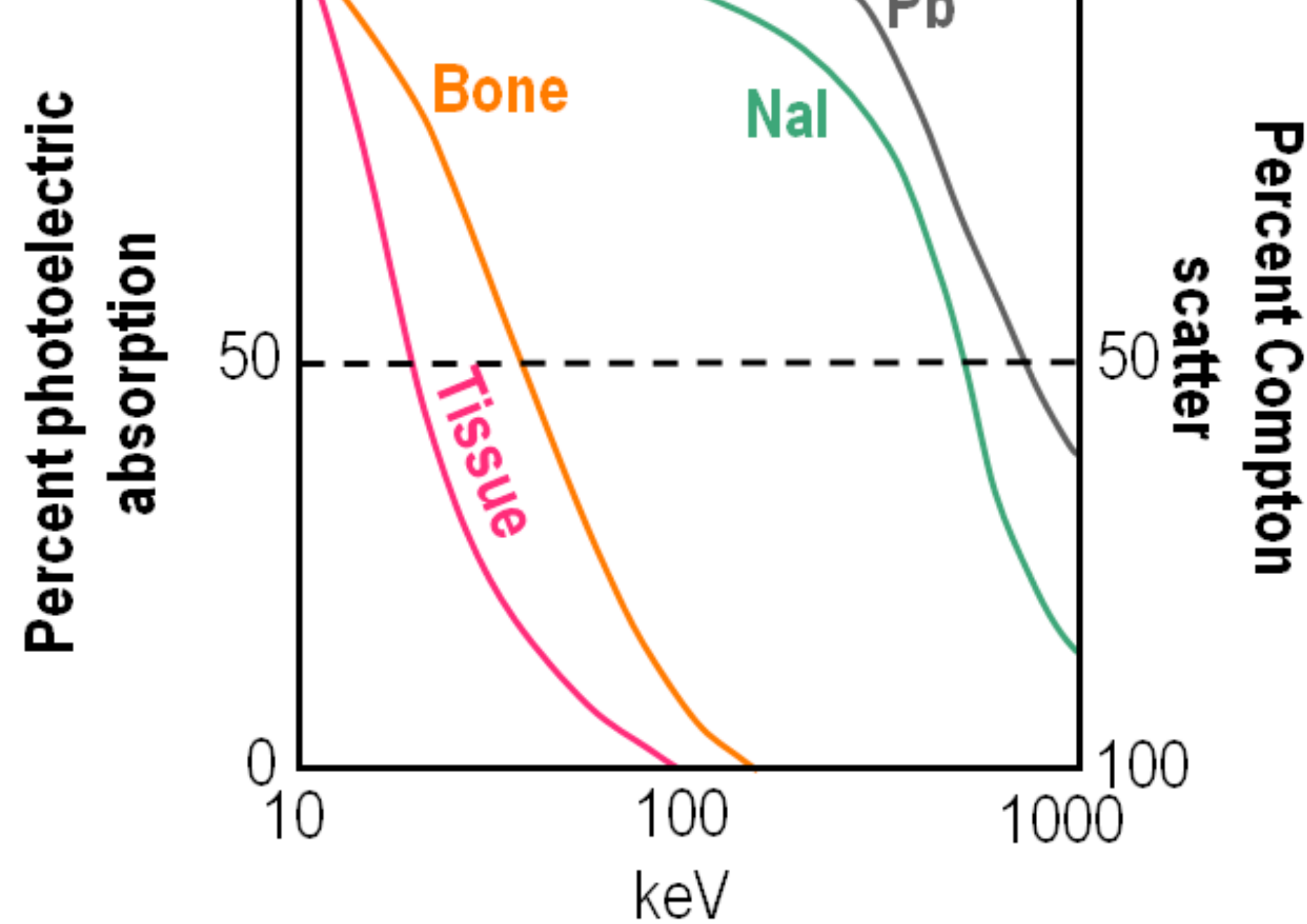


PE vs Comptons for c



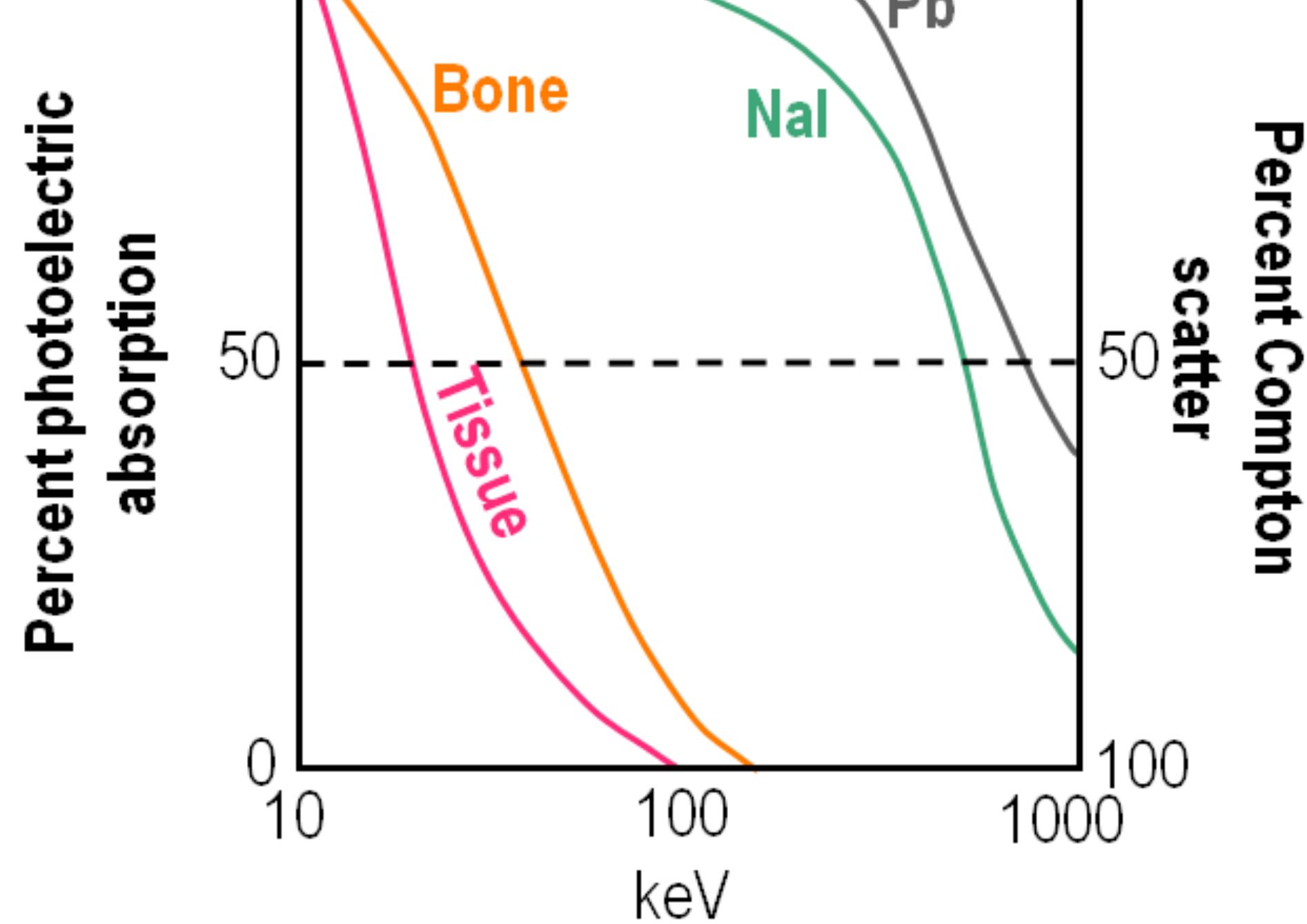
PE vs Comptons for different materials





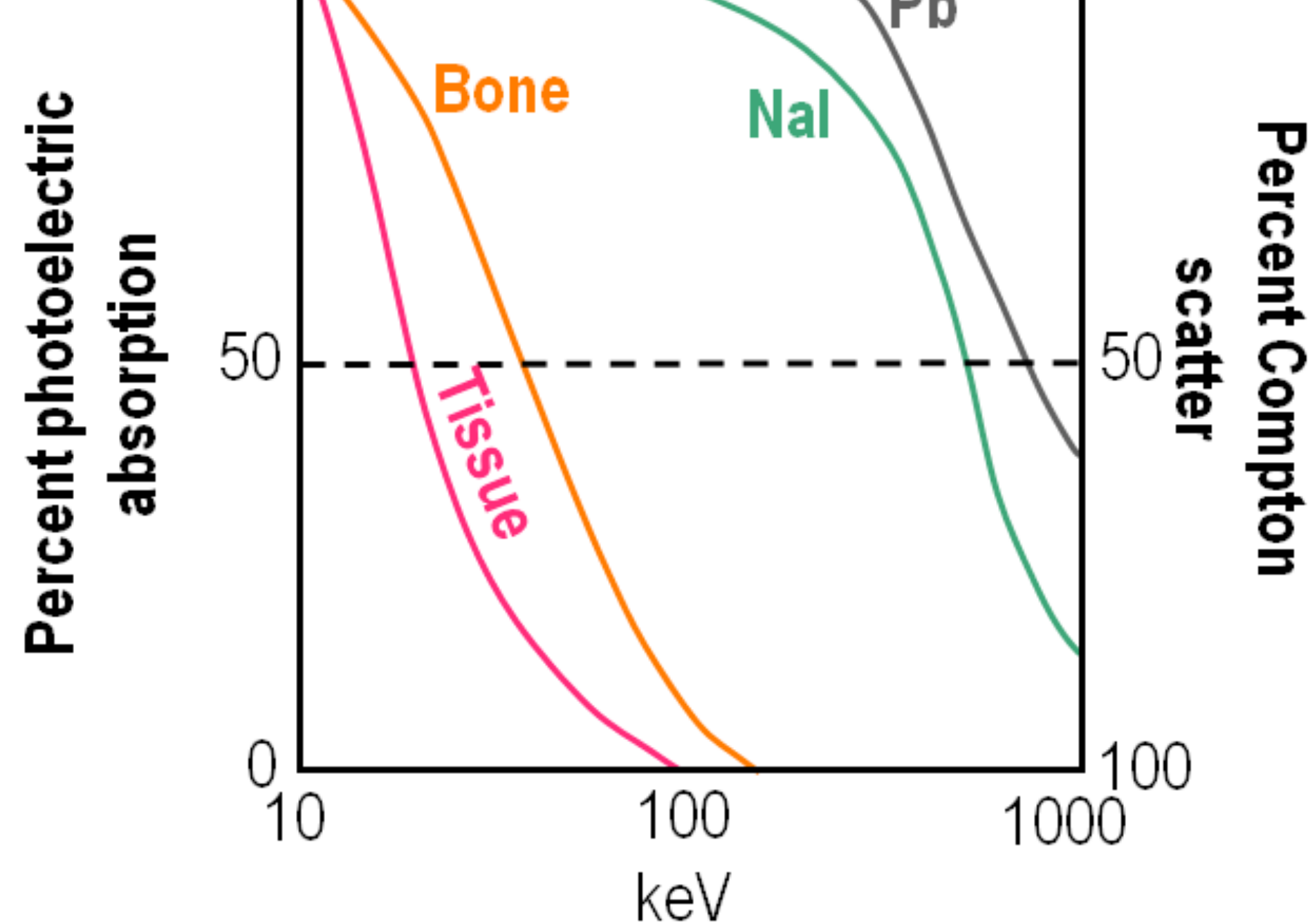
Which of the following energies would you choose for chest x-ray if you want to minimise Compton scattering?

1. 30 keV
2. 50 keV
3. 100 keV
4. anything between 120 keV and 200 keV will do
5. it doesn't matter, the probability of undergoing Compton scattering is a property of the material



Which of the following energies would you choose for chest x-ray if you want to maximise object contrast?

1. 30 keV
2. 50 keV
3. 100 keV
4. anything between 120 keV and 200 keV will do
5. it doesn't matter, the probability of undergoing Compton scattering is a property of the material



Which of the following energies would you choose for chest x-ray if you want to maximise number of primary photons reaching the detector (that is also minimising energy deposited on patient)?

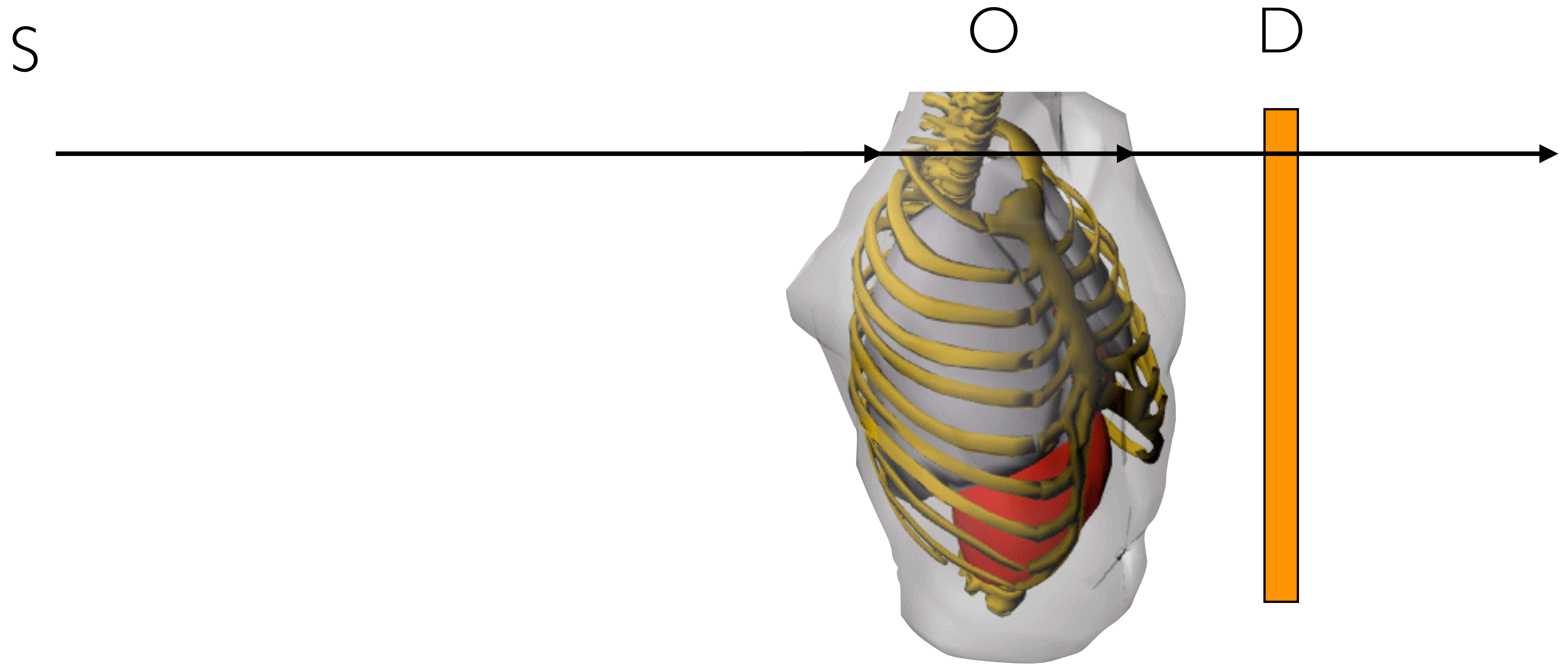
1. 30 keV
2. 50 keV
3. 100 keV
4. anything between 120 keV and 200 keV will do
5. it doesn't matter, the probability of undergoing Compton scattering is a property of the material

Lessons learned: Object contrast is maximum at low energy, Compton/PE fraction is minimal at low energy, detector signal is maximum at high energy, (dose to the patient is maximum at low energy)

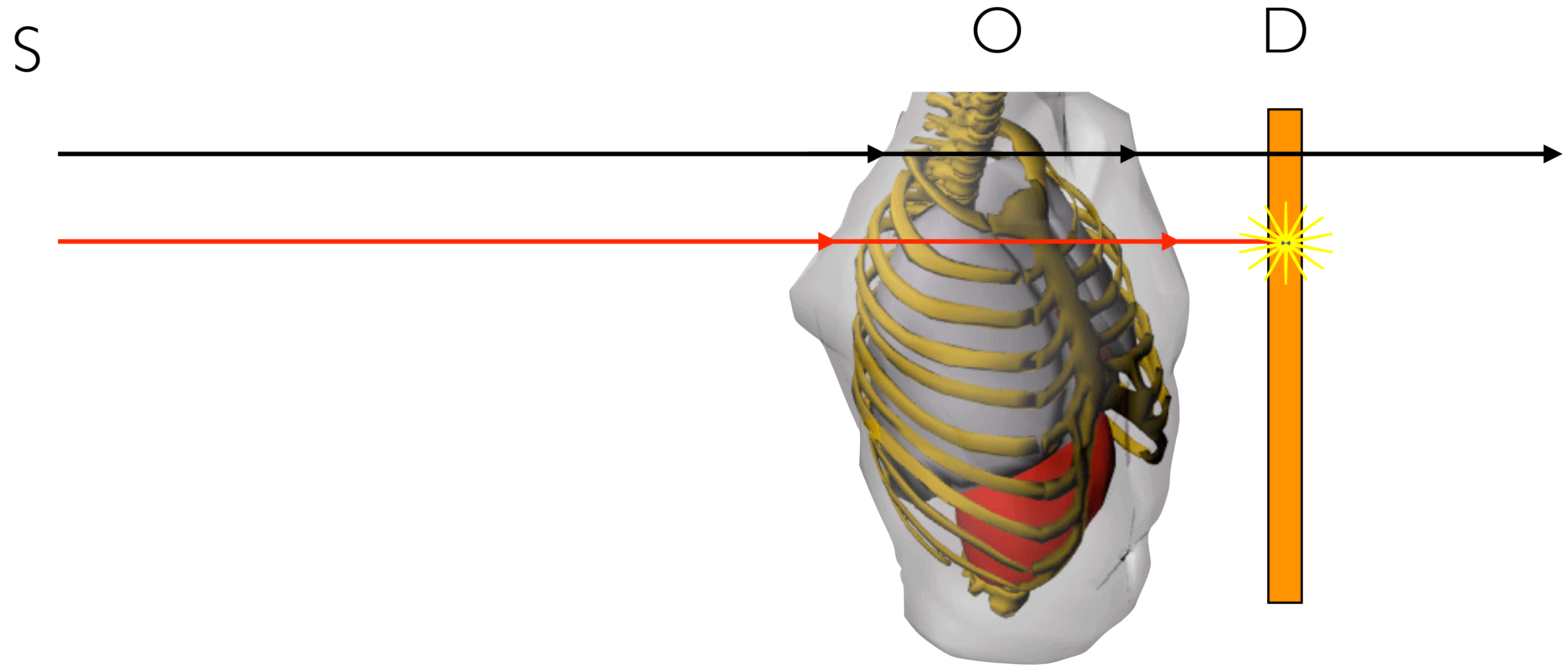
We need an engineer!

Quick jump to clinical
applications

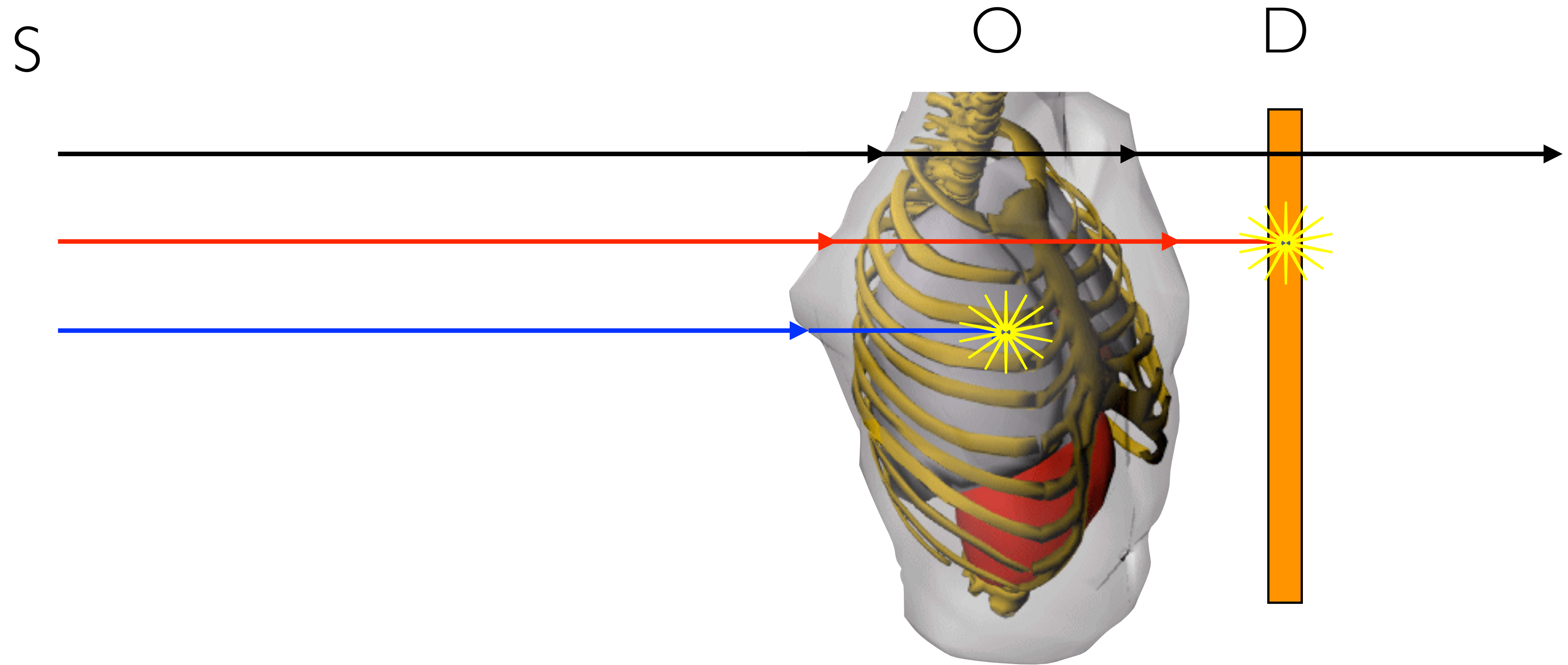
Possible outcomes



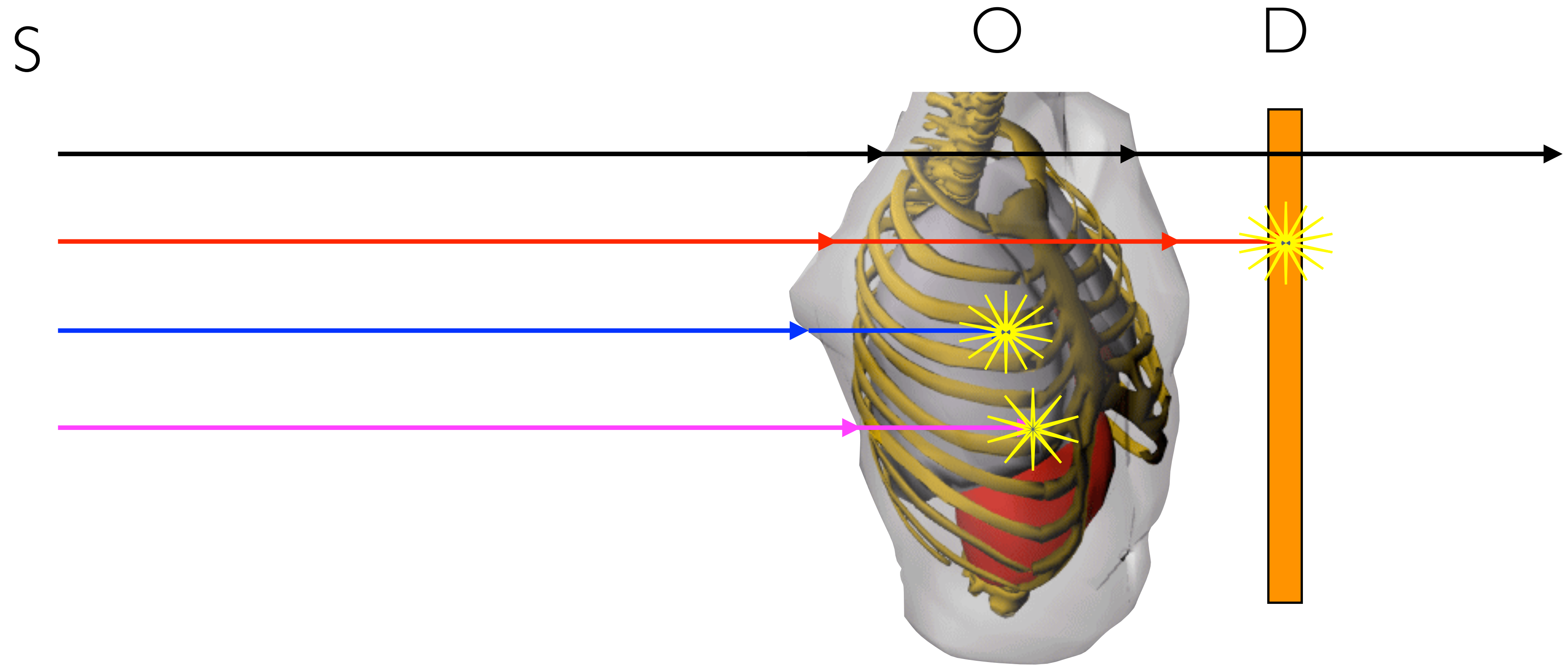
Possible outcomes



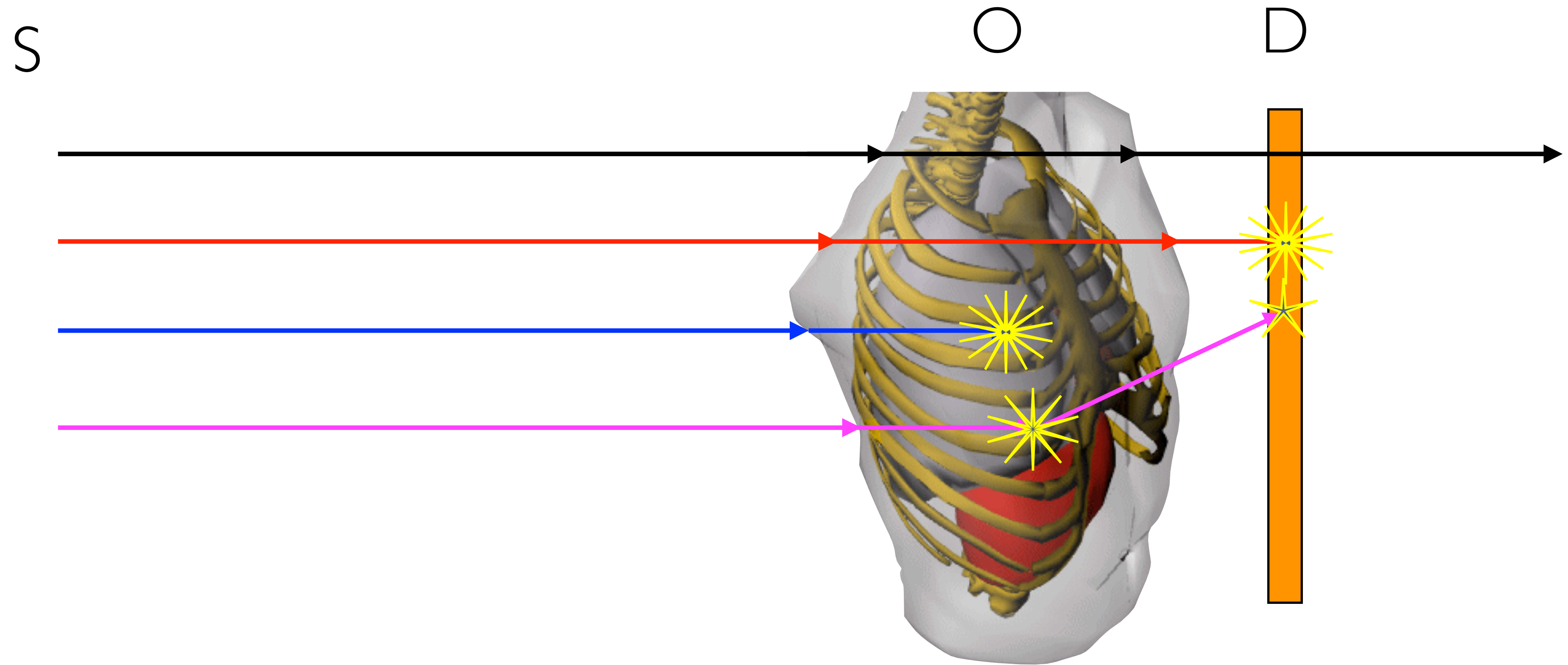
Possible outcomes



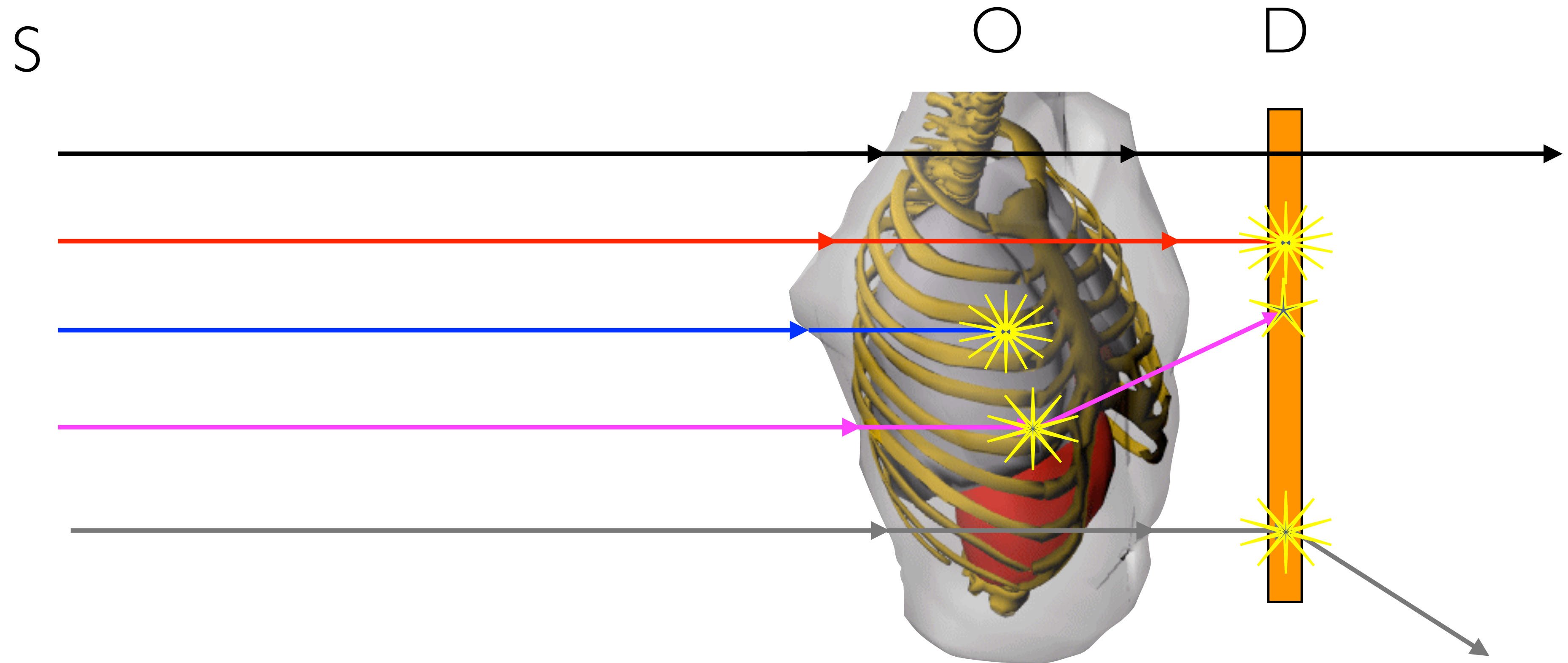
Possible outcomes



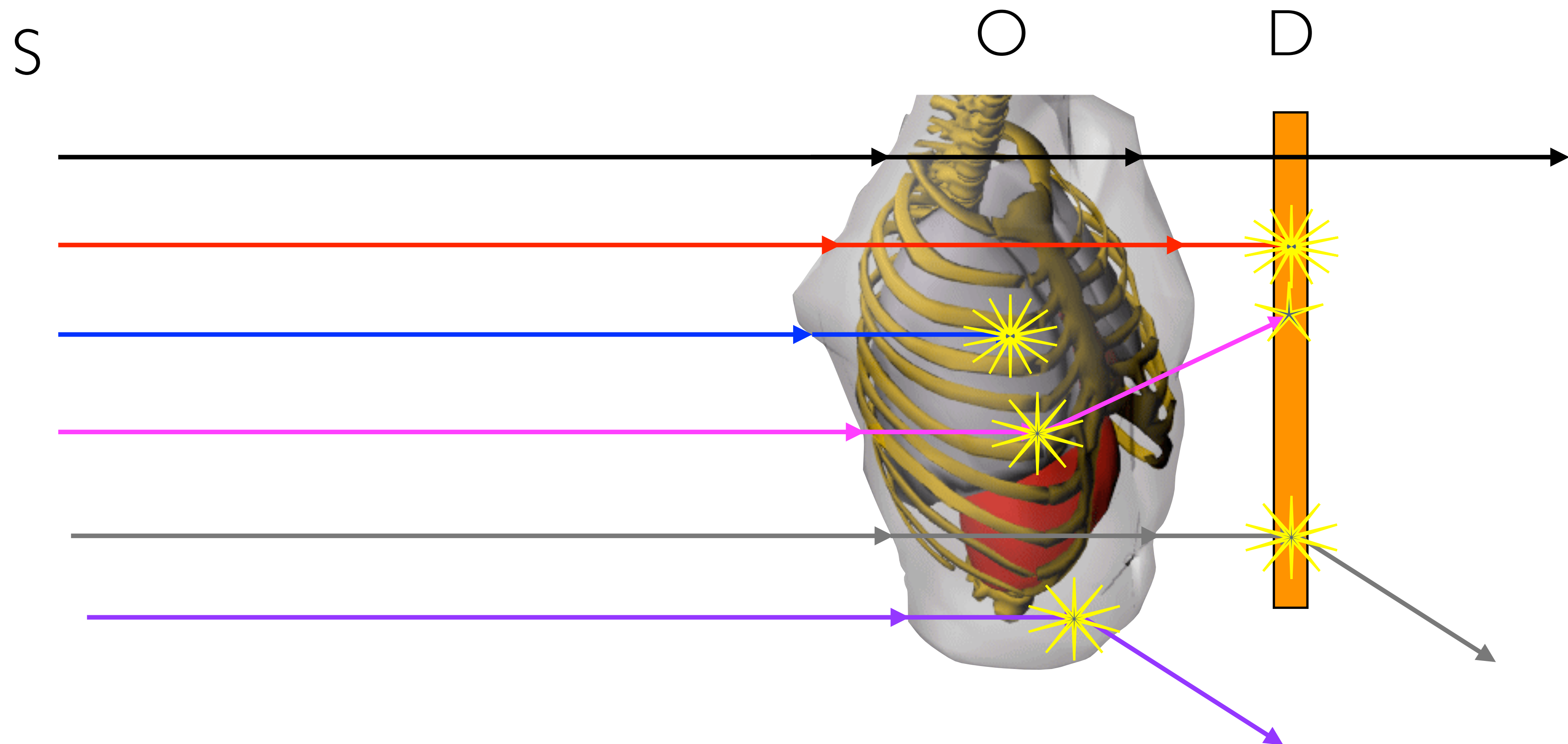
Possible outcomes



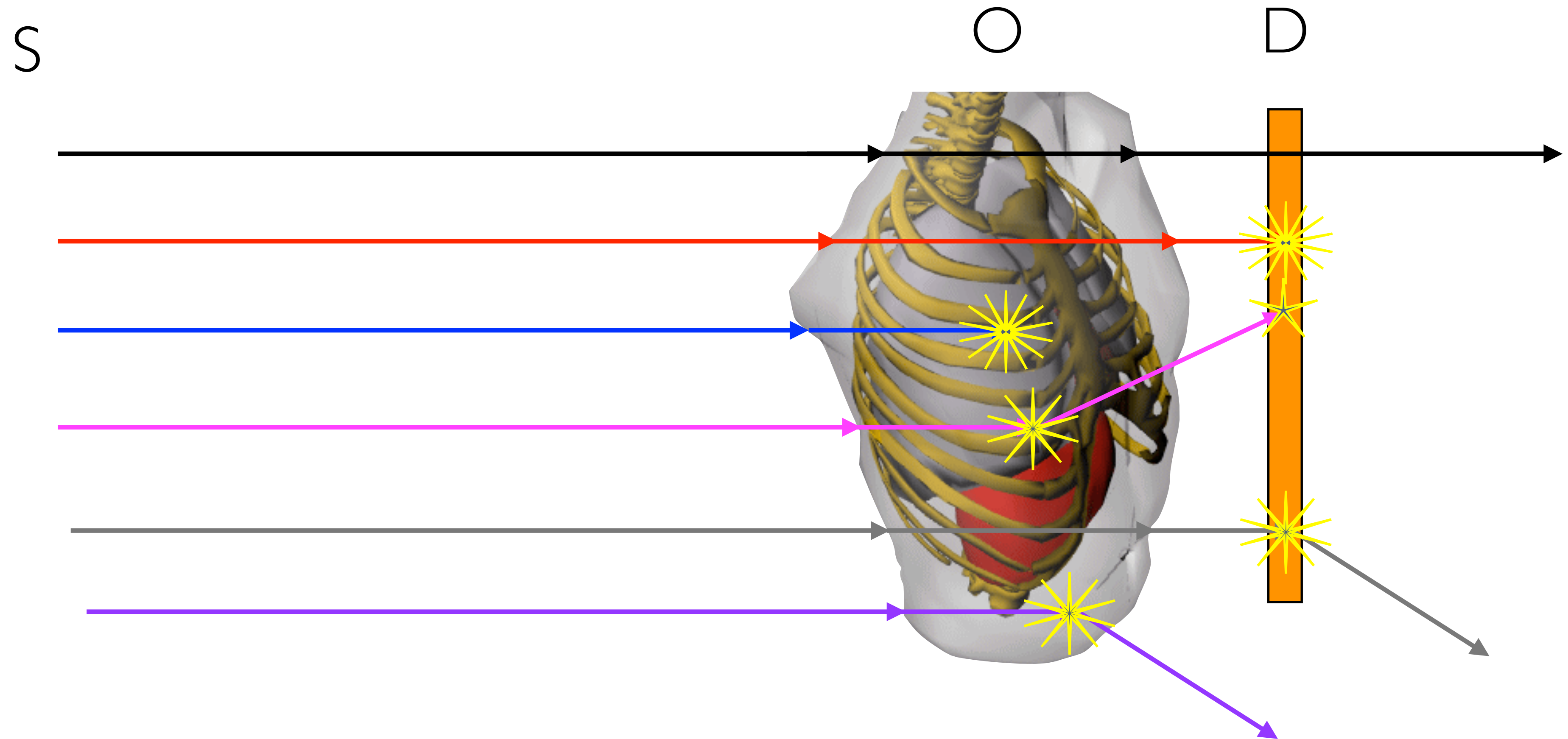
Possible outcomes



Possible outcomes

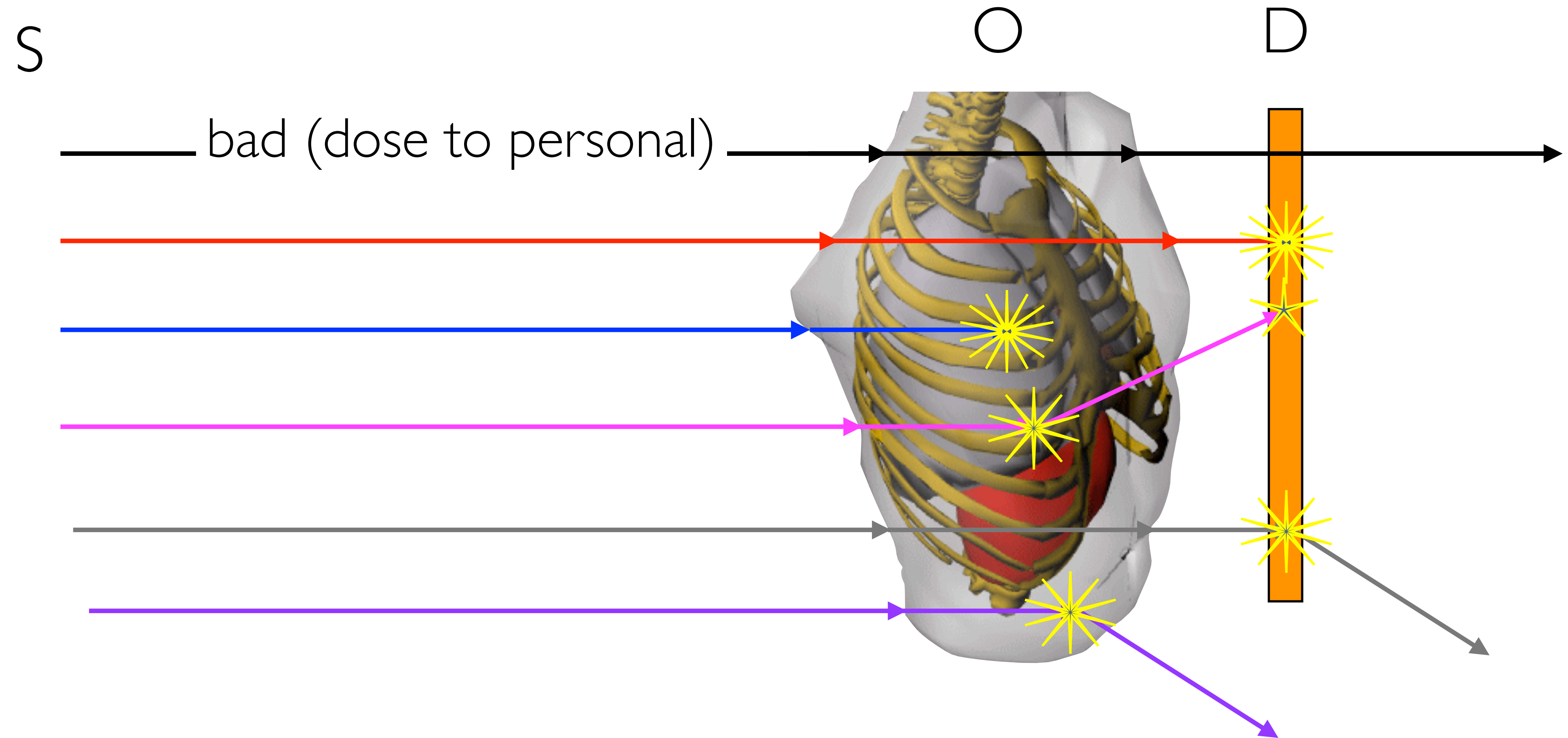


Possible outcomes



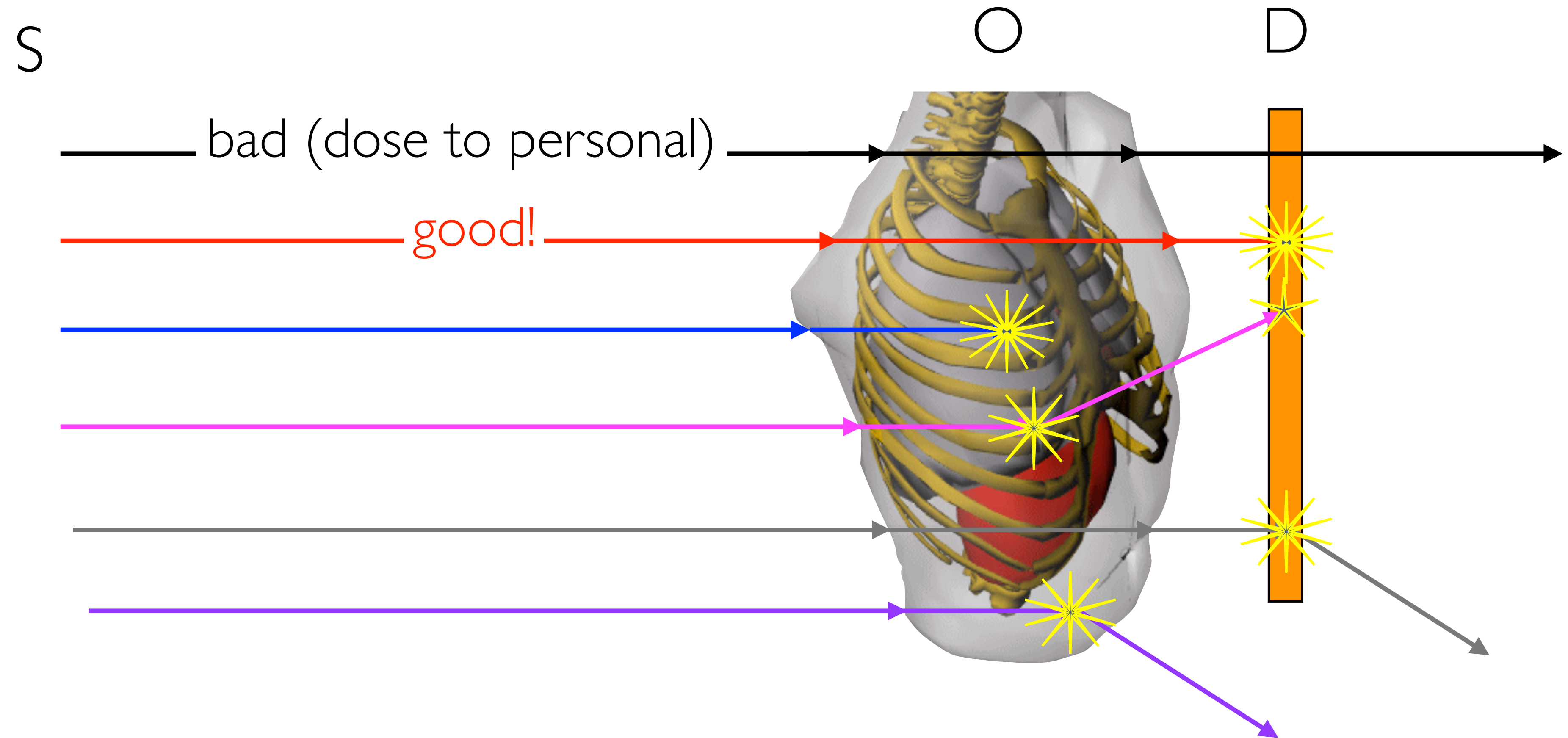
Which are the “good” ones and which are the “bad” ones?

Possible outcomes



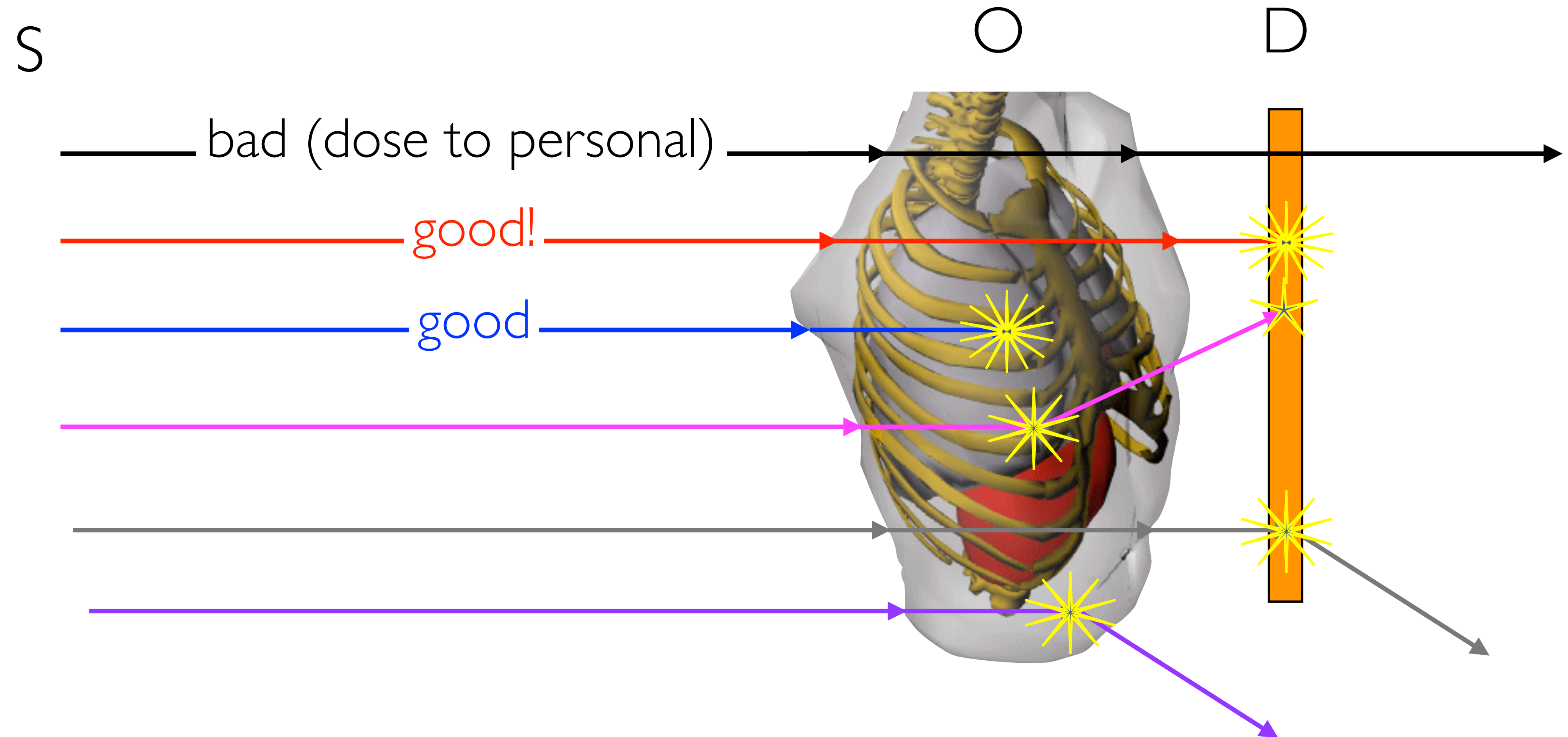
Which are the “good” ones and which are the “bad” ones?

Possible outcomes



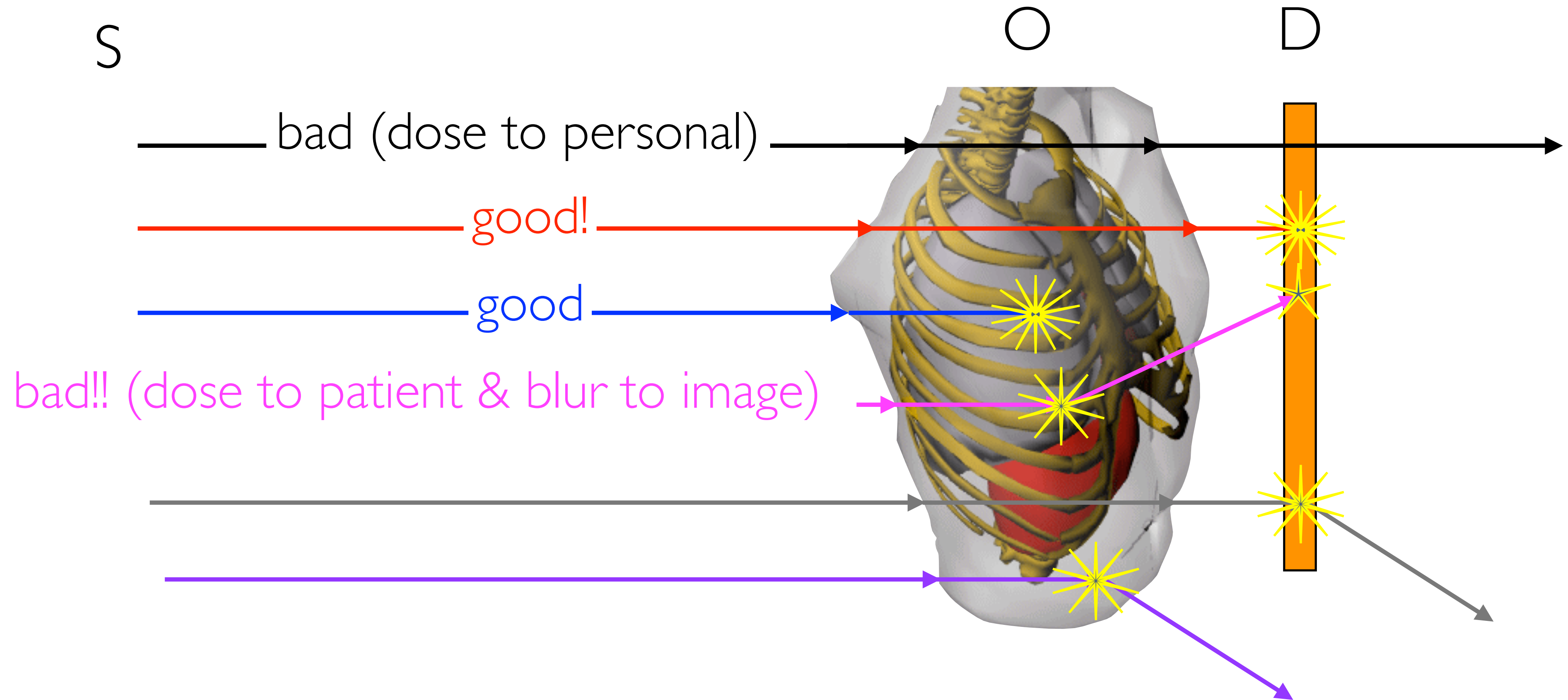
Which are the “good” ones and which are the “bad” ones?

Possible outcomes



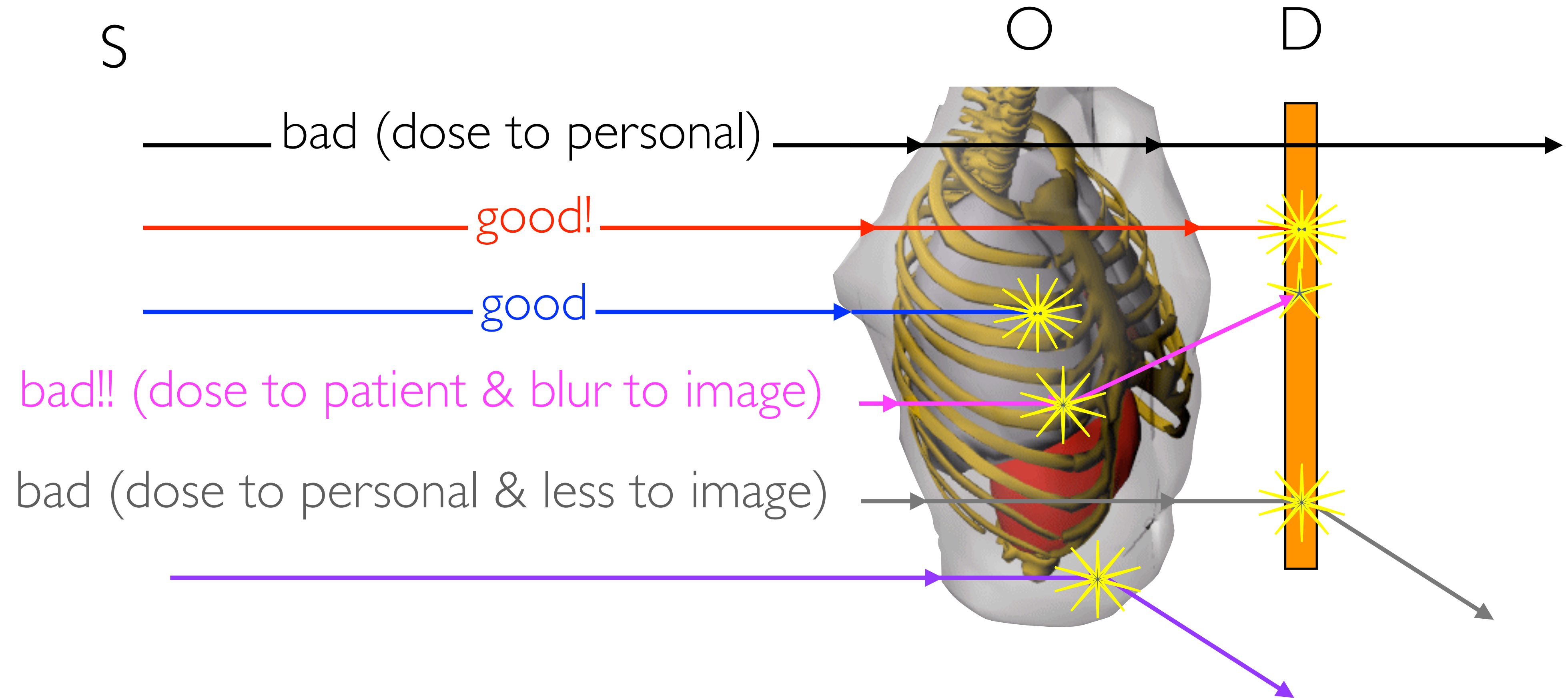
Which are the “good” ones and which are the “bad” ones?

Possible outcomes



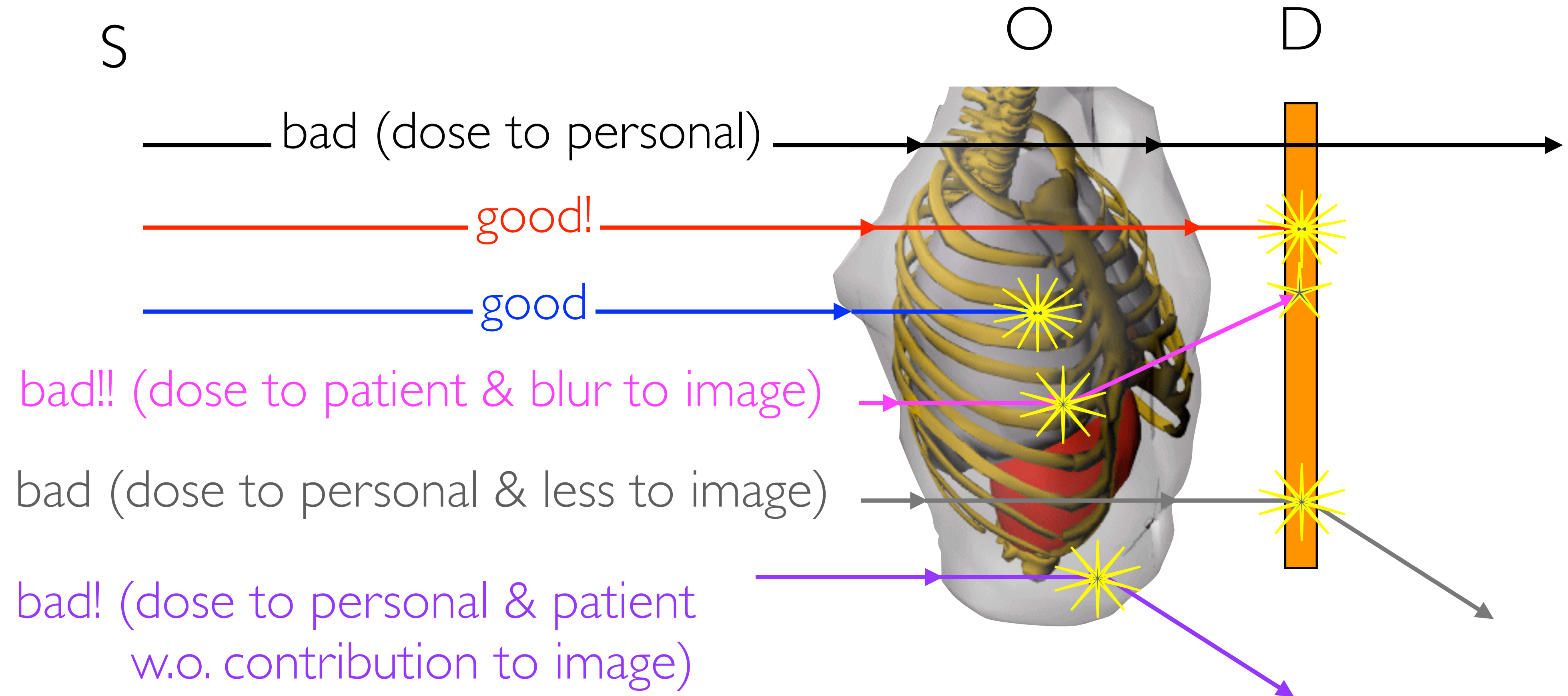
Which are the “good” ones and which are the “bad” ones?

Possible outcomes

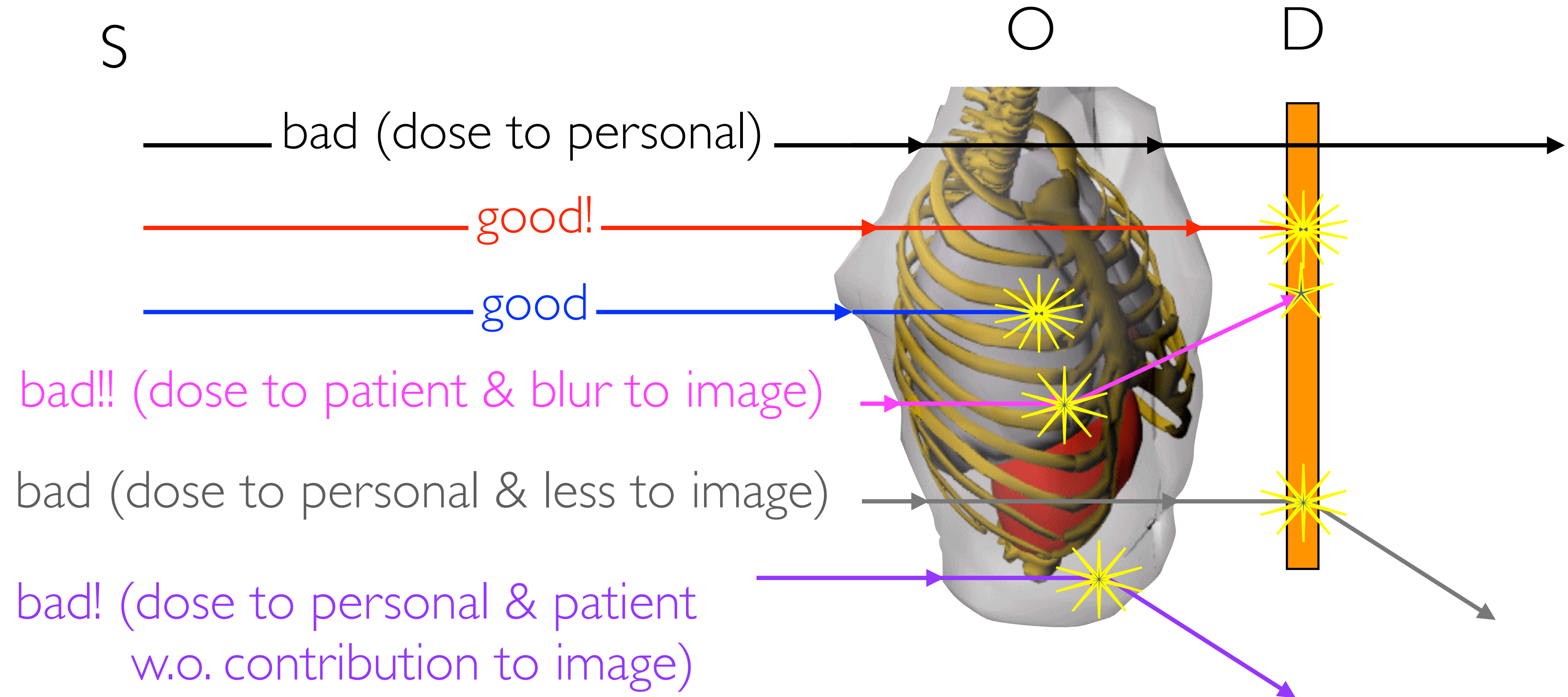


Which are the “good” ones and which are the “bad” ones?

Possible outcomes



Possible outcomes



Lessons to learn:

1. avoid compton in patient and detector!
2. x-rays should absorb ~100% in detector

Next we will talk about the elephant in the
room: Noise!

This will add yet another factor to consider
and will be an important player in distinguishing
between object contrast and image contrast.

Object contrast vs image contrast

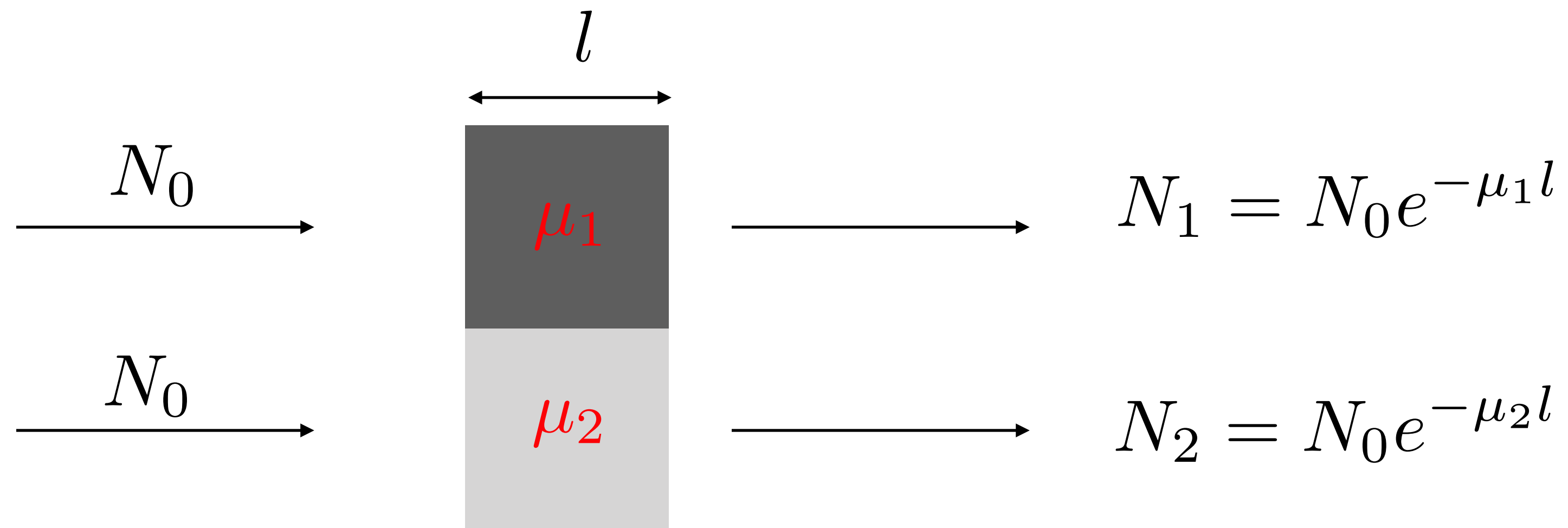
Object contrast: relative difference in attenuation between object and background

————→ it is a property of the object and only the object!

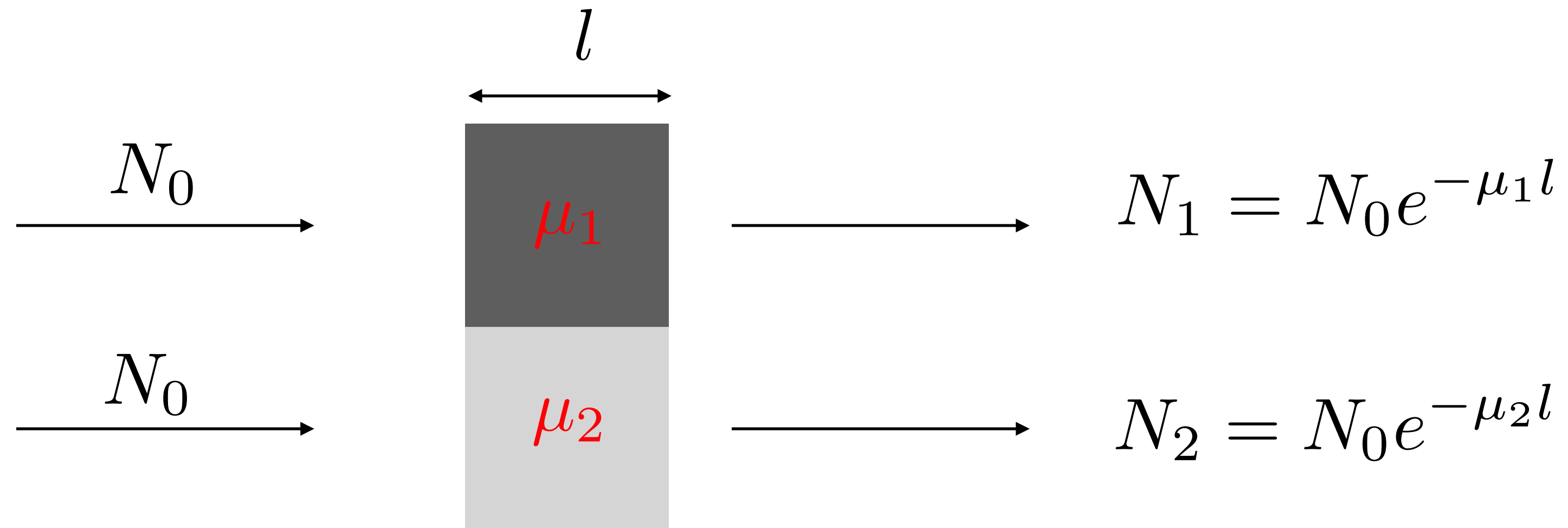
Image contrast: relative difference in grey scale or pixel value in the image between object and background

————→ depends on the object **and** imaging set up (protocol + imaging system)

Let's discuss a simple example

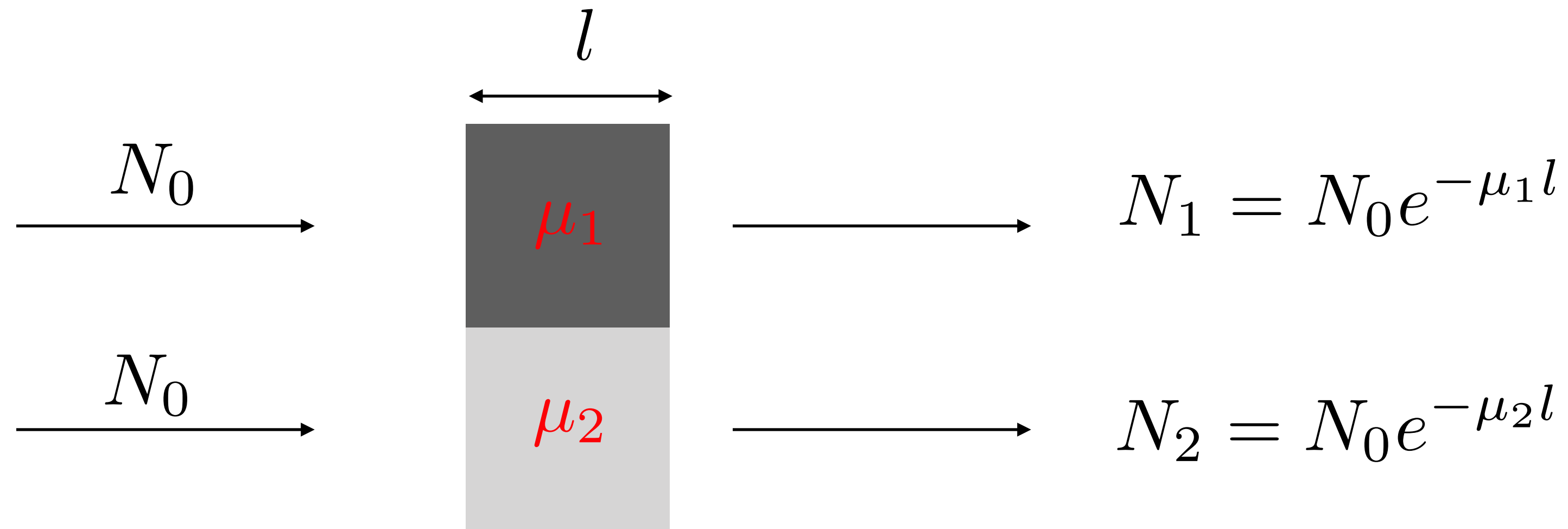


Let's discuss a simple example



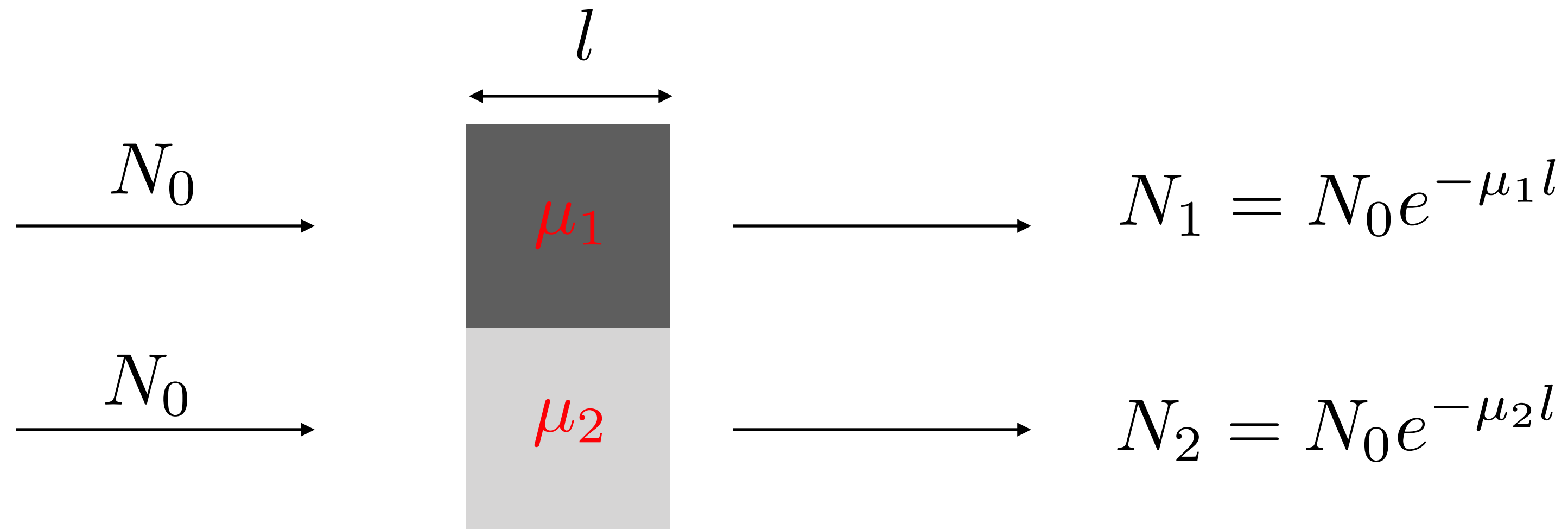
$$OC = \frac{N_1 - N_2}{N_1} = \frac{N_0(e^{-\mu_1 l} - e^{-\mu_2 l})}{N_0 e^{-\mu_1 l}} = 1 - e^{-(\mu_2 - \mu_1)l}$$

Let's discuss a simple example



$$OC = \frac{N_1 - N_2}{N_1} = \frac{N_0(e^{-\mu_1 l} - e^{-\mu_2 l})}{N_0 e^{-\mu_1 l}} = 1 - e^{-(\mu_2 - \mu_1) \overset{\text{size of object}}{\textcircled{l}}}$$

Let's discuss a simple example

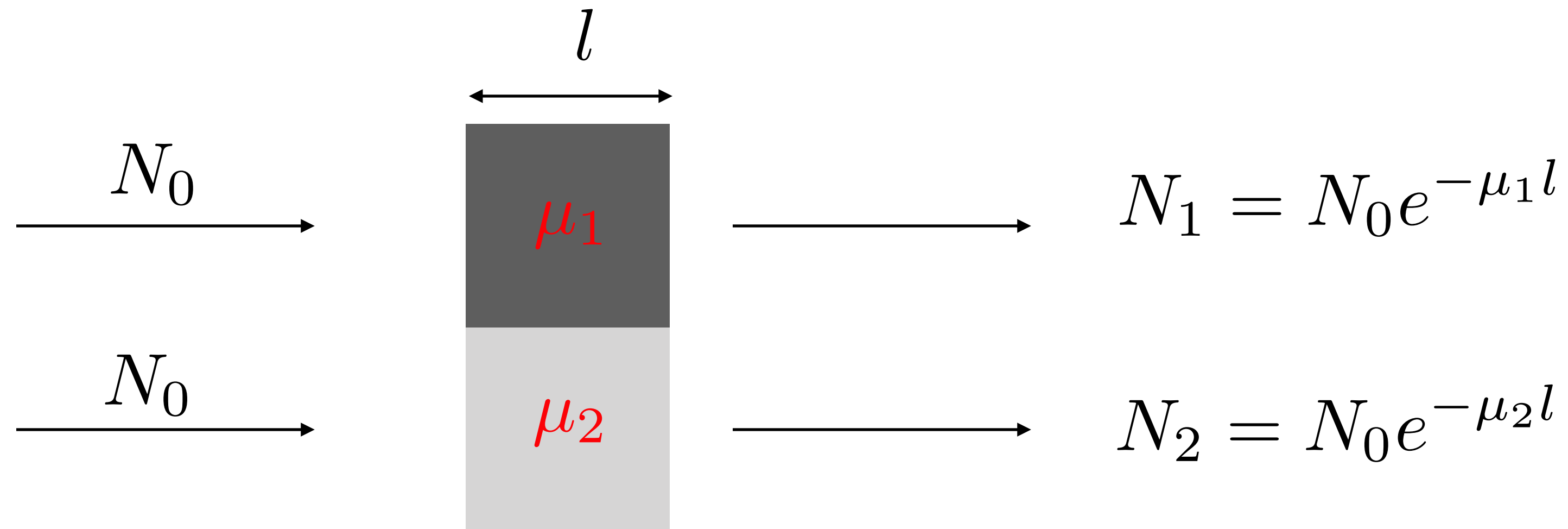


$$OC = \frac{N_1 - N_2}{N_1} = \frac{N_0 (e^{-\mu_1 l} - e^{-\mu_2 l})}{N_0 e^{-\mu_1 l}} = 1 - e^{-(\mu_2 - \mu_1)l}$$

size of object

attenuation difference

Let's discuss a simple example



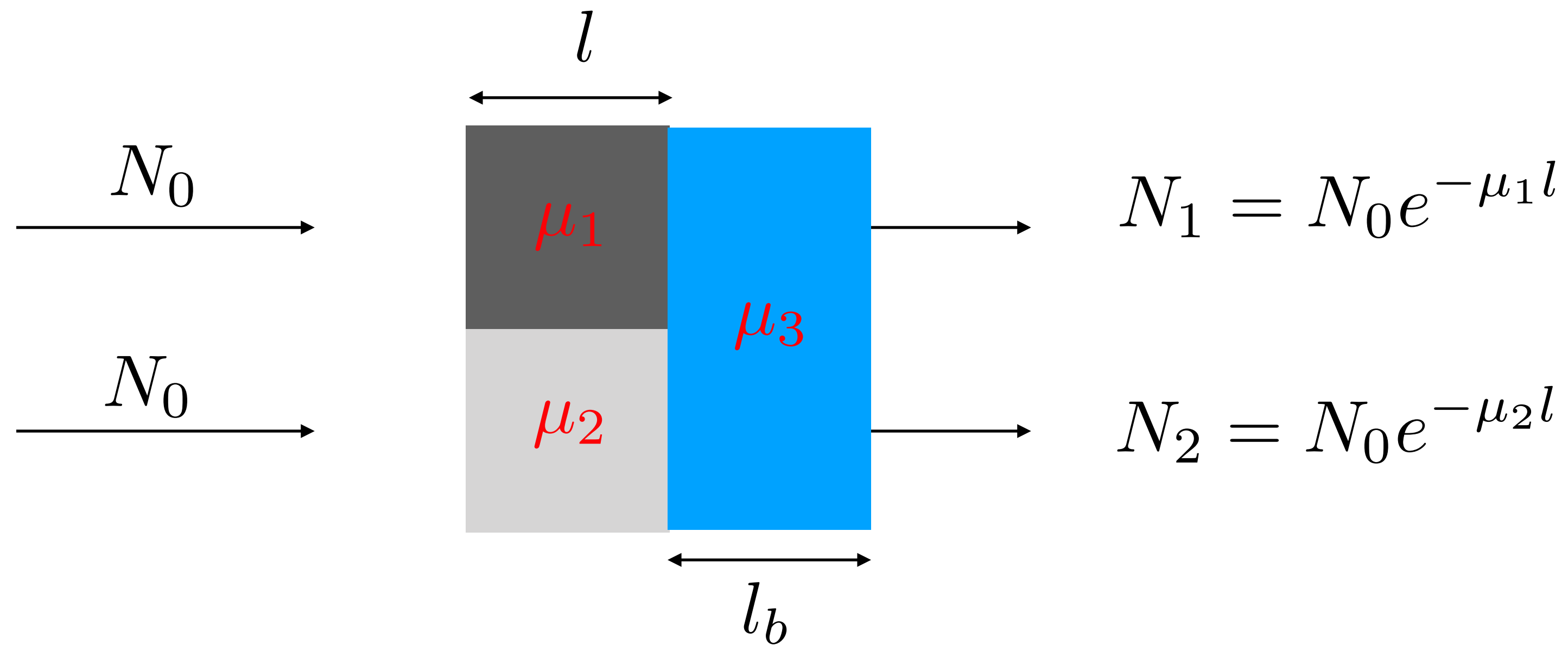
$$OC = \frac{N_1 - N_2}{N_1} = \frac{N_0(e^{-\mu_1 l} - e^{-\mu_2 l})}{N_0 e^{-\mu_1 l}} = 1 - e^{-(\mu_2 - \mu_1)l}$$

size of object

attenuation difference

independent on number of photons!

Let's discuss a simple example



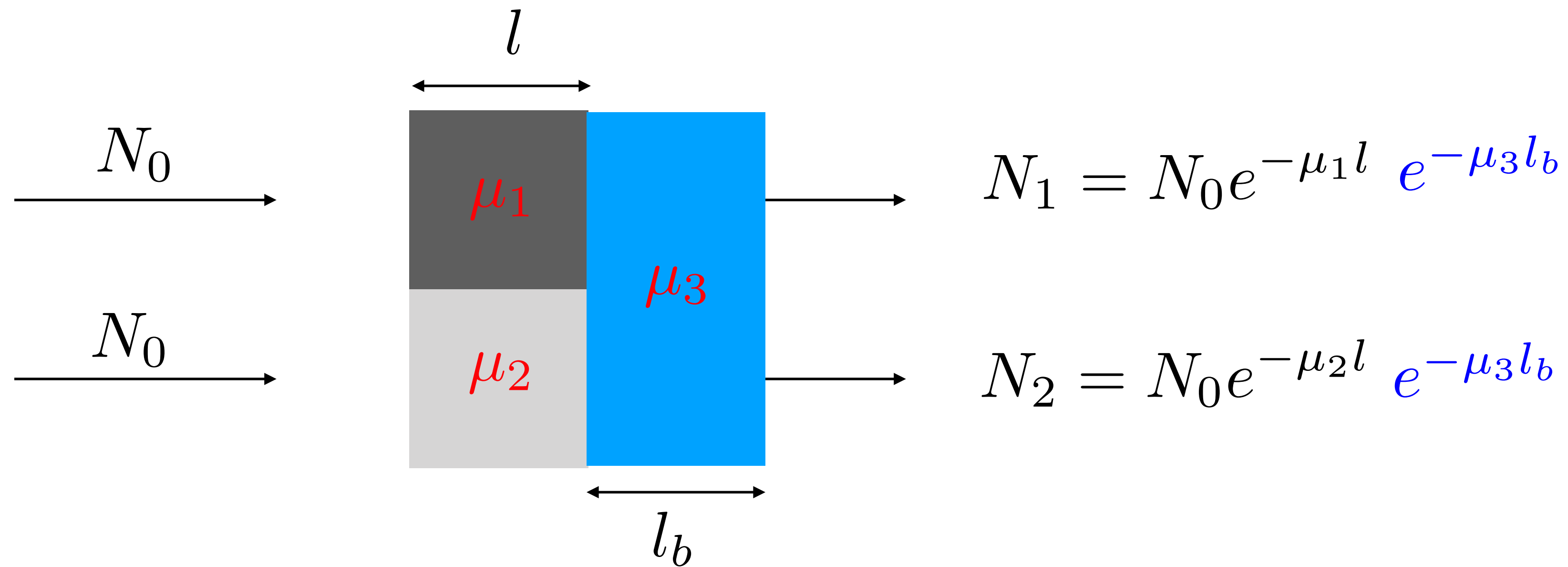
$$OC = \frac{N_1 - N_2}{N_1} = \frac{N_0(e^{-\mu_1 l} - e^{-\mu_2 l})}{N_0 e^{-\mu_1 l}} = 1 - e^{-(\mu_2 - \mu_1)l}$$

size of object

attenuation difference

independent on number of photons!

Let's discuss a simple example



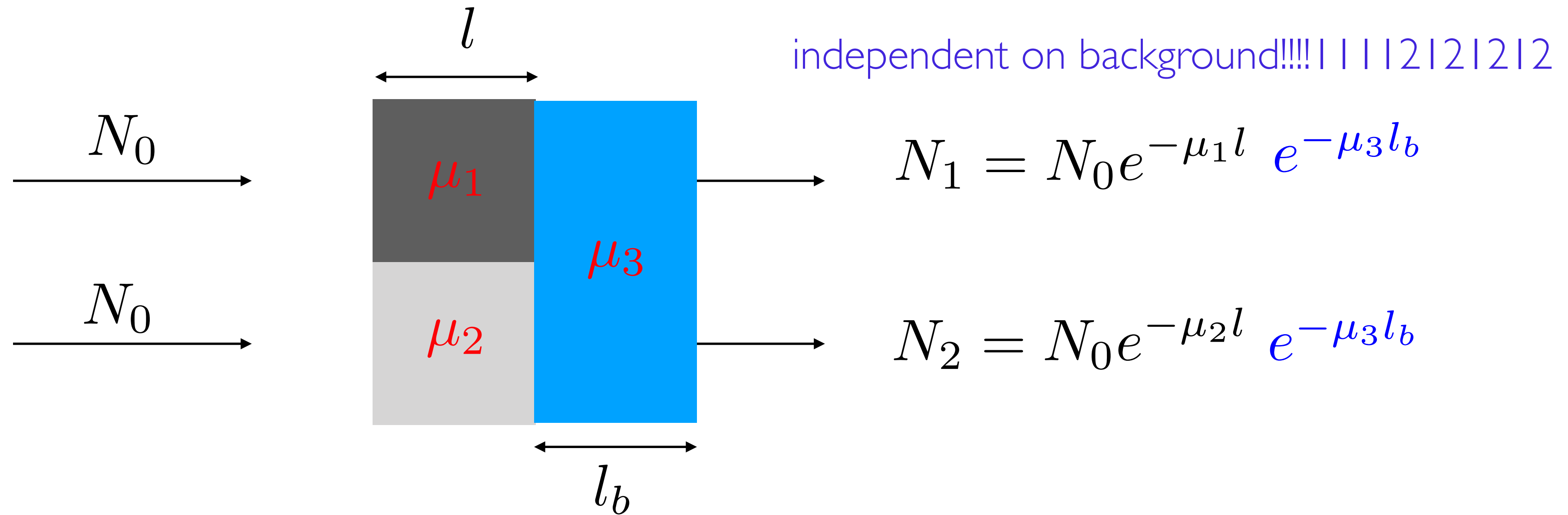
$$OC = \frac{N_1 - N_2}{N_1} = \frac{N_0(e^{-\mu_1 l} - e^{-\mu_2 l})}{N_0 e^{-\mu_1 l}} = 1 - e^{-(\mu_2 - \mu_1)l}$$

size of object

attenuation difference

independent on number of photons!

Let's discuss a simple example



$$OC = \frac{N_1 - N_2}{N_1} = \frac{N_0 (e^{-\mu_1 l} - e^{-\mu_2 l})}{N_0 e^{-\mu_1 l}} = 1 - e^{-(\mu_2 - \mu_1) l}$$

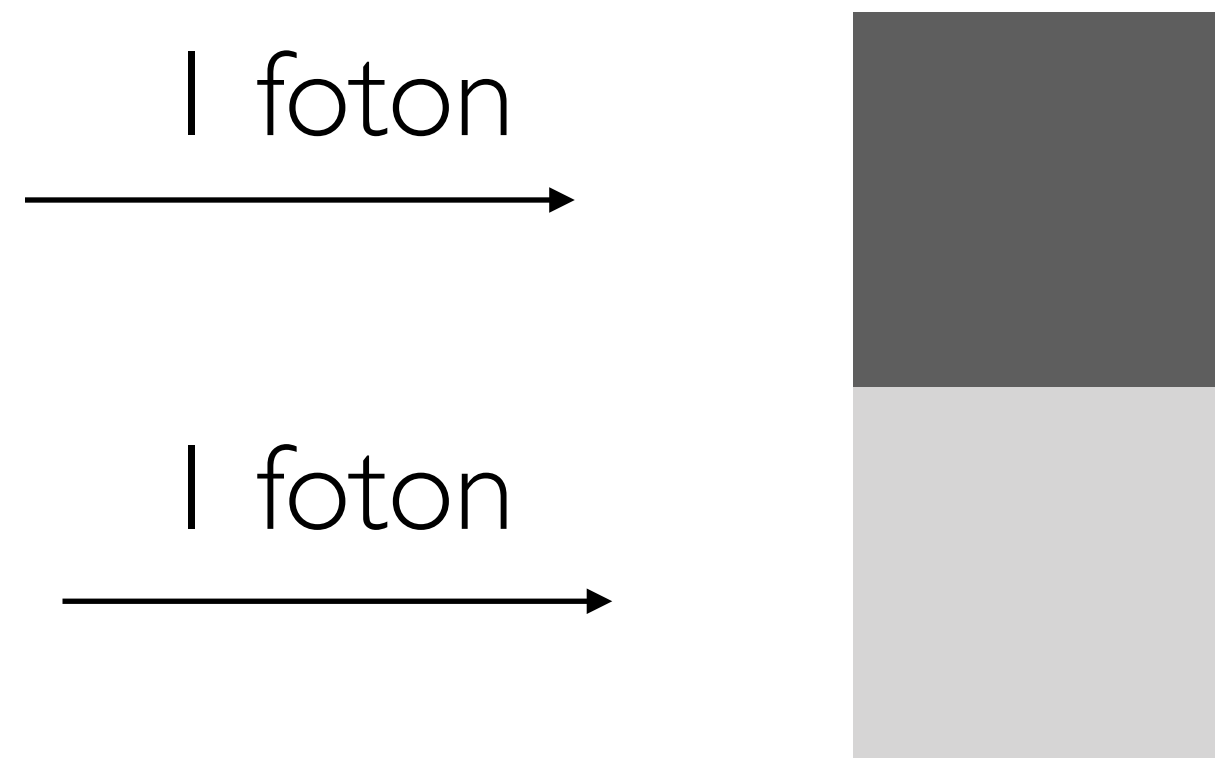
size of object

attenuation difference

independent on number of photons!

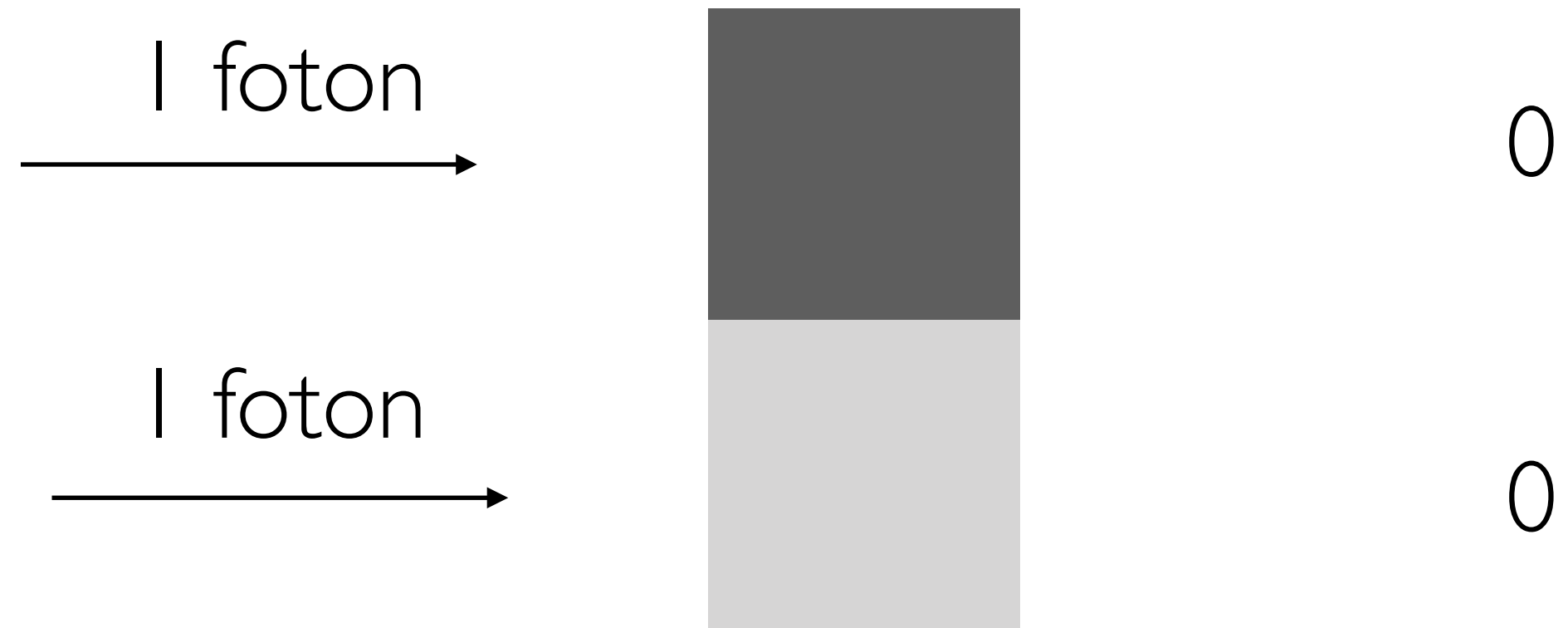
Let's do a “*reality check*”

possible outcomes



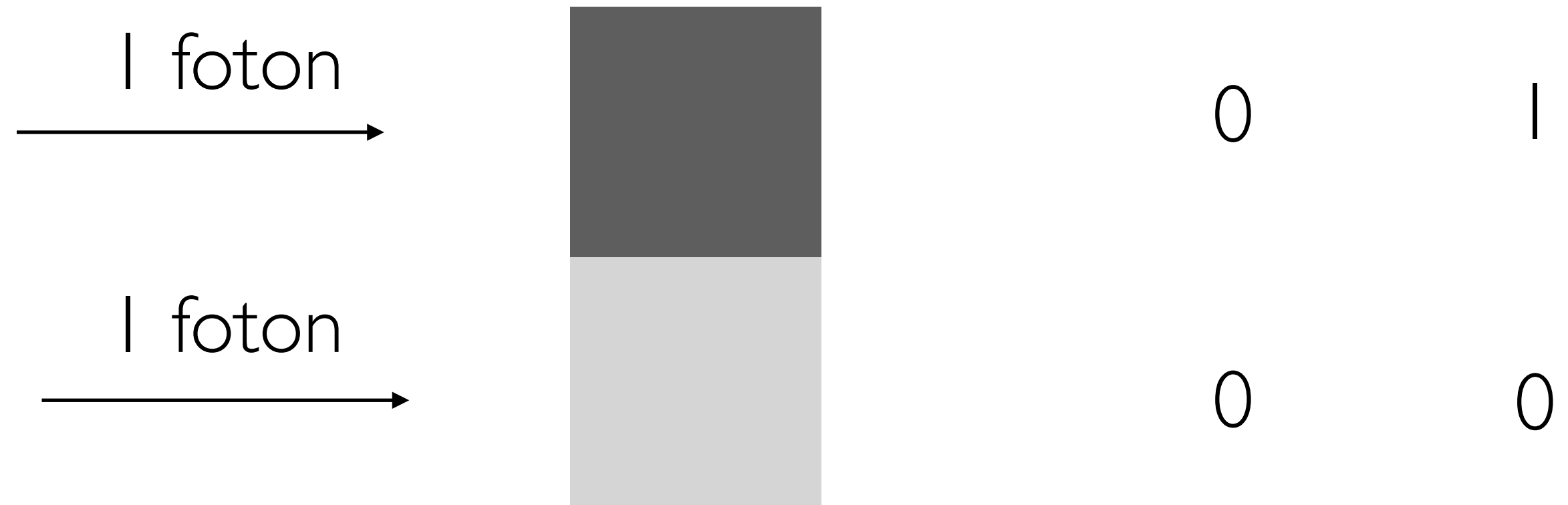
Let's do a “*reality check*”

possible outcomes



Let's do a “*reality check*”

possible outcomes



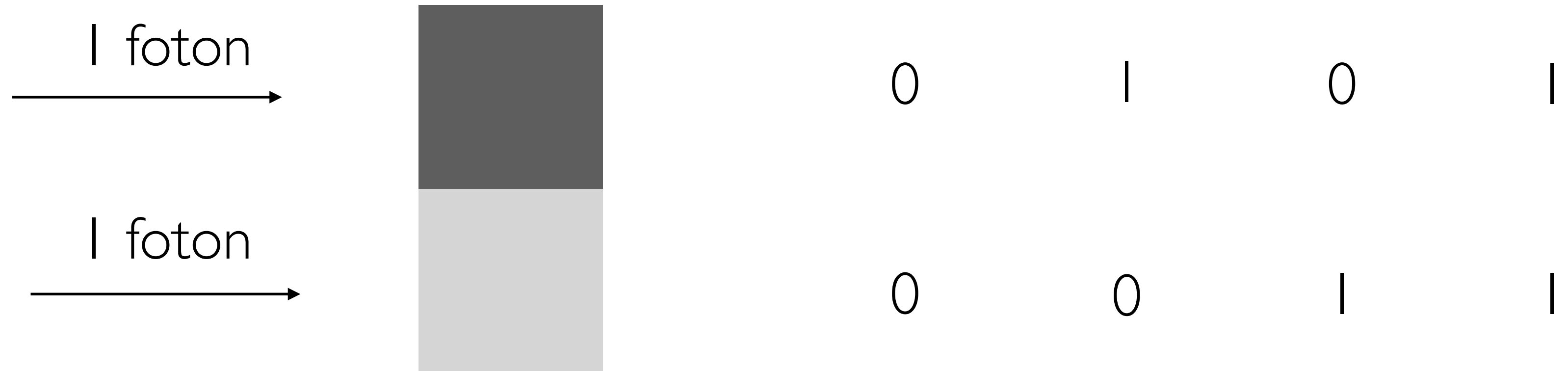
Let's do a “*reality check*”

possible outcomes



Let's do a “*reality check*”

possible outcomes



Let's do a “*reality check*”

possible outcomes



Let's do a "reality check"

possible outcomes



Since the two plates have different attenuation $\Rightarrow$ the 4 outcomes have not the same probability, but with measurement this will not show!

simulation time!

object

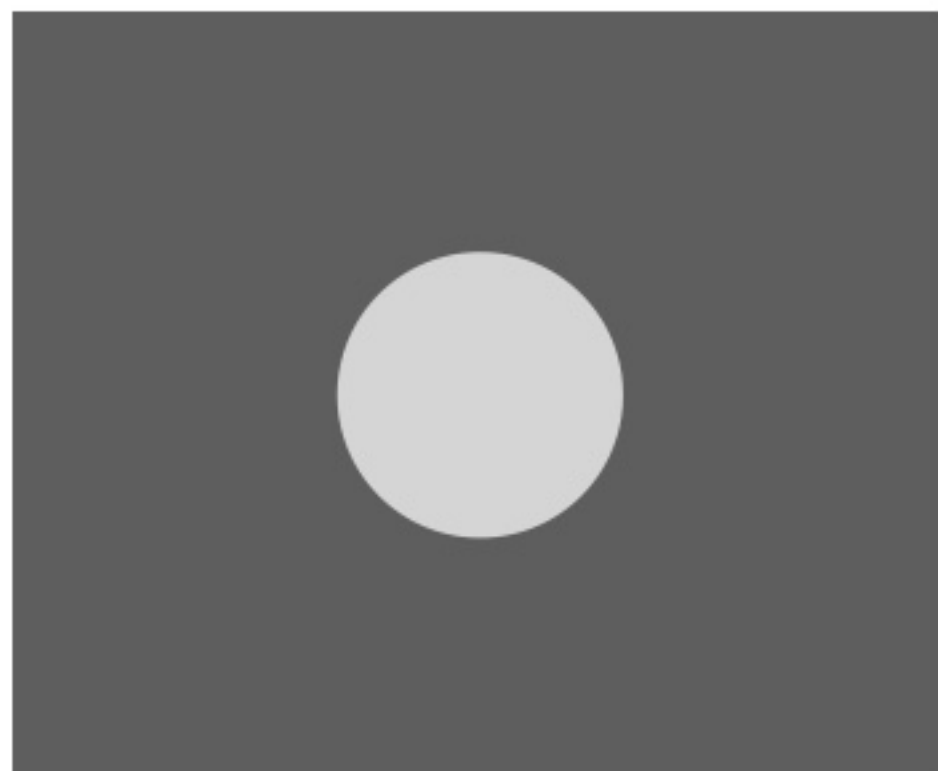


Image A

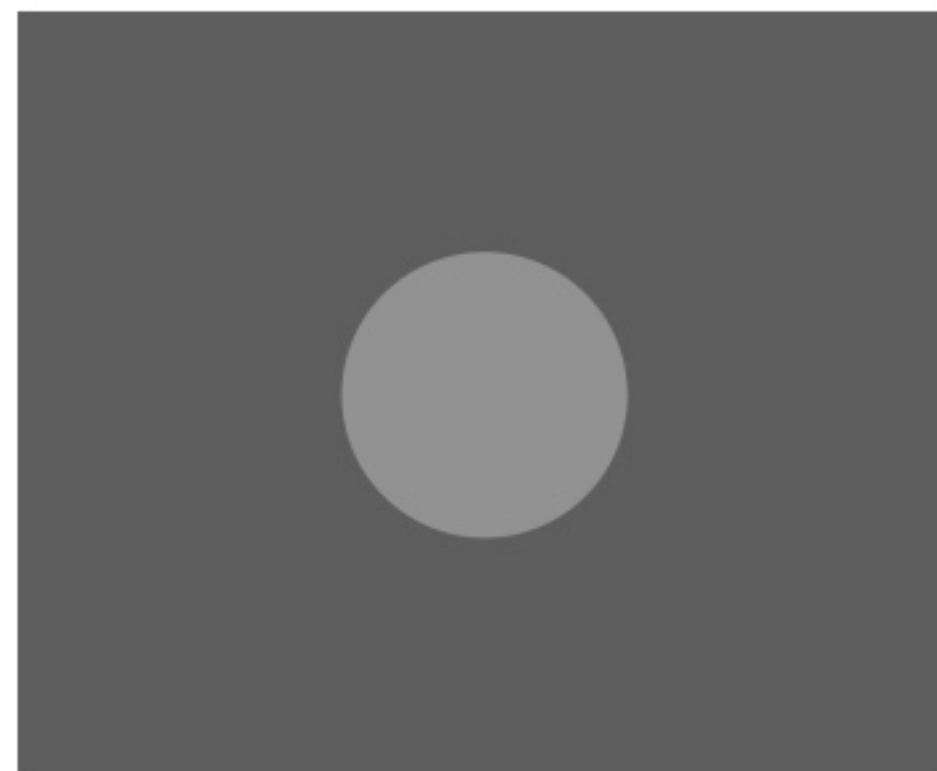


Image B

All have the same object contrast!

images

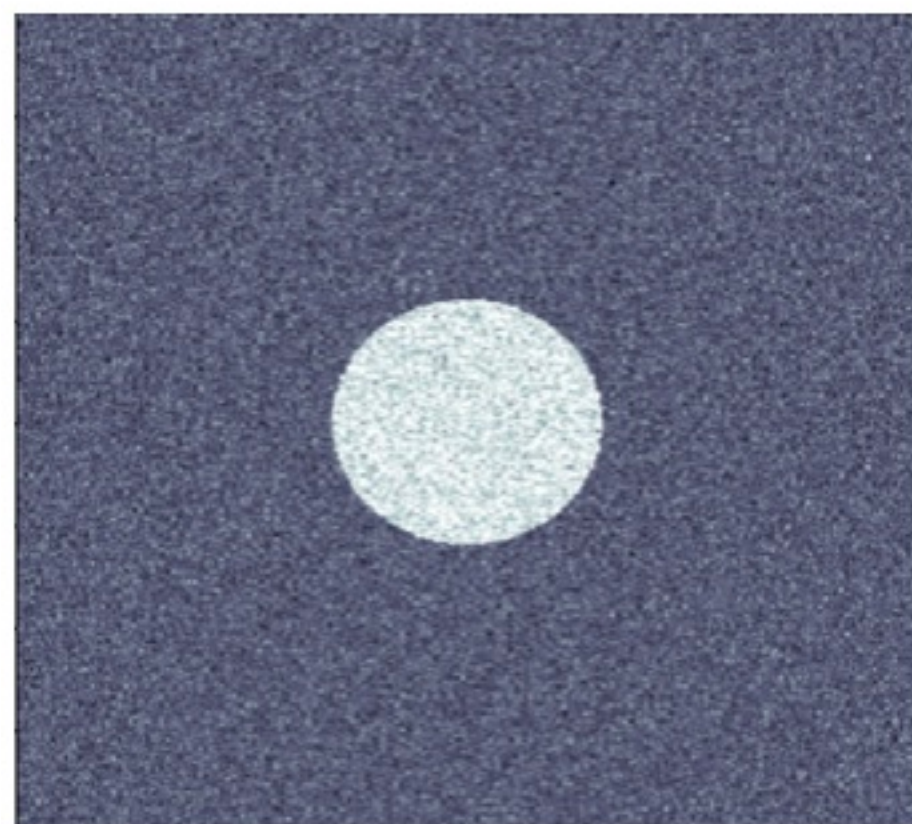


Image A

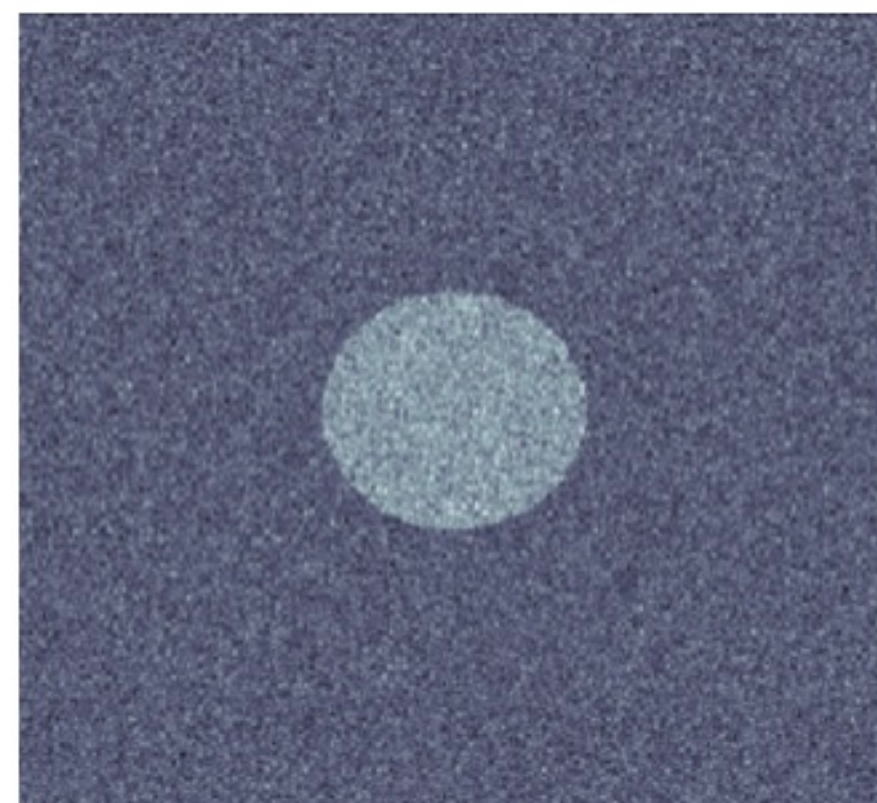


Image B

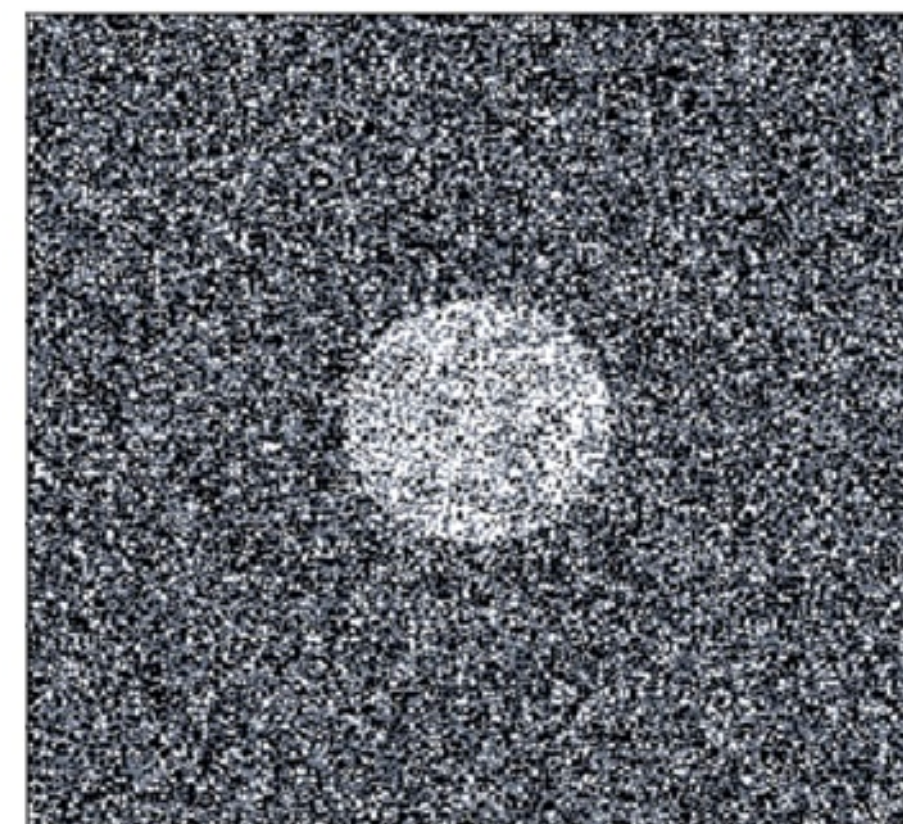


Image A

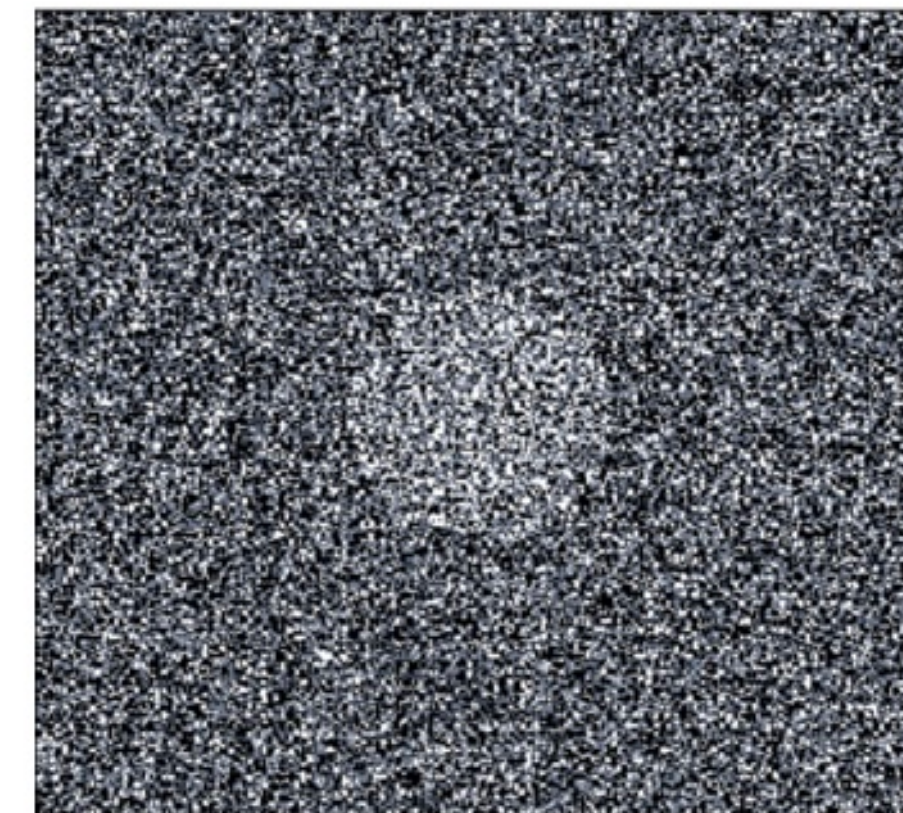


Image B

object

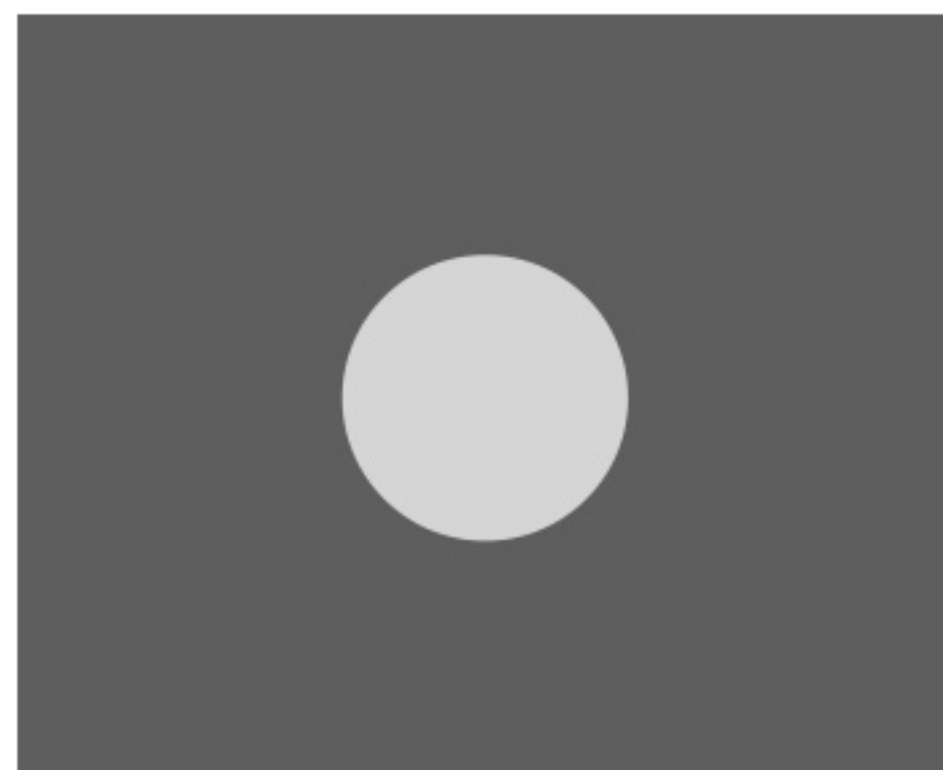


Image A

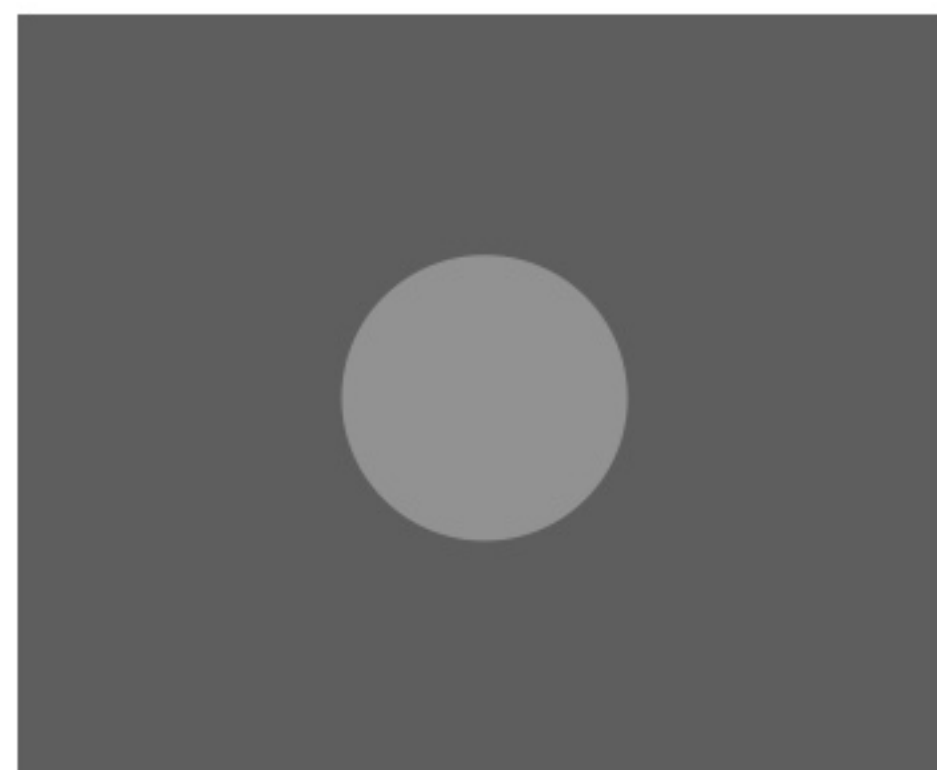


Image B

infinite number of photons

All have the same object contrast!

images

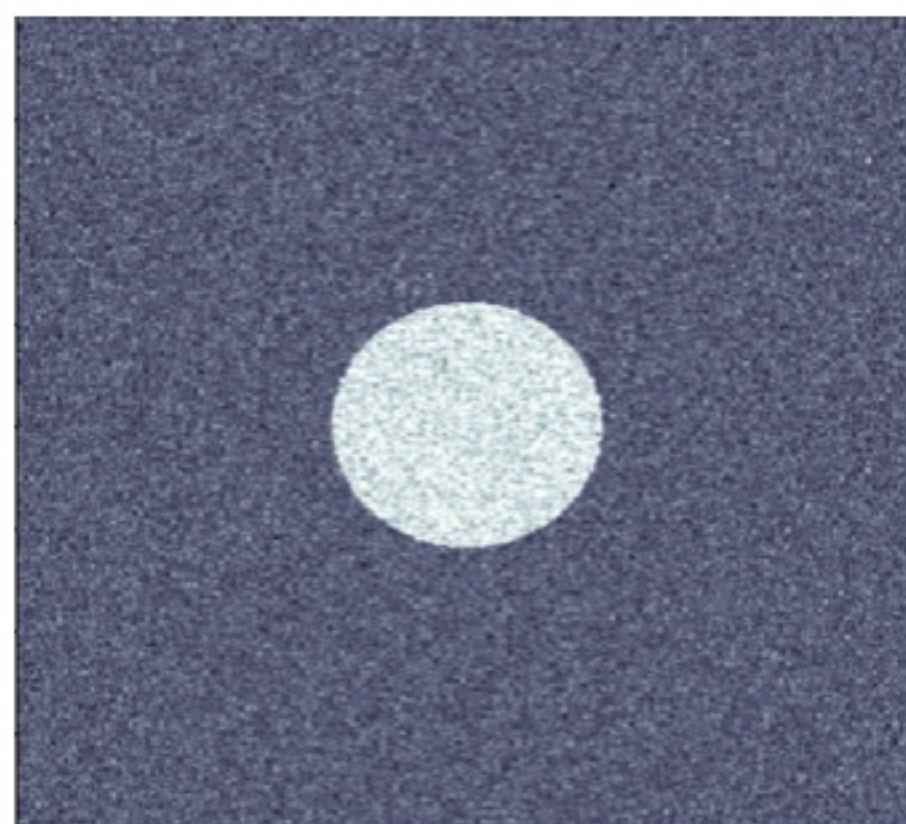


Image A

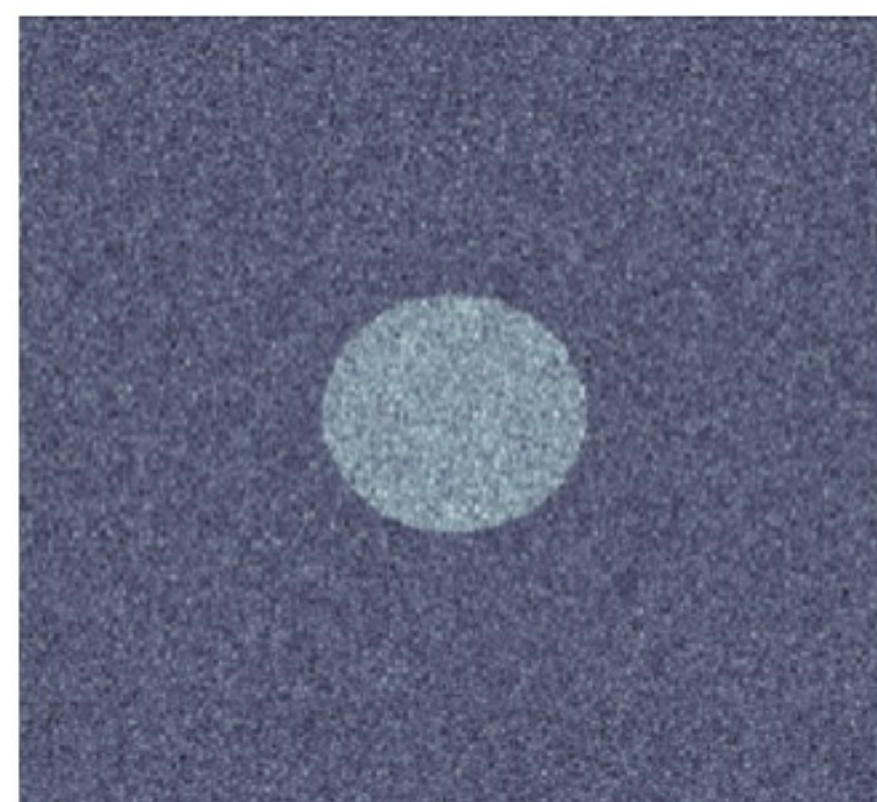


Image B

“many” photons

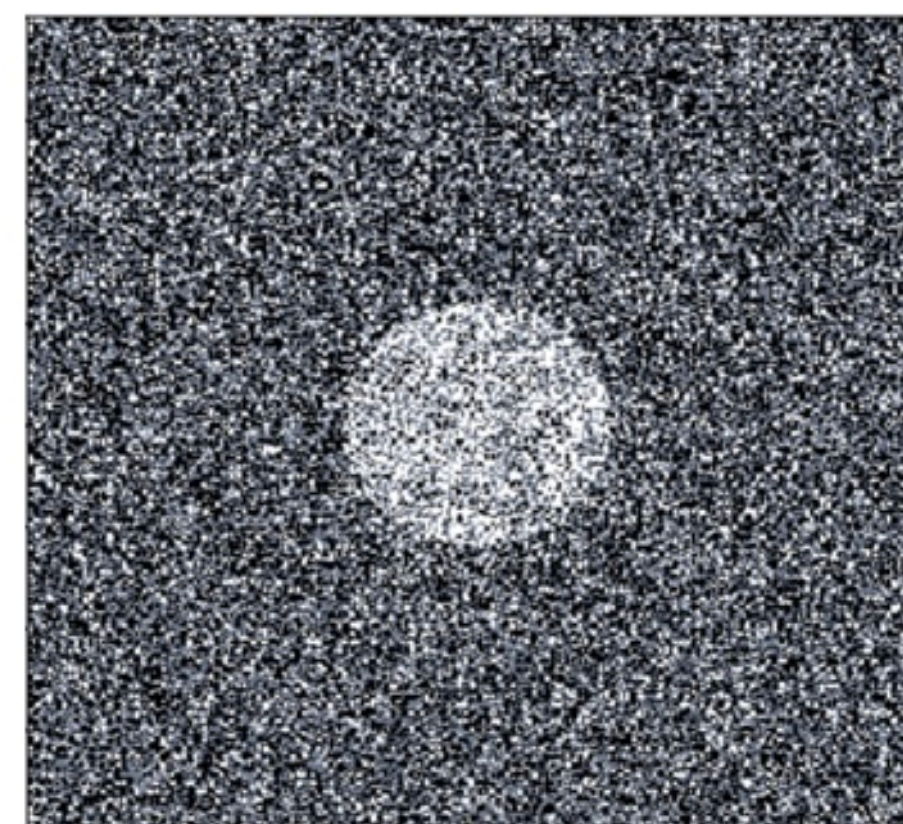


Image A

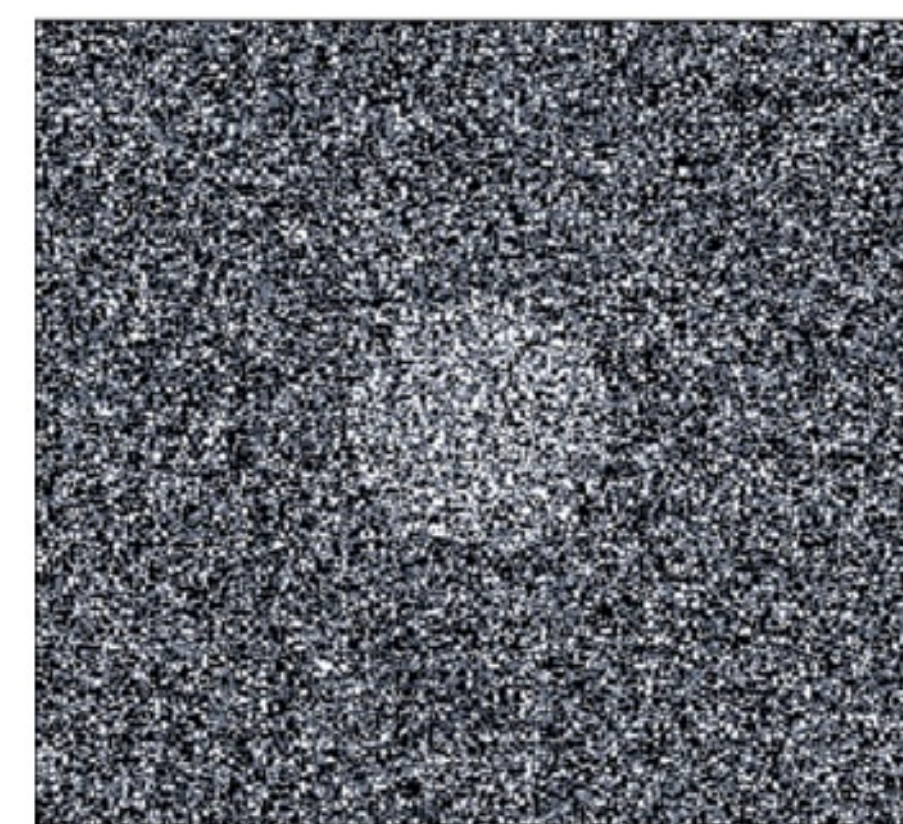


Image B

“few” photons

Poisson distribution and counting statistics

Each time one has events happening randomly at a fixed rate and independently on the past then the number of events follows:

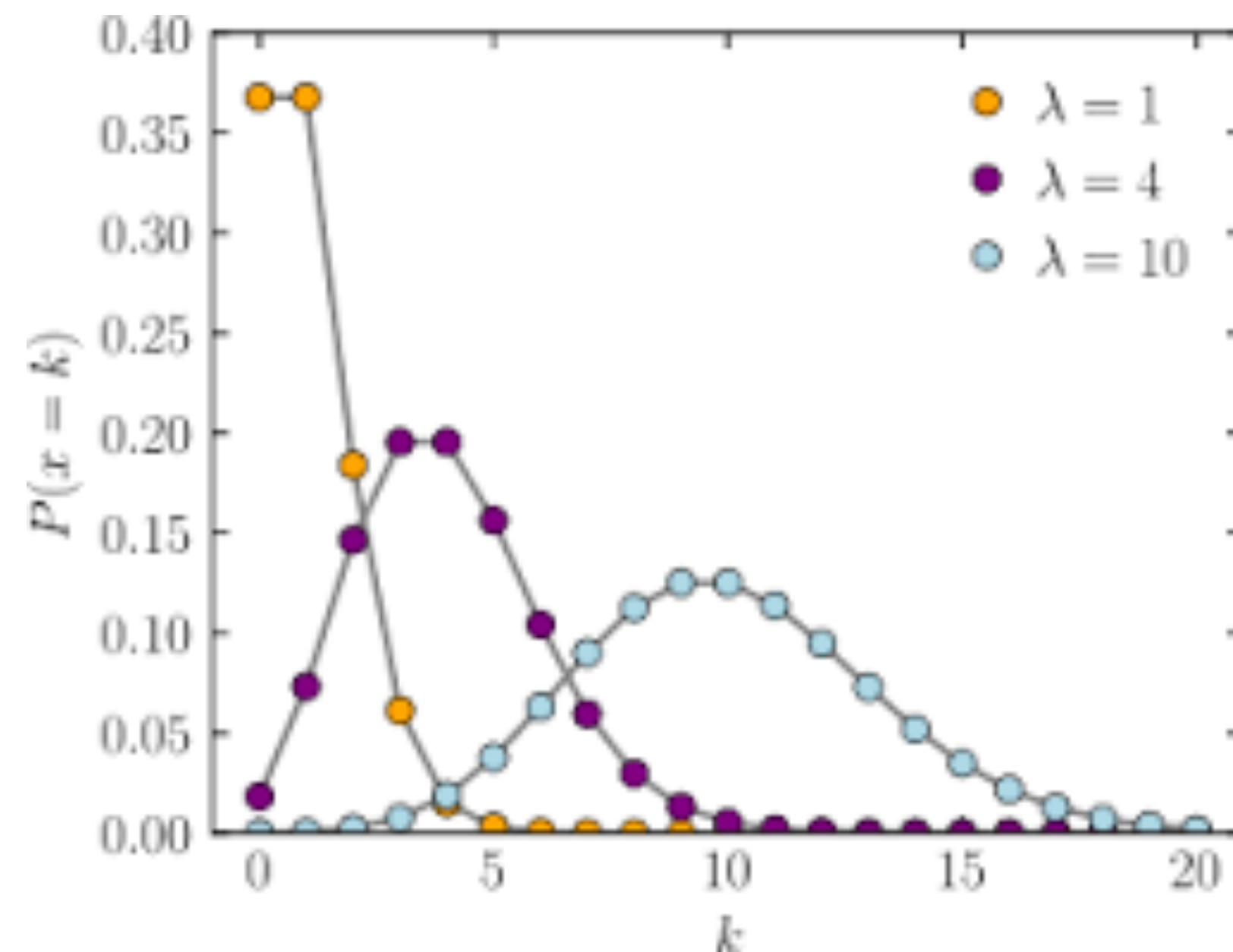


Poisson distribution

$$Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

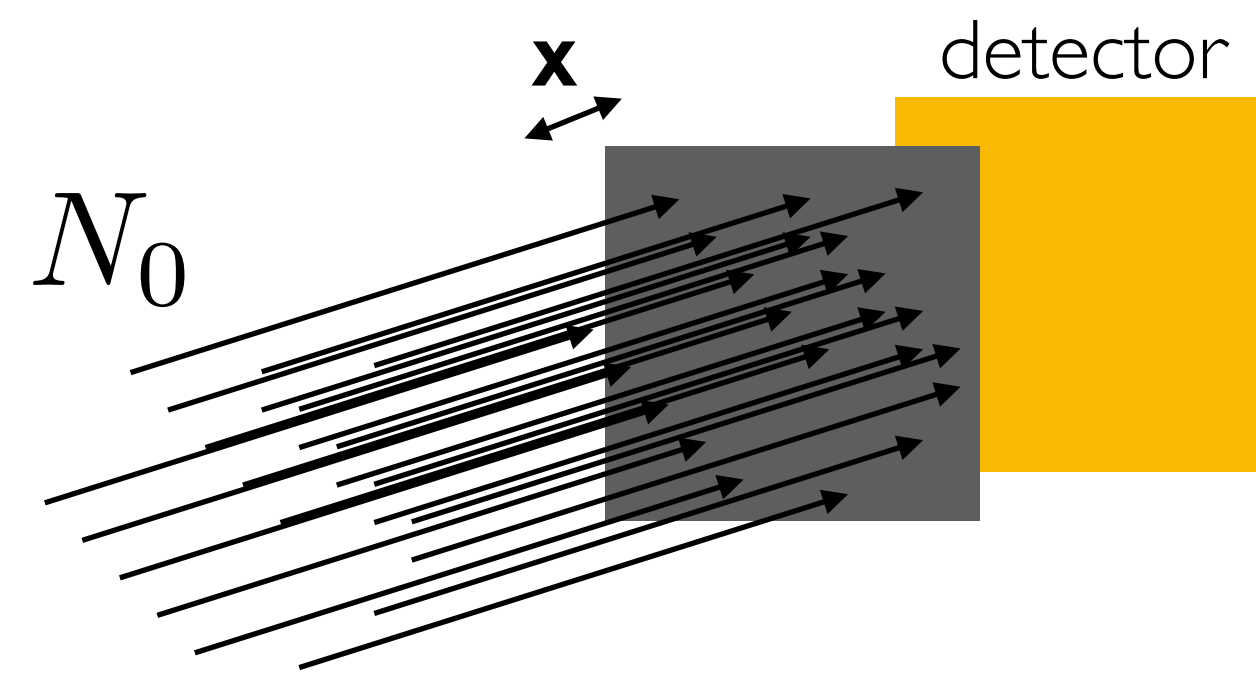
Most relevant for us

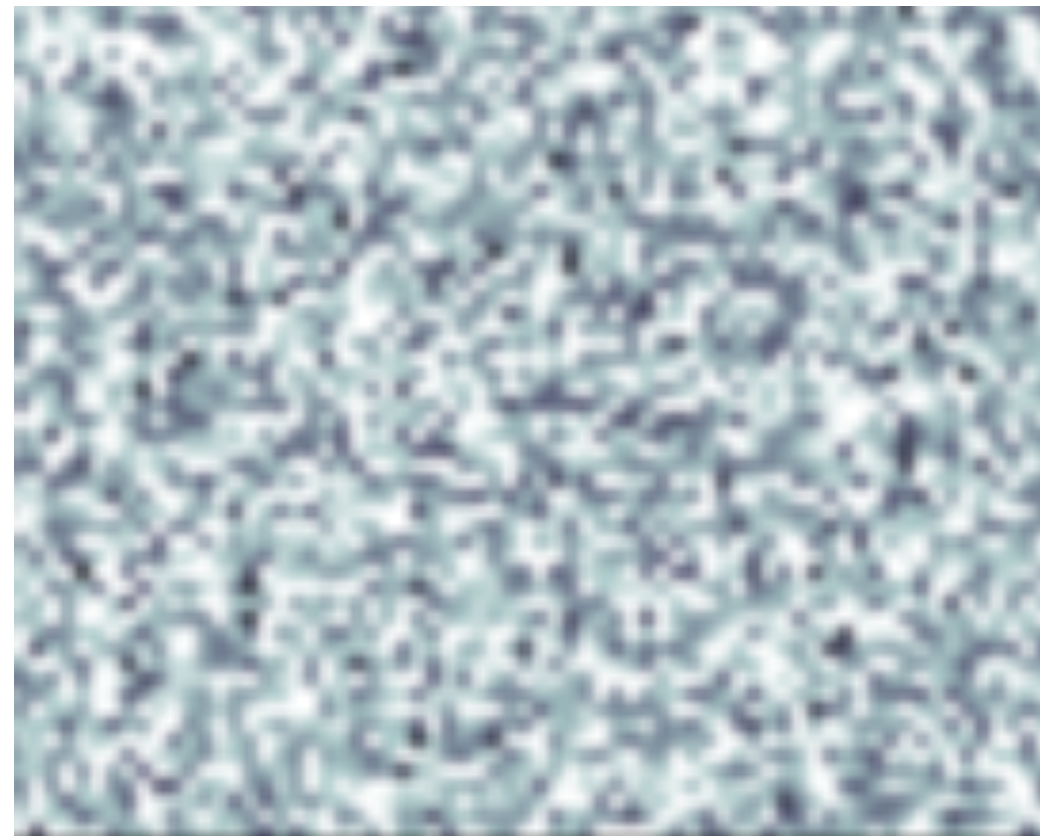
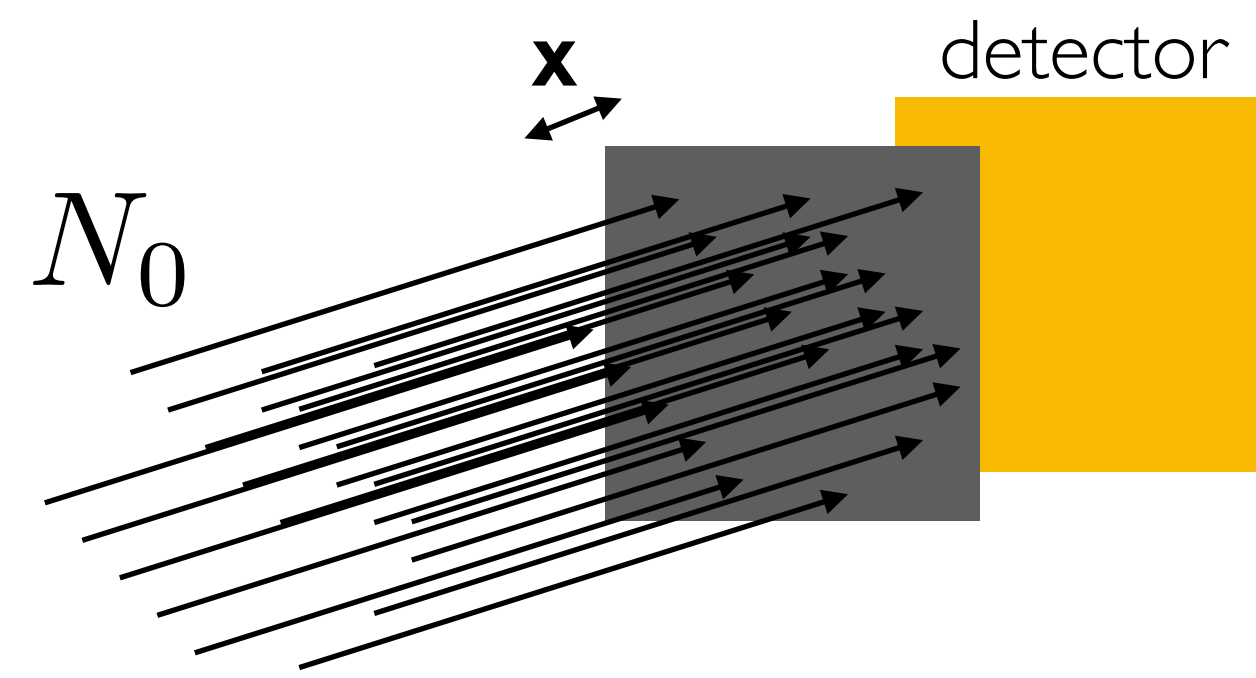
In a Poisson distribution the variance equals the expected value



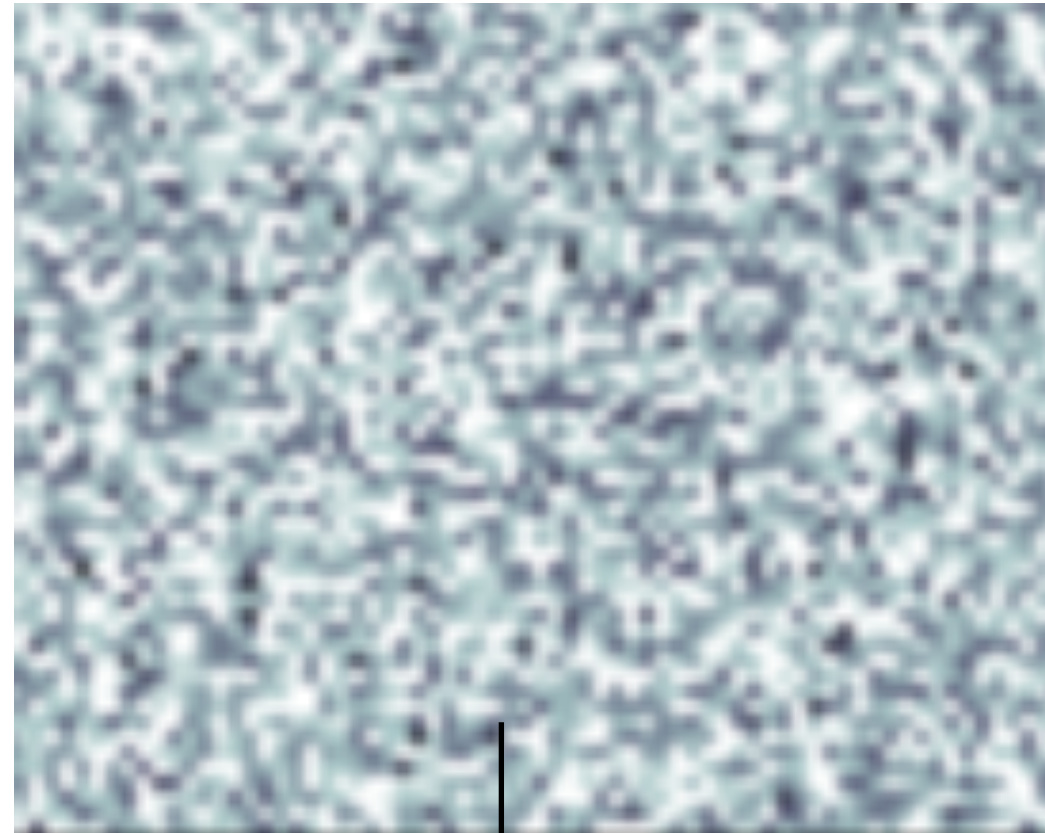
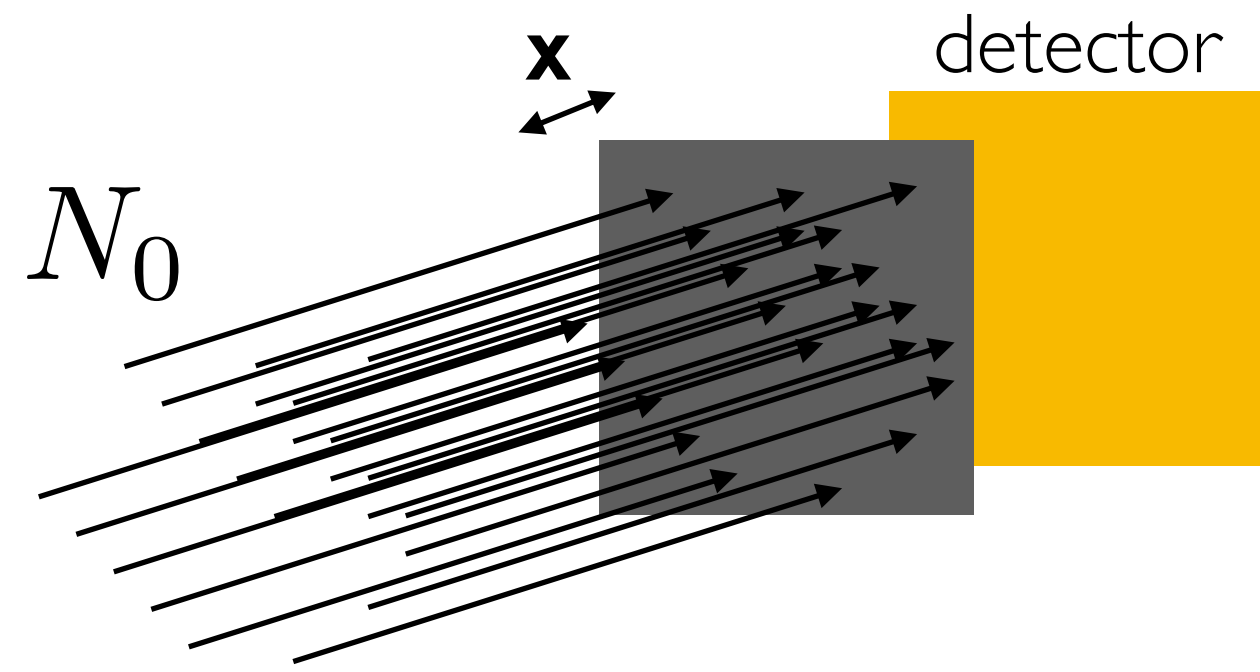
If the expected value of the number of photons through an object is N , and I repeat the experiment many times I should expect:

1. The average number of photons through the object is N
2. The standard deviation is $\sqrt{N}$
3. In around 67% of the cases the measured number of photons will be in the interval $N \pm \sqrt{N}$
4. In around 95% of the cases the measured number of photons will be in the interval $N \pm 2\sqrt{N}$
5. In around 99% of the cases the measured number of photons will be in the interval $N \pm 3\sqrt{N}$





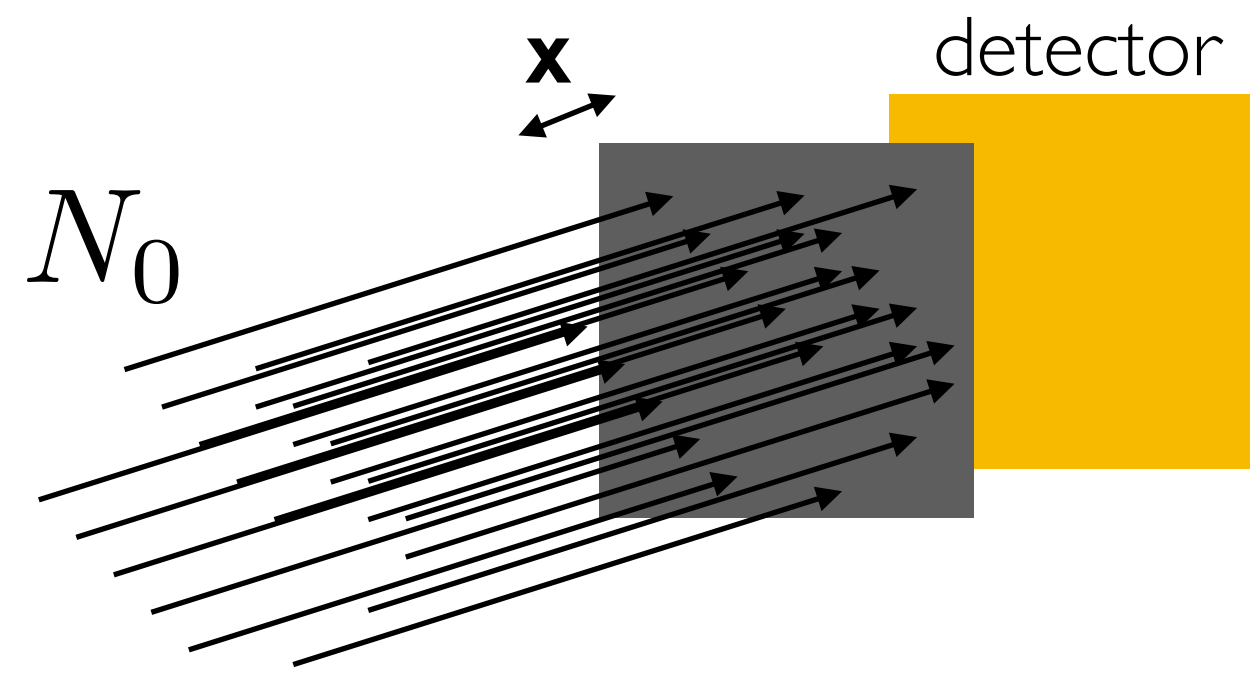
An image is an array $N_x \times N_y$ (e.g. $256 \times 256, 512 \times 512, \dots$)



I_{11} I_{12} $\dots$ $\dots$ I_{1N_x}
 I_{21} I_{22} $\dots$ $\dots$ I_{2N_x}

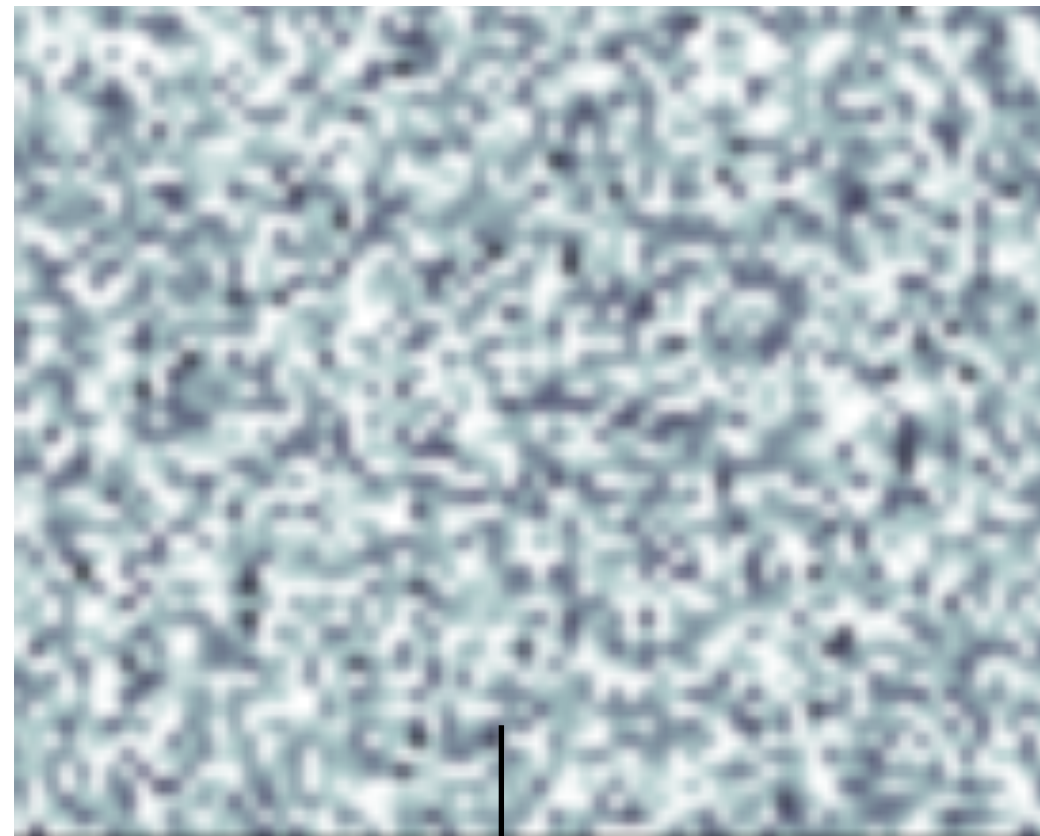
I_{N_y1} I_{N_y2} $\dots$ $\dots$ $I_{N_yN_x}$

An image is an array $N_x \times N_y$ (e.g. $256 \times 256, 512 \times 512, \dots$)



expected number of photons
on each pixel:

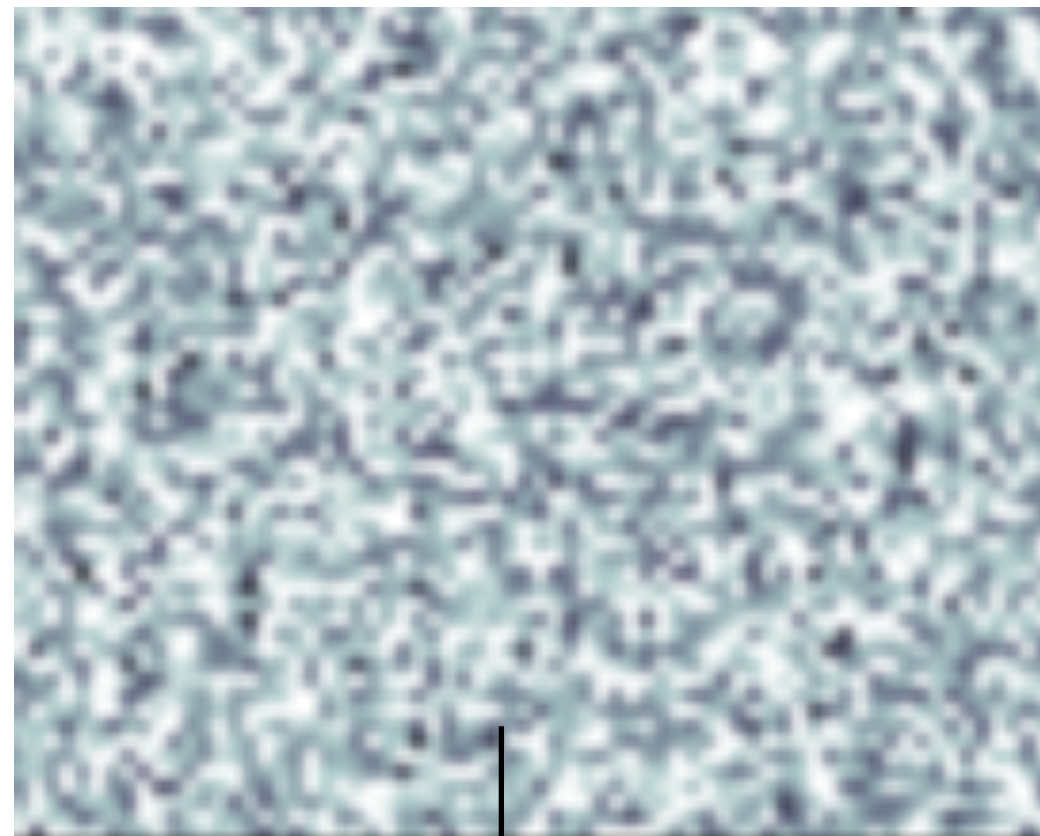
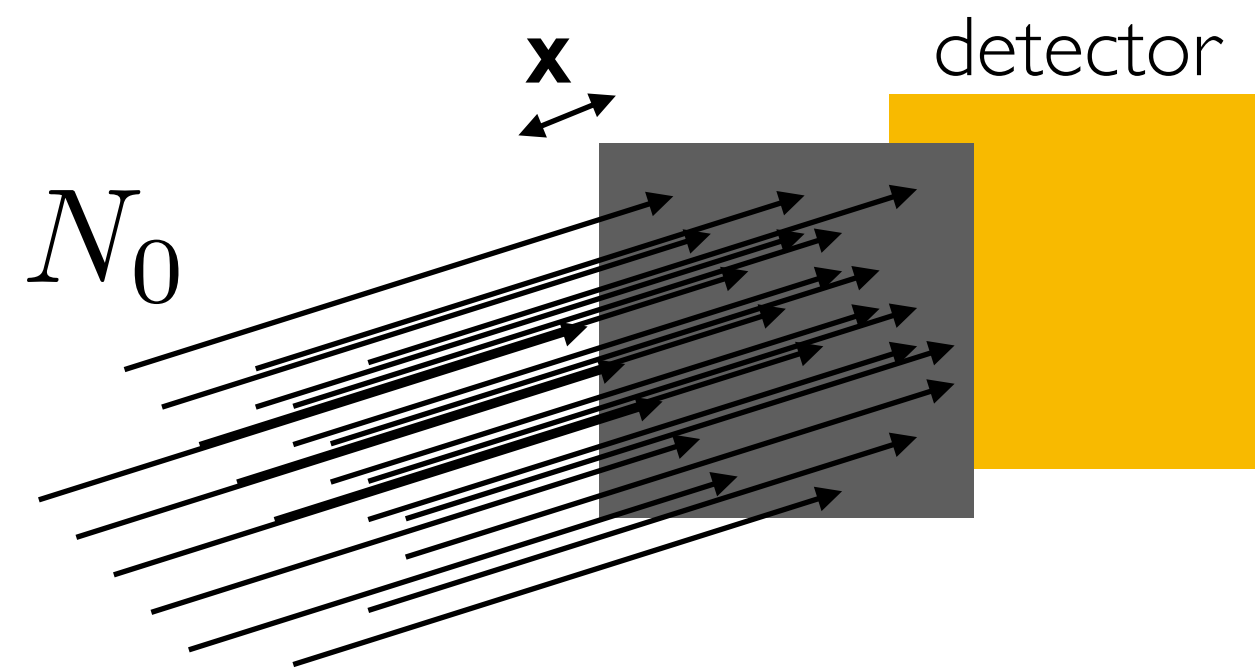
$$N = N_0 e^{-\mu x}$$



I_{11}	I_{12}	$\dots$	$\dots$	I_{1N_x}
I_{21}	I_{22}	$\dots$	$\dots$	I_{2N_x}

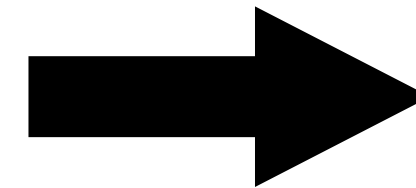
I_{N_y1}	I_{N_y2}	$\dots$	$\dots$	$I_{N_yN_x}$
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An image is an array $N_x \times N_y$ (e.g. $256 \times 256, 512 \times 512, \dots$)



I_{11} I_{12} ... $\dots$ I_{1N_x}
 I_{21} I_{22} ... $\dots$ I_{2N_x}

I_{N_y1} I_{N_y2} ... $\dots$ $I_{N_yN_x}$



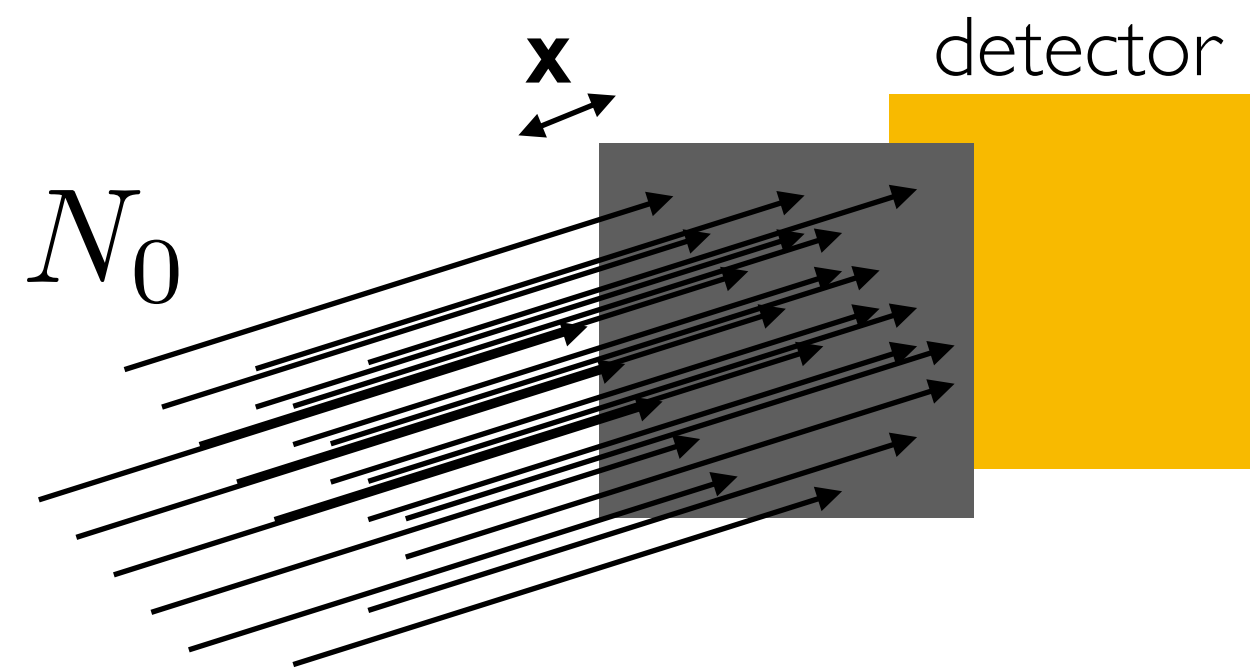
expected number of photons
on each pixel:

$$N = N_0 e^{-\mu x}$$

average value: $\frac{\sum_{ij} I_{ij}}{N_x N_y} \approx N$

variance: $\frac{\sum_{ij} (I_{ij} - N)^2}{N_x N_y} \approx N$

An image is an array $N_x \times N_y$ (e.g. $256 \times 256, 512 \times 512, \dots$)

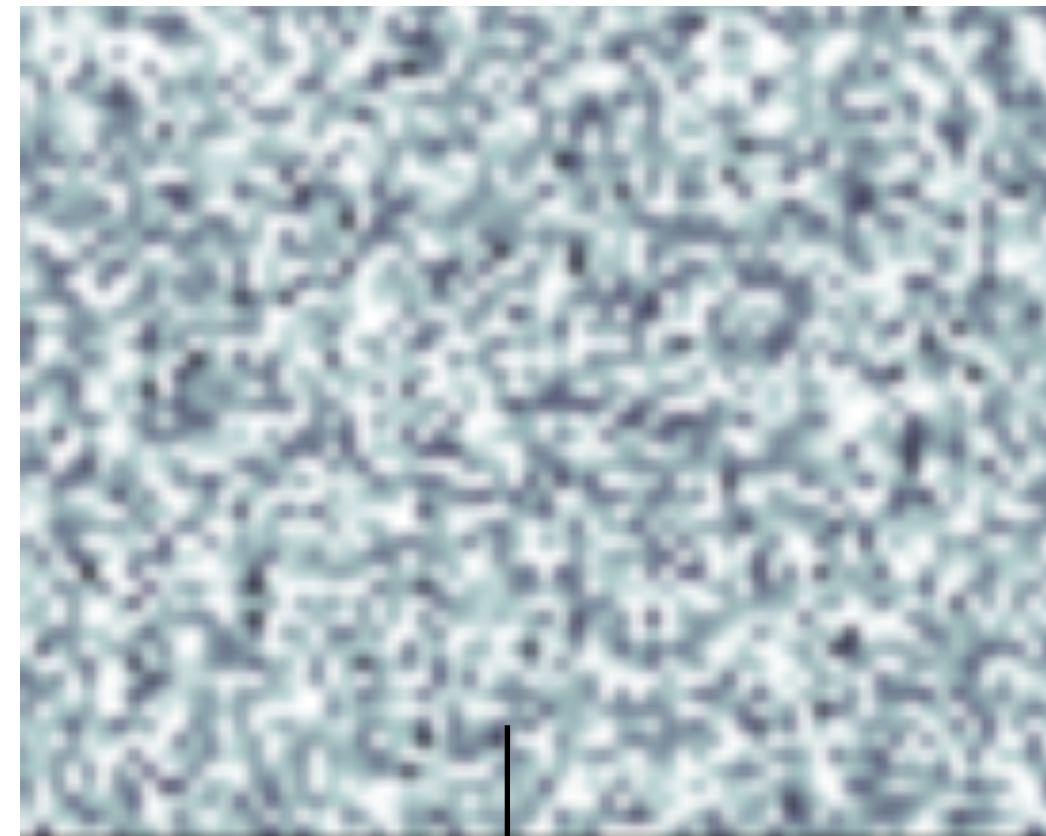


expected number of photons
on each pixel: $N = N_0 e^{-\mu x}$

average value: $\frac{\sum_{ij} I_{ij}}{N_x N_y} \approx N$

Poisson!

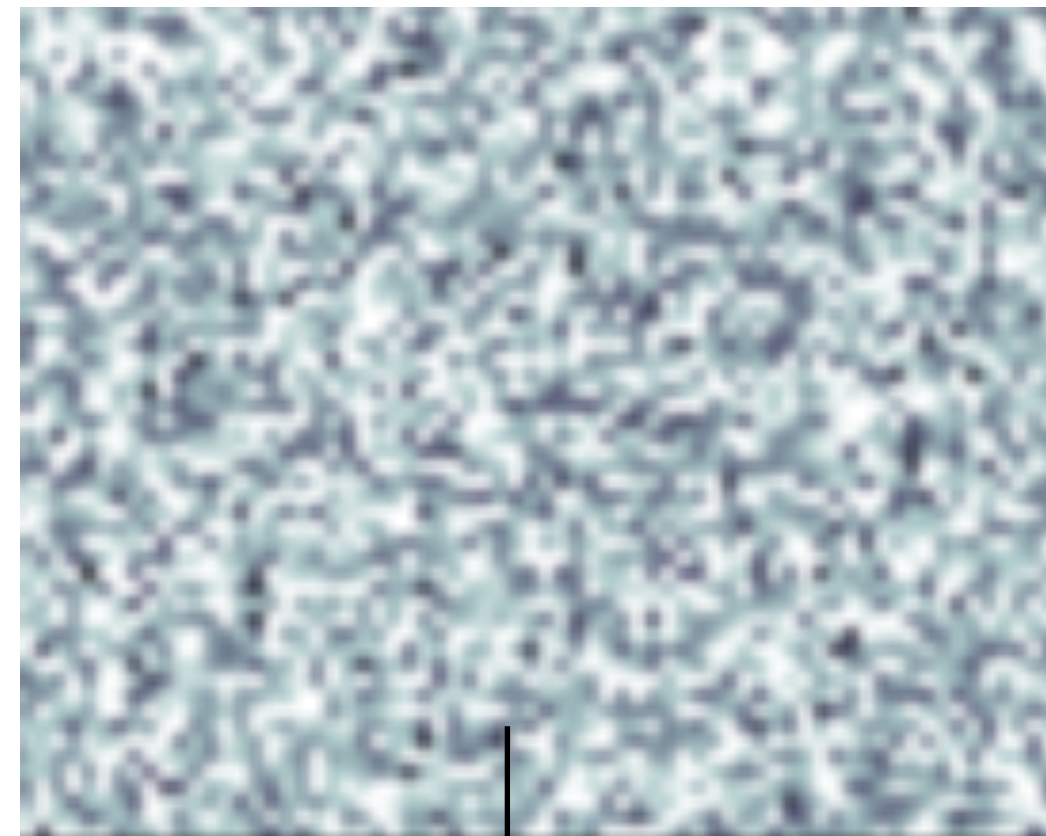
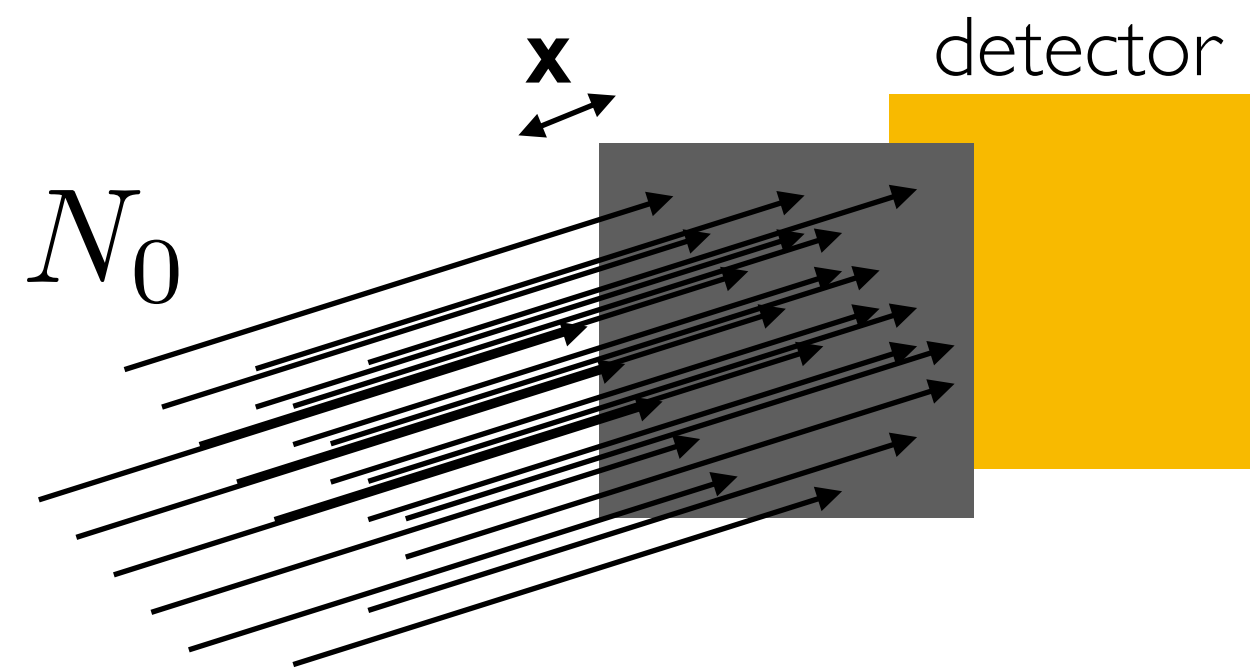
variance: $\frac{\sum_{ij} (I_{ij} - N)^2}{N_x N_y} \approx N$



$I_{11} \quad I_{12} \quad \dots \quad \dots \quad I_{1N_x}$
 $I_{21} \quad I_{22} \quad \dots \quad \dots \quad I_{2N_x}$

$I_{N_y 1} \quad I_{N_y 2} \quad \dots \quad \dots \quad I_{N_y N_x}$

An image is an array $N_x \times N_y$ (e.g. $256 \times 256, 512 \times 512, \dots$)



I_{11} I_{12} ... $\dots$ I_{1N_x}
 I_{21} I_{22} ... $\dots$ I_{2N_x}

I_{N_y1} I_{N_y2} ... $\dots$ $I_{N_yN_x}$

expected number of photons
on each pixel:

$$N = N_0 e^{-\mu x}$$

average value: $\frac{\sum_{ij} I_{ij}}{N_x N_y} \approx N$

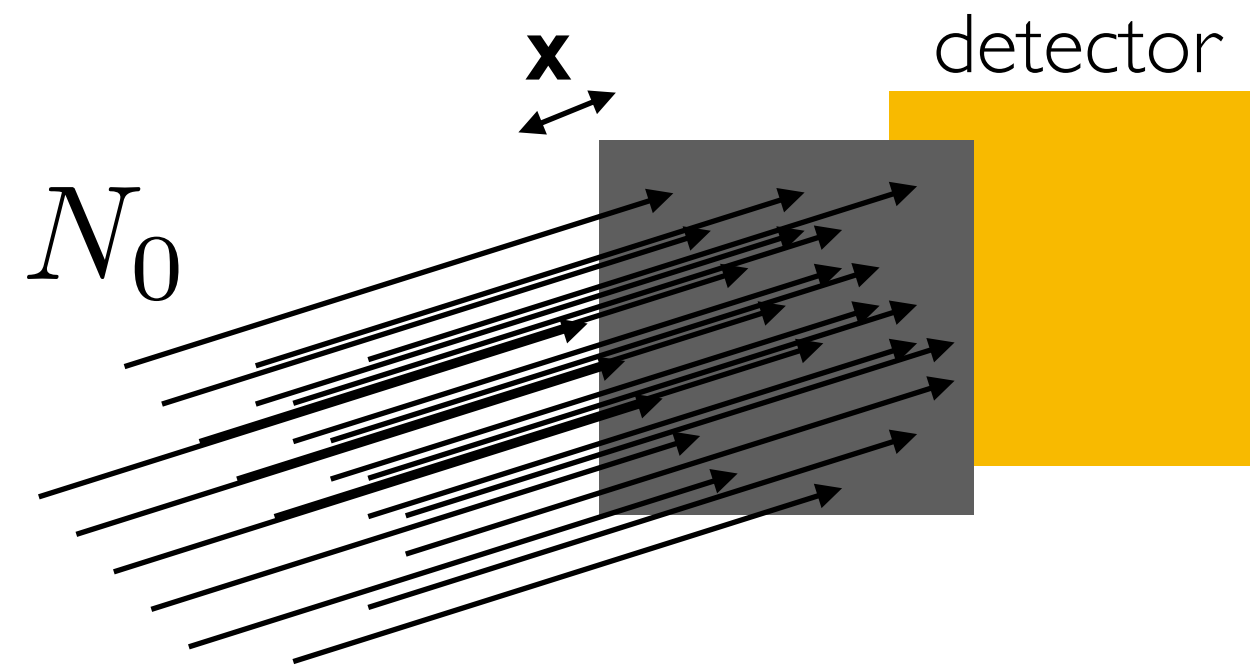
variance: $\frac{\sum_{ij} (I_{ij} - N)^2}{N_x N_y} \approx N$

Poisson!



 pixel value

An image is an array $N_x \times N_y$ (e.g. $256 \times 256, 512 \times 512, \dots$)

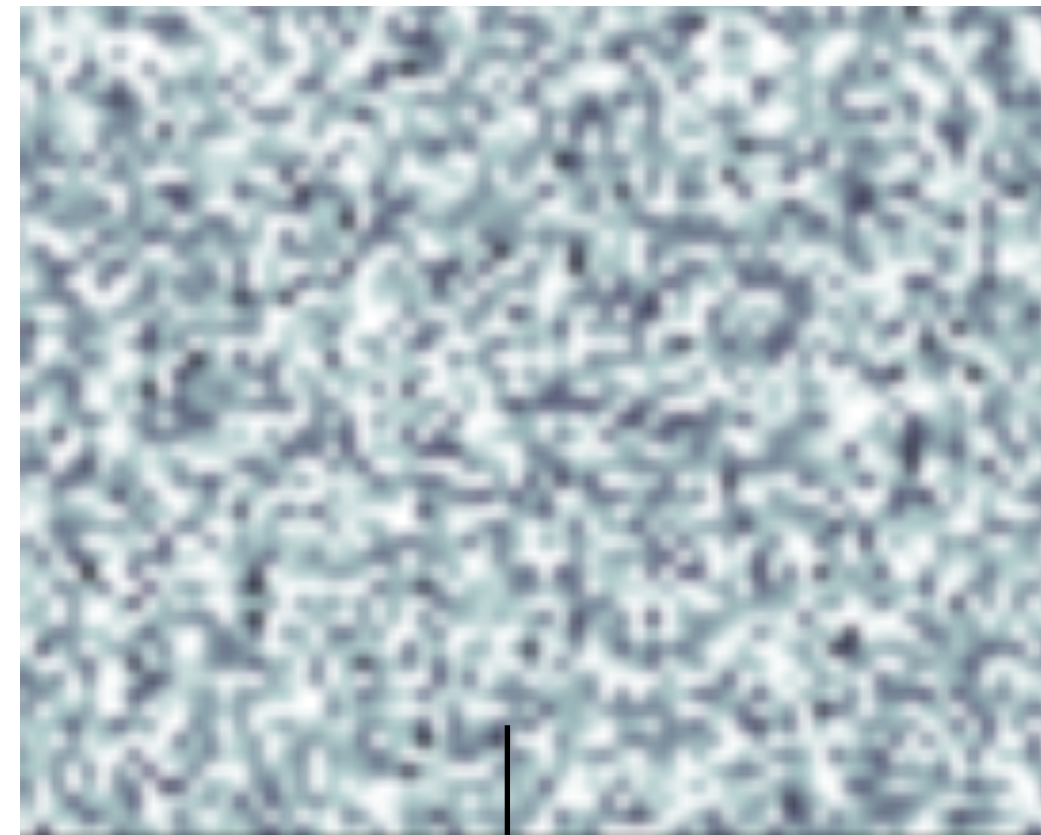


expected number of photons on each pixel: $N = N_0 e^{-\mu x}$

average value: $\frac{\sum_{ij} I_{ij}}{N_x N_y} \approx N$

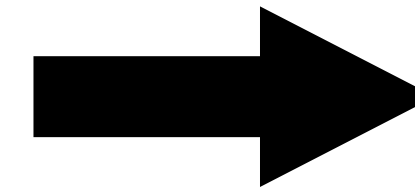
Poisson!

variance: $\frac{\sum_{ij} (I_{ij} - N)^2}{N_x N_y} \approx N$

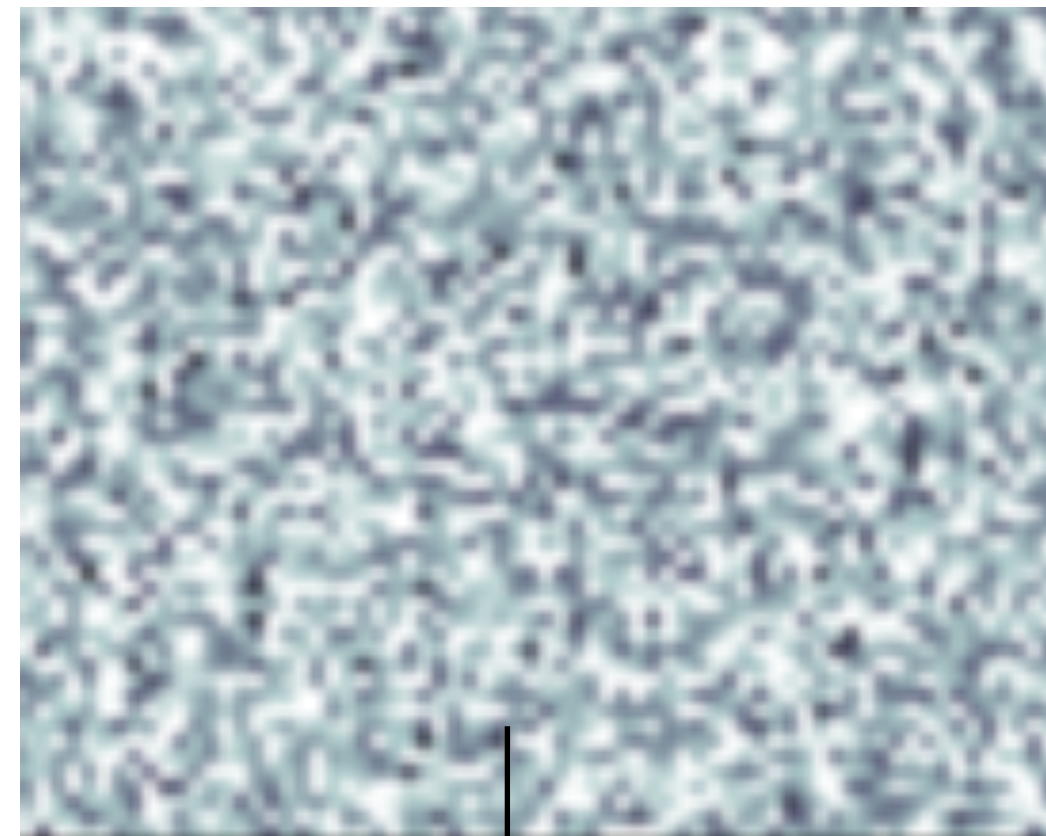
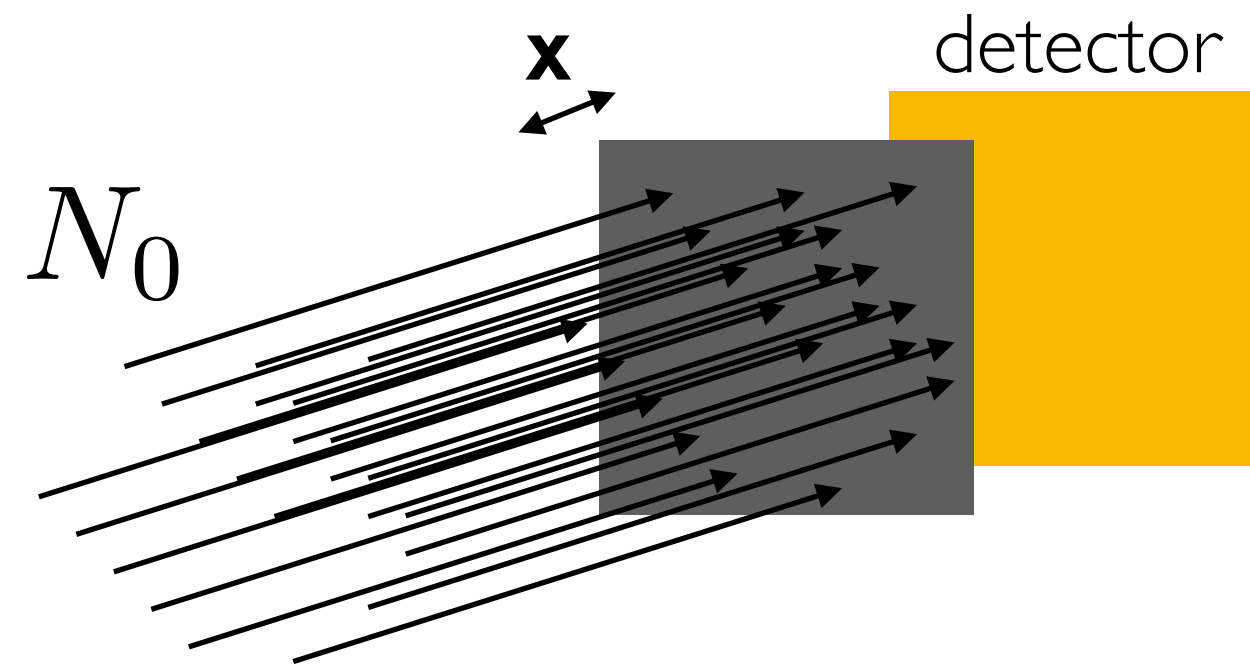


$I_{11} \quad I_{12} \quad \dots \quad \dots \quad I_{1N_x}$
 $I_{21} \quad I_{22} \quad \dots \quad \dots \quad I_{2N_x}$

$I_{N_y 1} \quad I_{N_y 2} \quad \dots \quad \dots \quad I_{N_y N_x}$

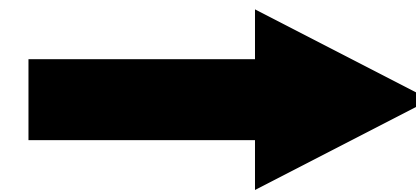


An image is an array $N_x \times N_y$ (e.g. $256 \times 256, 512 \times 512, \dots$)



$I_{11} \quad I_{12} \quad \dots \quad \dots \quad I_{1N_x}$
 $I_{21} \quad I_{22} \quad \dots \quad \dots \quad I_{2N_x}$

$I_{N_y1} \quad I_{N_y2} \quad \dots \quad \dots \quad I_{N_yN_x}$



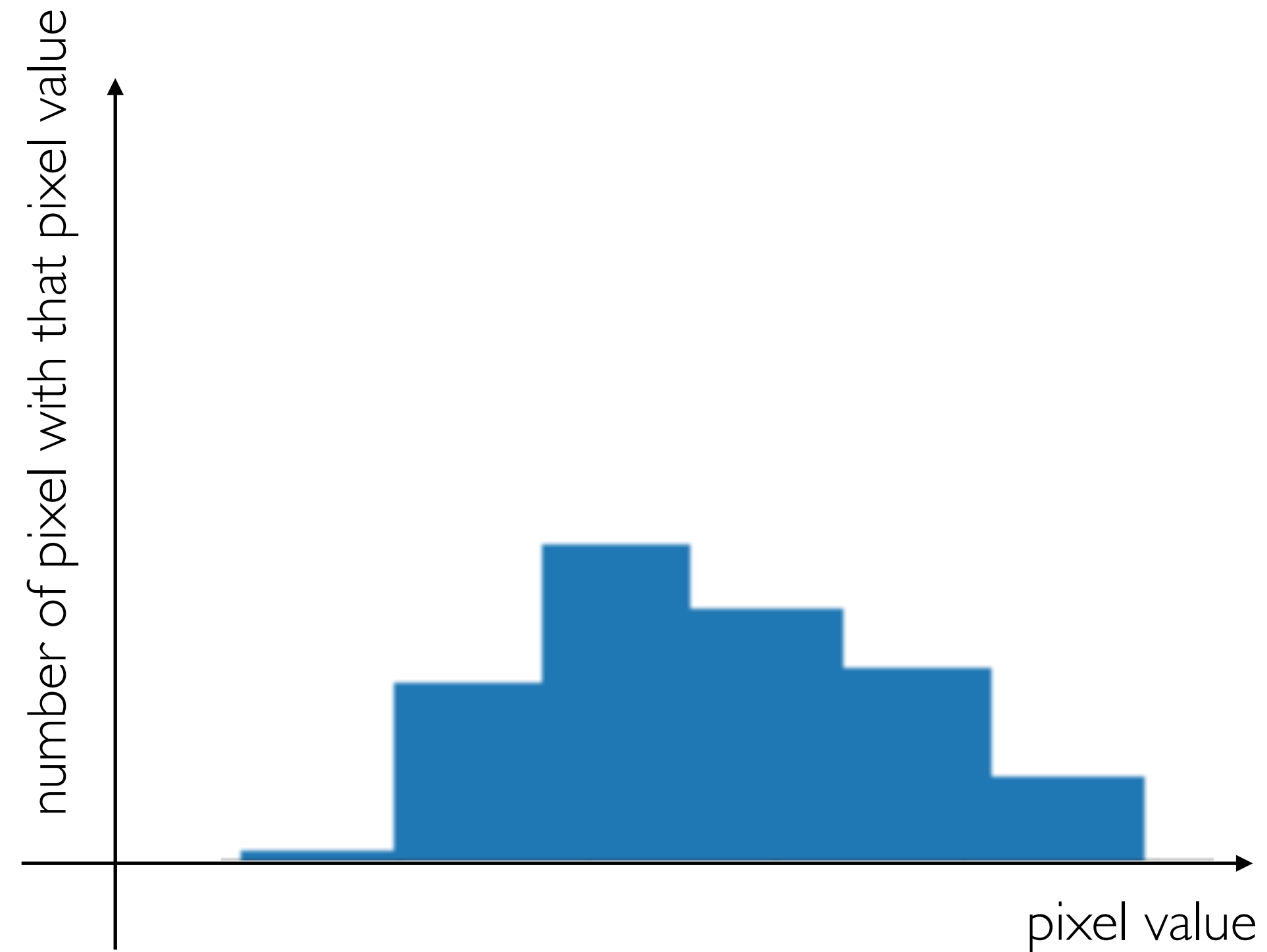
expected number of photons
on each pixel:

$$N = N_0 e^{-\mu x}$$

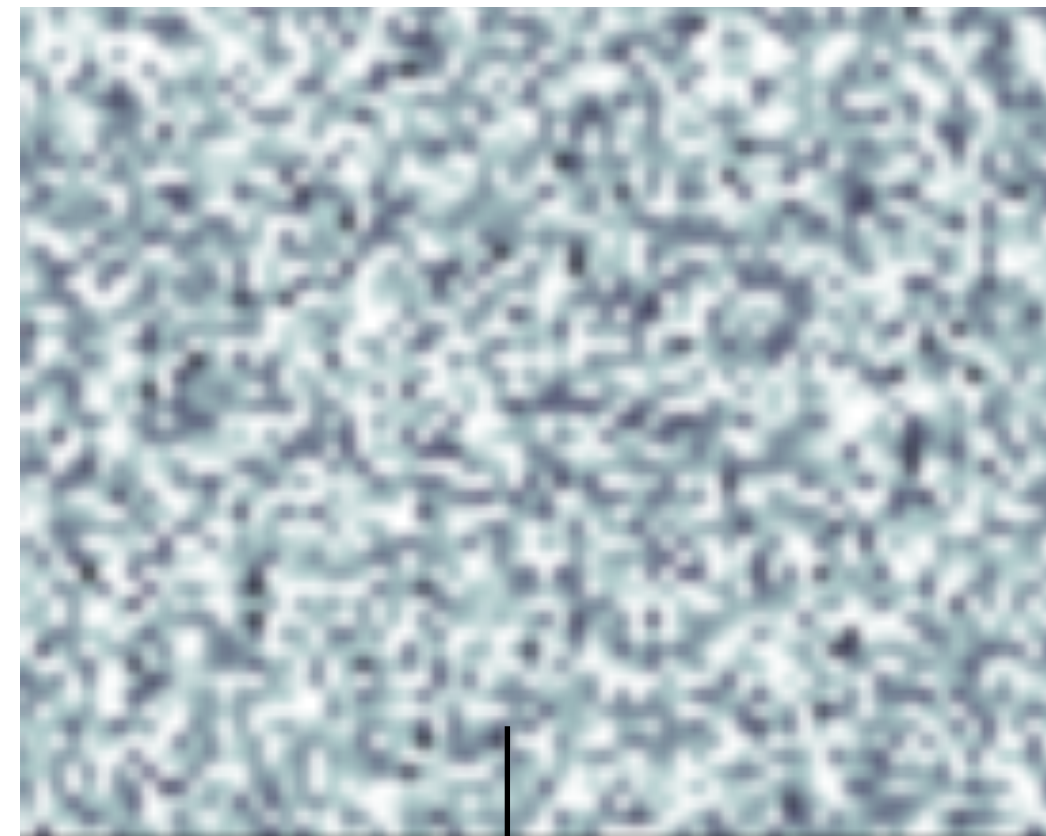
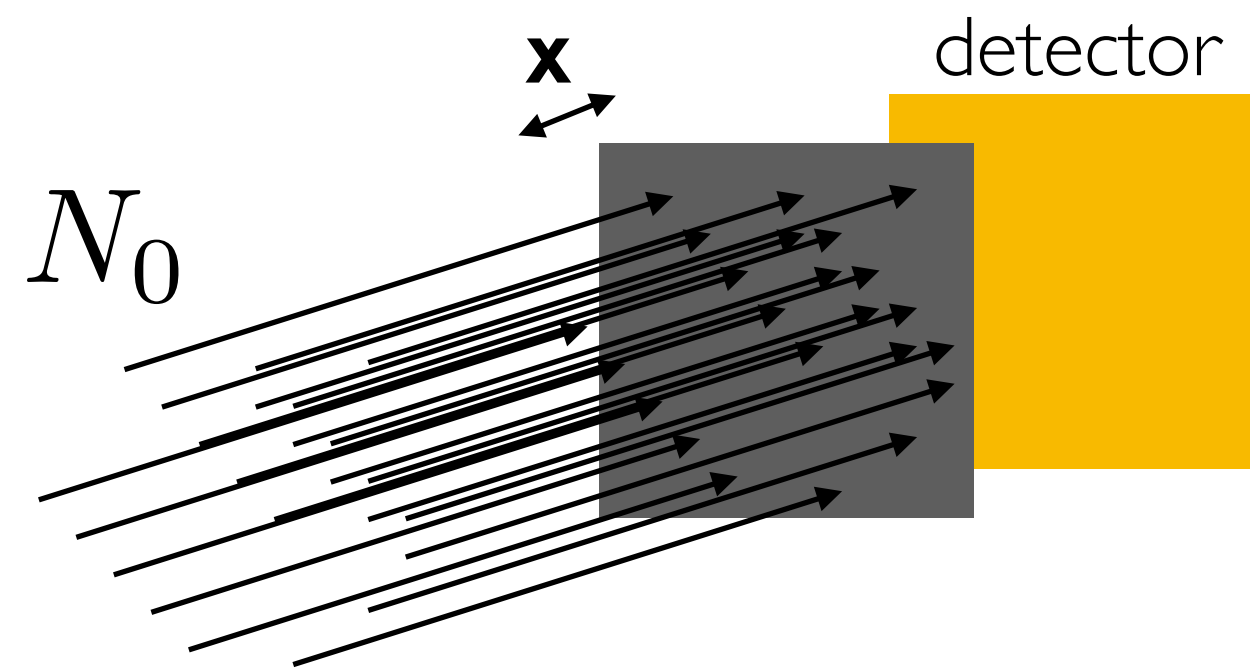
average value: $\frac{\sum_{ij} I_{ij}}{N_x N_y} \approx N$

Poisson!

variance: $\frac{\sum_{ij} (I_{ij} - N)^2}{N_x N_y} \approx N$



An image is an array $N_x \times N_y$ (e.g. $256 \times 256, 512 \times 512, \dots$)



$I_{11} \quad I_{12} \quad \dots \quad \dots \quad I_{1N_x}$
 $I_{21} \quad I_{22} \quad \dots \quad \dots \quad I_{2N_x}$

$I_{N_y1} \quad I_{N_y2} \quad \dots \quad \dots \quad I_{N_yN_x}$

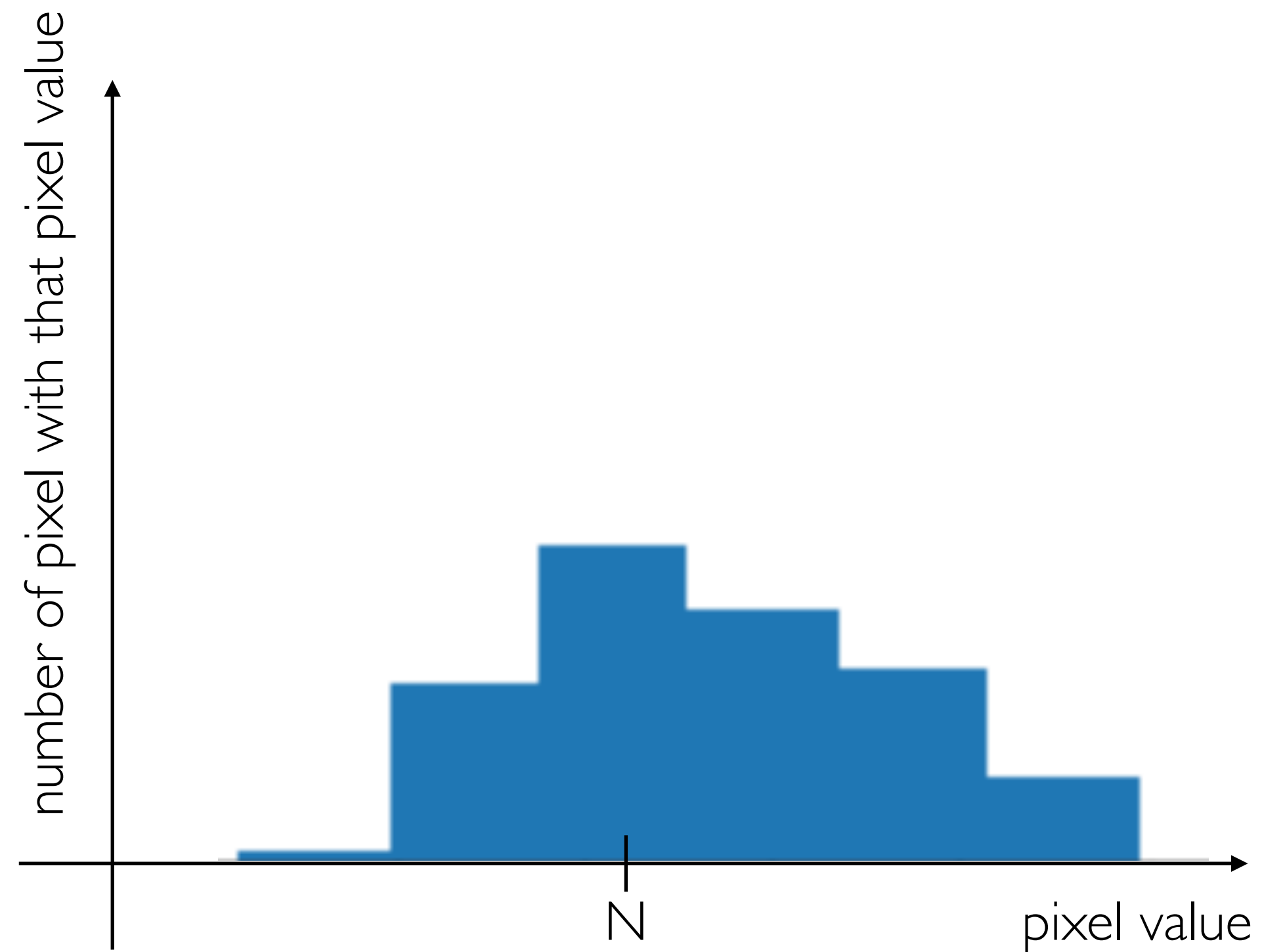
expected number of photons
on each pixel:

$$N = N_0 e^{-\mu x}$$

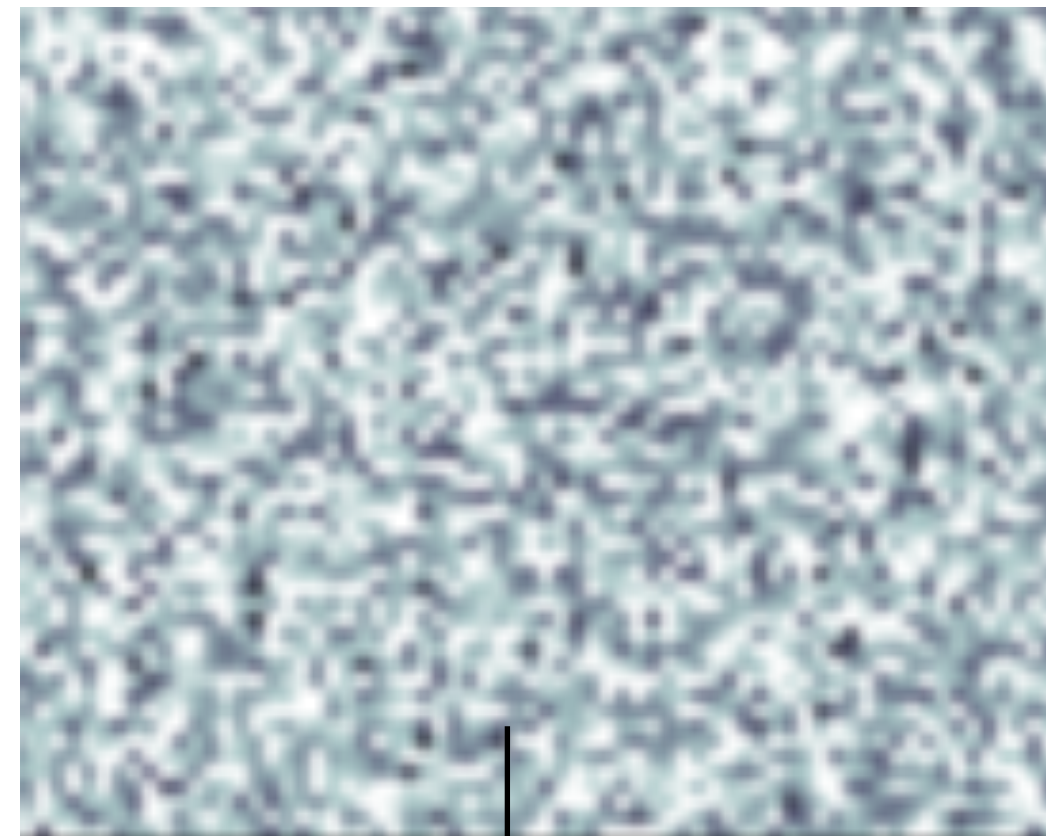
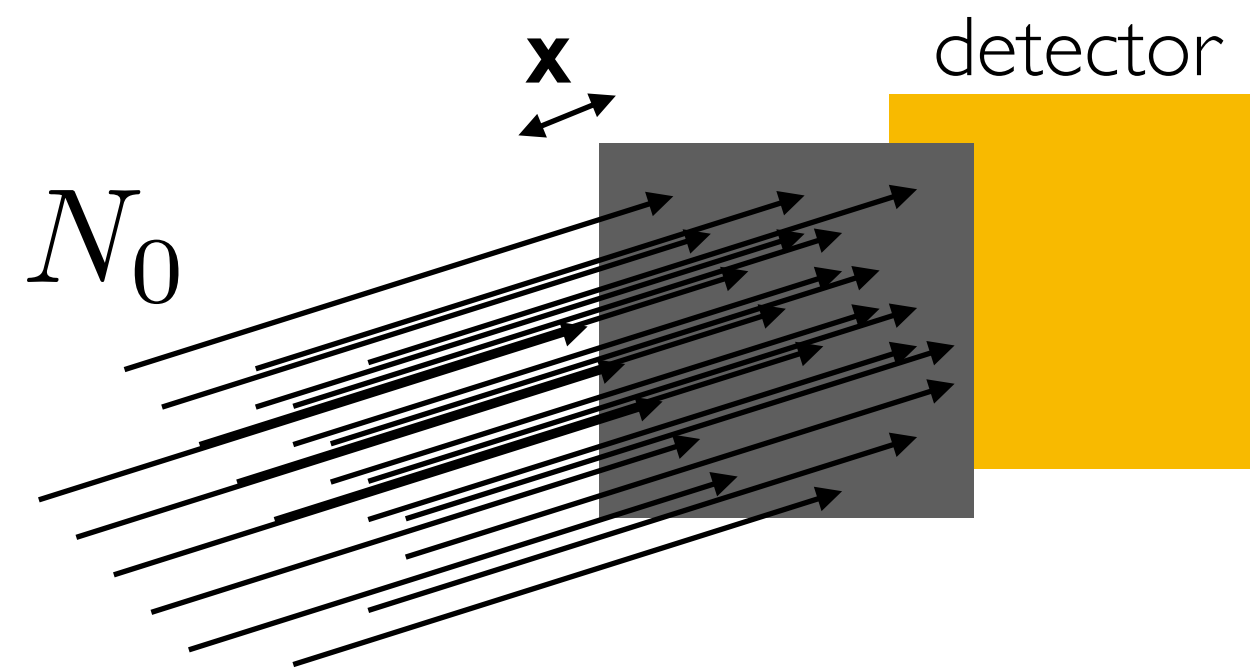
average value: $\frac{\sum_{ij} I_{ij}}{N_x N_y} \approx N$

variance: $\frac{\sum_{ij} (I_{ij} - N)^2}{N_x N_y} \approx N$

Poisson!



An image is an array $N_x \times N_y$ (e.g. $256 \times 512 \times 512, \dots$)



$I_{11} \quad I_{12} \quad \dots \quad \dots \quad I_{1N_x}$
 $I_{21} \quad I_{22} \quad \dots \quad \dots \quad I_{2N_x}$

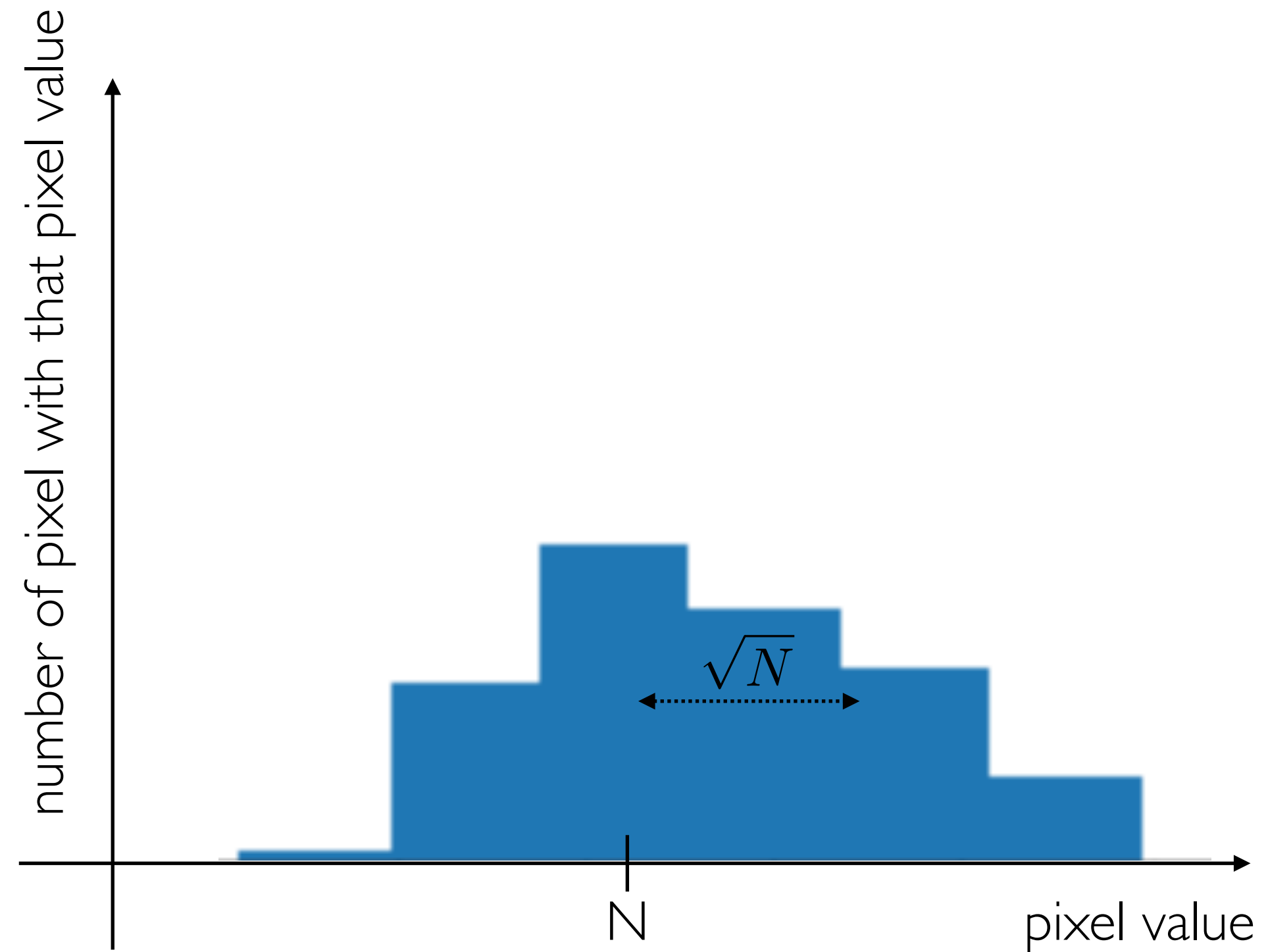
$I_{N_y1} \quad I_{N_y2} \quad \dots \quad \dots \quad I_{N_yN_x}$

expected number of photons
on each pixel: $N = N_0 e^{-\mu x}$

average value: $\frac{\sum_{ij} I_{ij}}{N_x N_y} \approx N$

variance: $\frac{\sum_{ij} (I_{ij} - N)^2}{N_x N_y} \approx N$

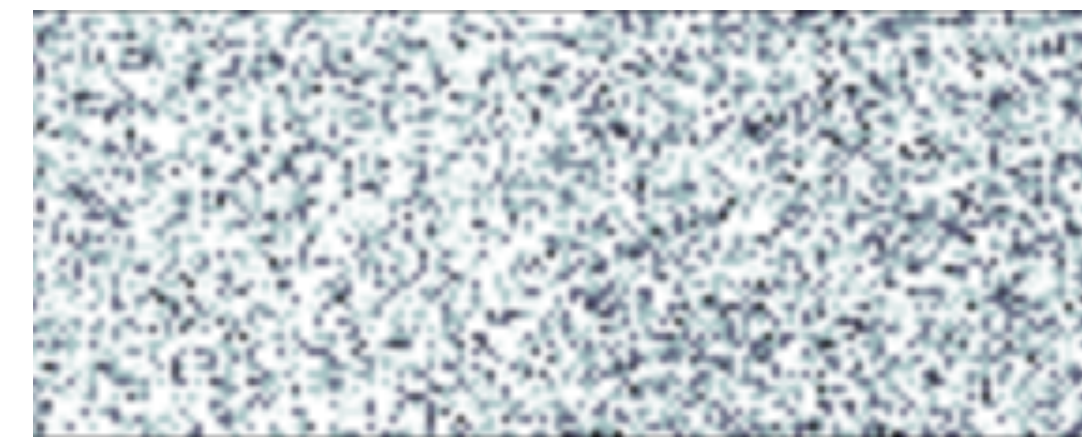
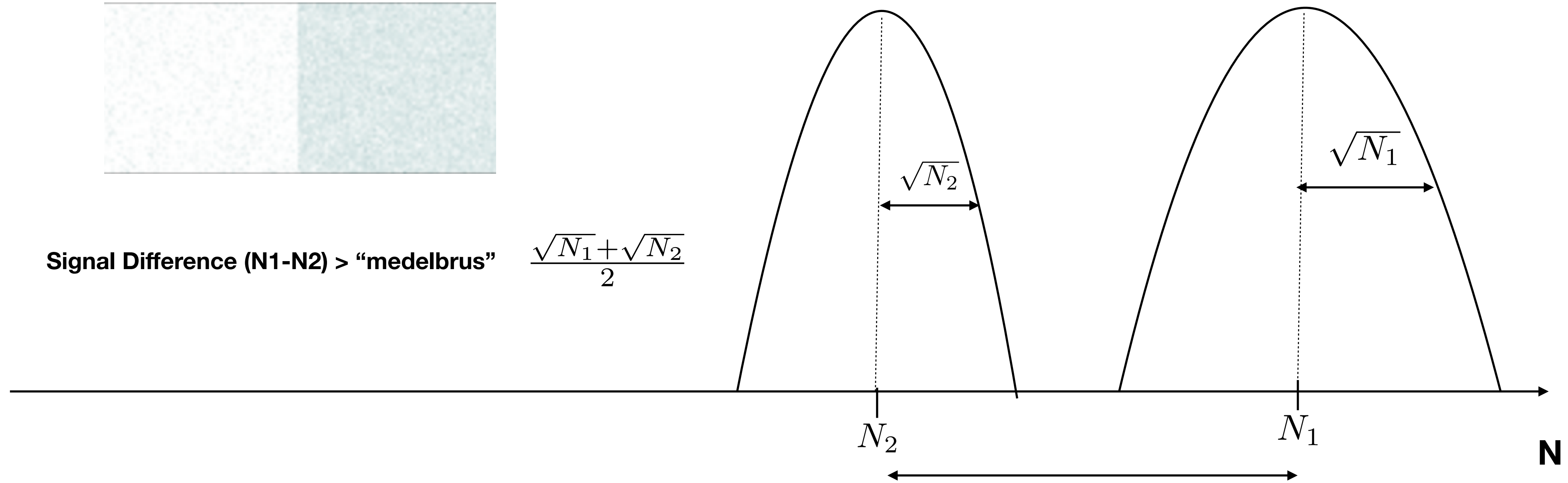
Poisson!



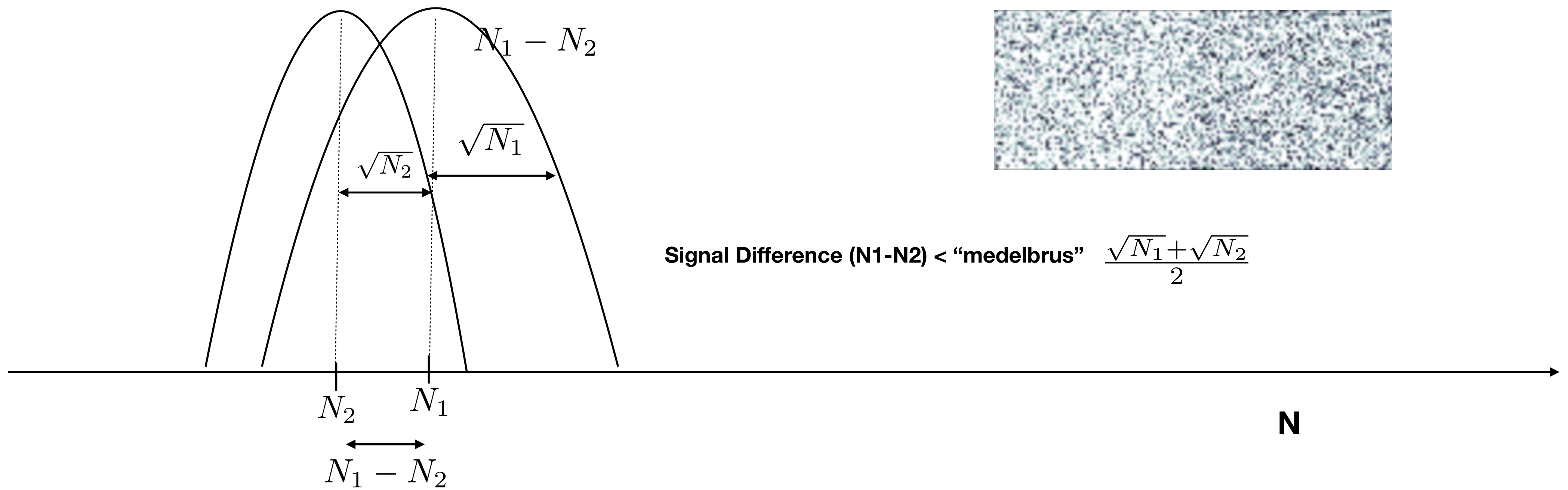


Signal Difference (N1-N2) > “medelbrus”

$$\frac{\sqrt{N_1} + \sqrt{N_2}}{2}$$



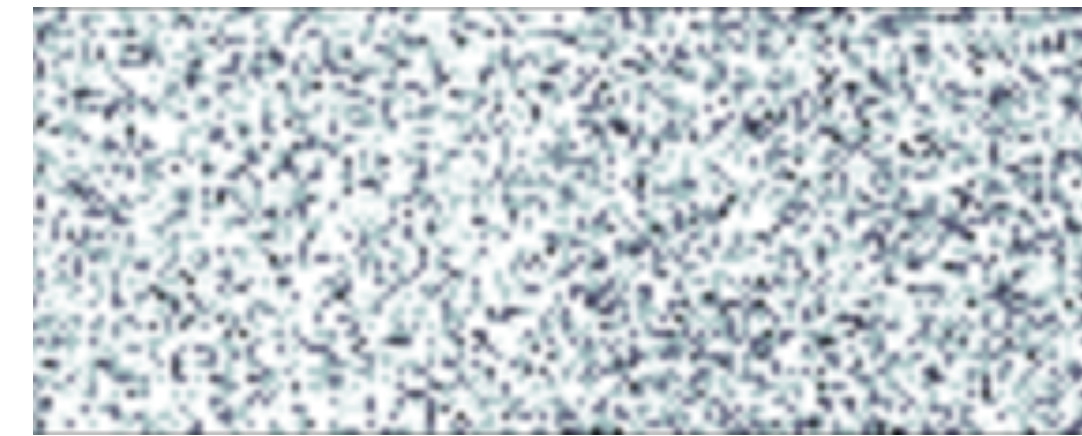
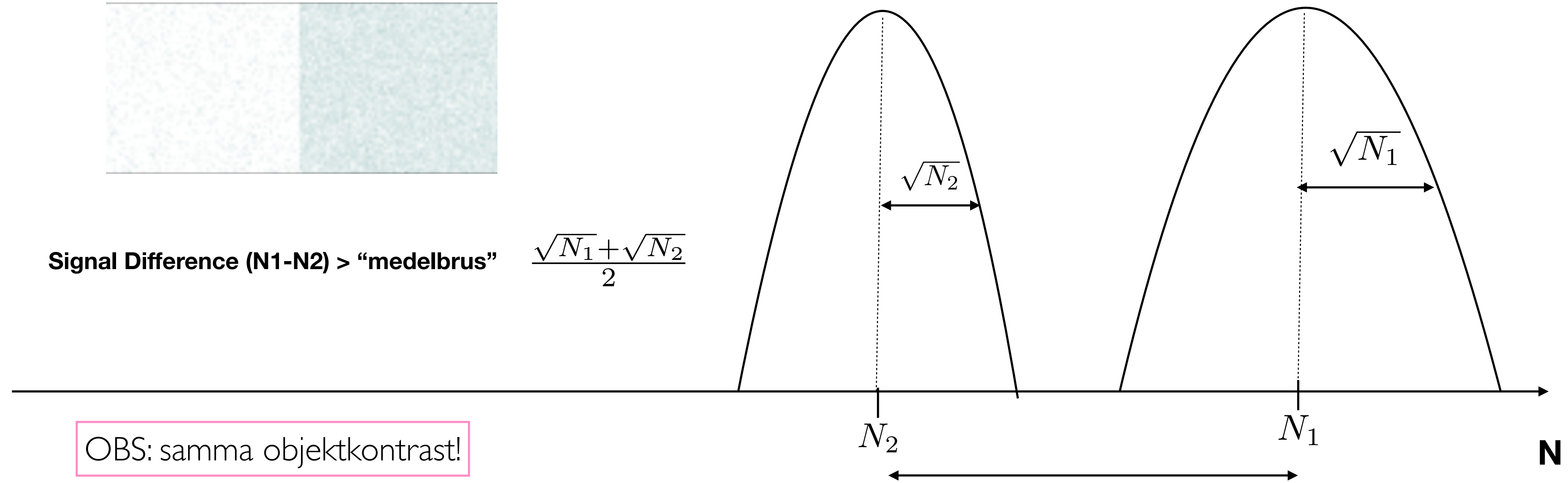
Signal Difference (N1-N2) < “medelbrus” $\frac{\sqrt{N_1} + \sqrt{N_2}}{2}$



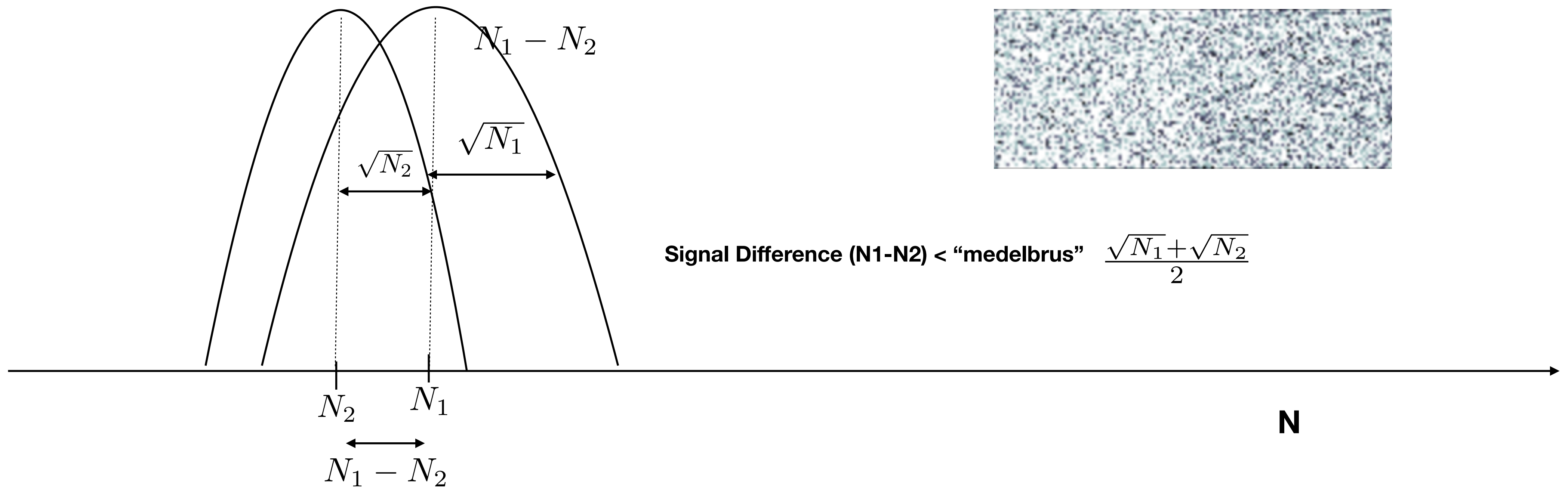


Signal Difference (N1-N2) > “medelbrus”

$$\frac{\sqrt{N_1} + \sqrt{N_2}}{2}$$



Signal Difference (N1-N2) < “medelbrus” $\frac{\sqrt{N_1} + \sqrt{N_2}}{2}$

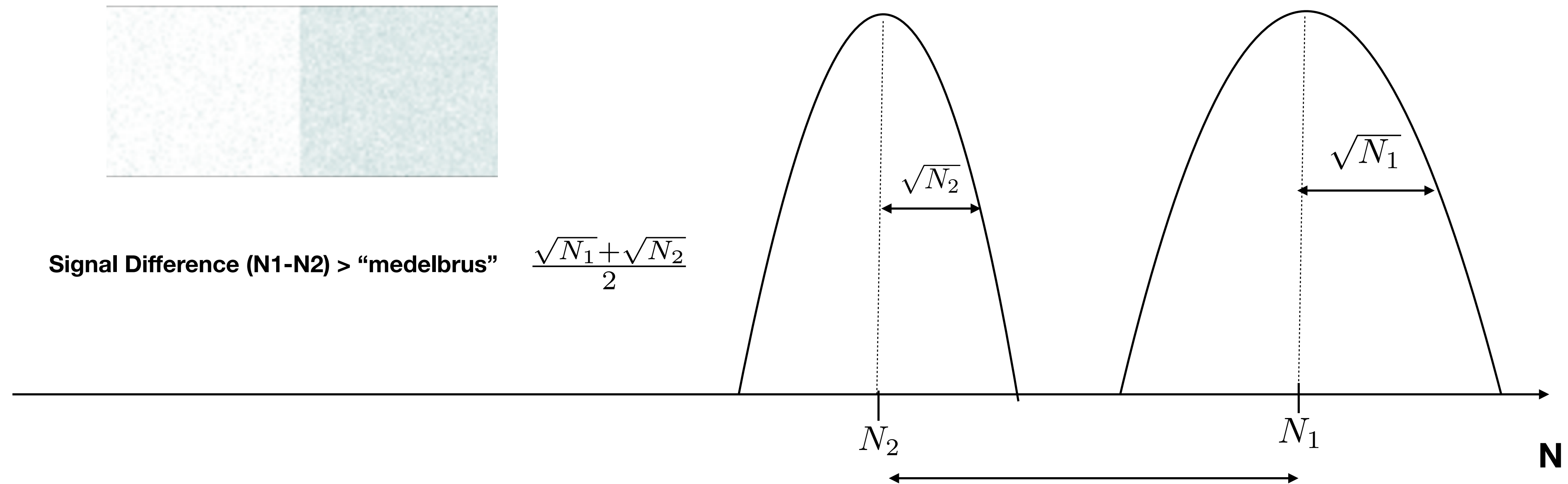


We create a quantity that tells us about the relation between
Signal Difference and Noise:

$$\text{SDNR} = \frac{N_1 - N_2}{\sqrt{N_1} + \sqrt{N_2}}$$

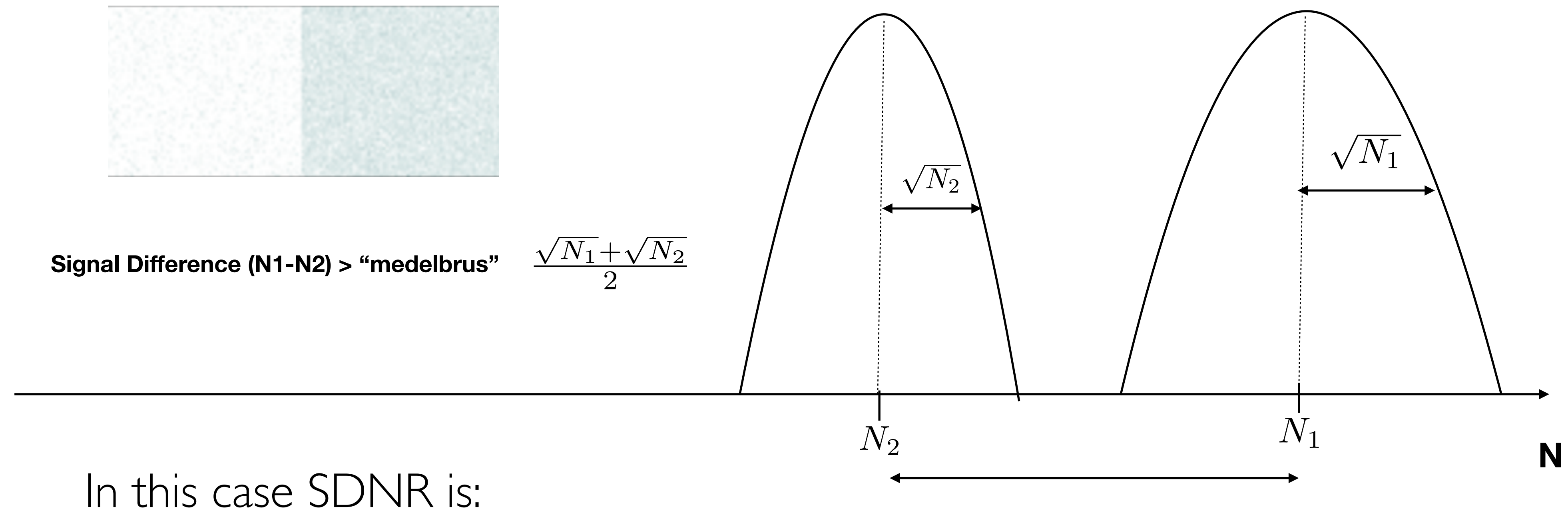
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Signal Difference and Noise:

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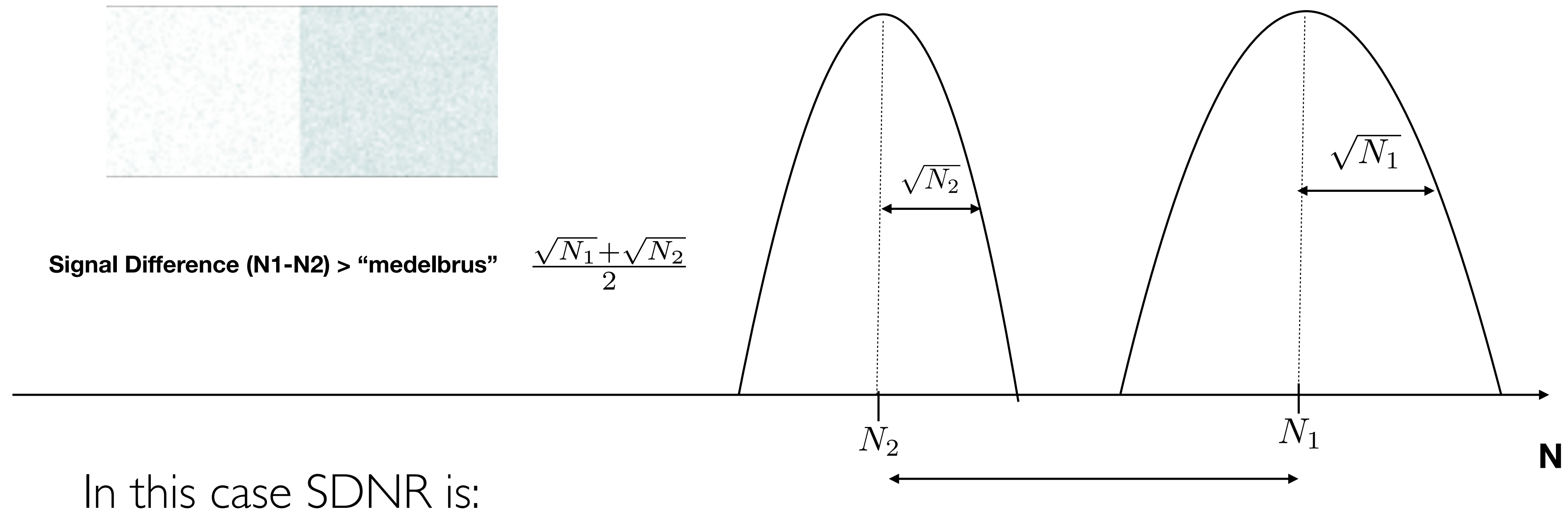


In this case SDNR is:

- | | |
|-------------------------------------|-------------------|
| 1. around 0 | 3. around 1 |
| 2. greater than 0, but lower than 1 | 4. greater than 1 |

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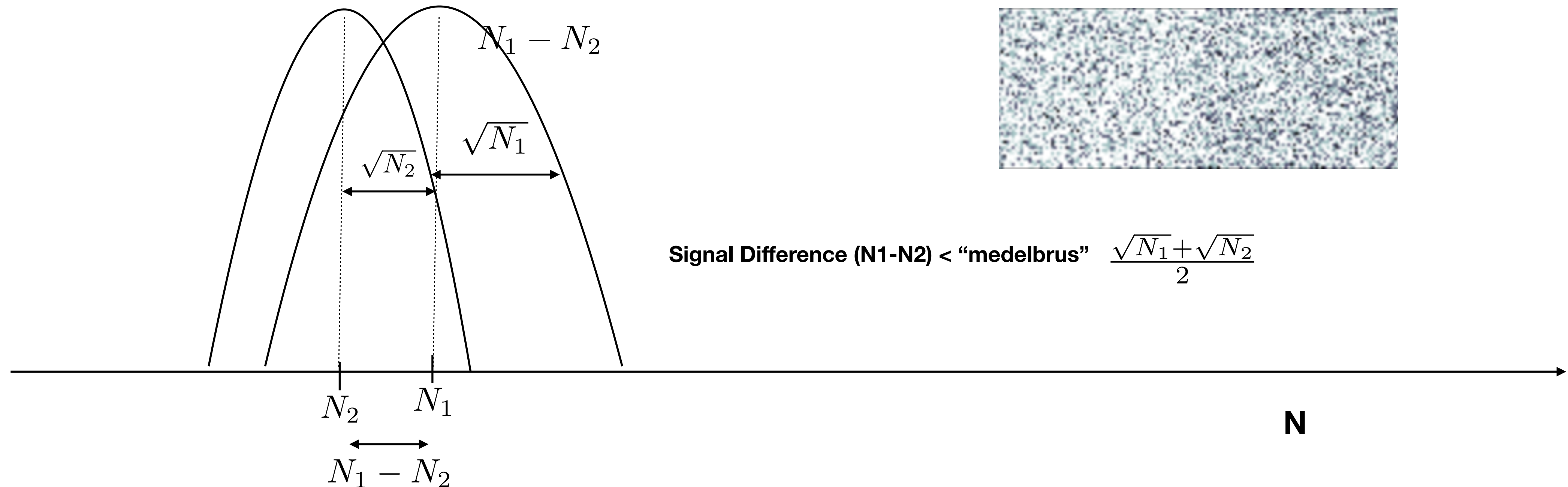


In this case SDNR is:

1. around 0
2. greater than 0, but lower than 1
3. around 1
4. greater than 1

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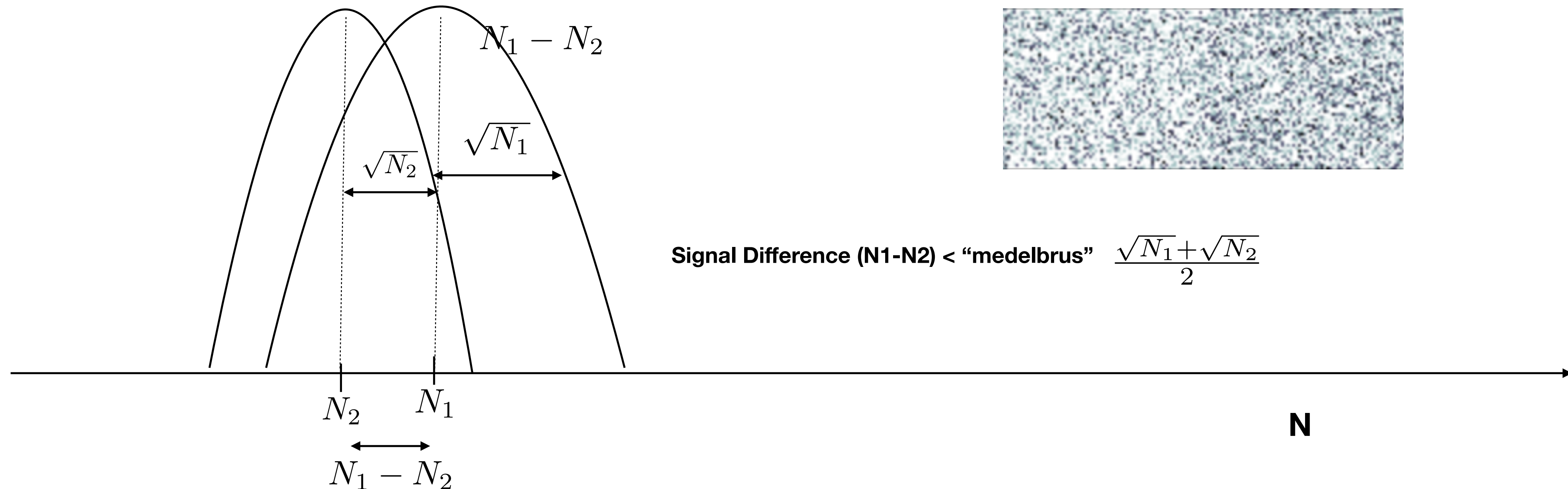


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1. around 0
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3. around 1
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Rose model: An object is visible if $\text{SDNR} \geq 5$

(Semi-empiric model, value 5 from experiment)

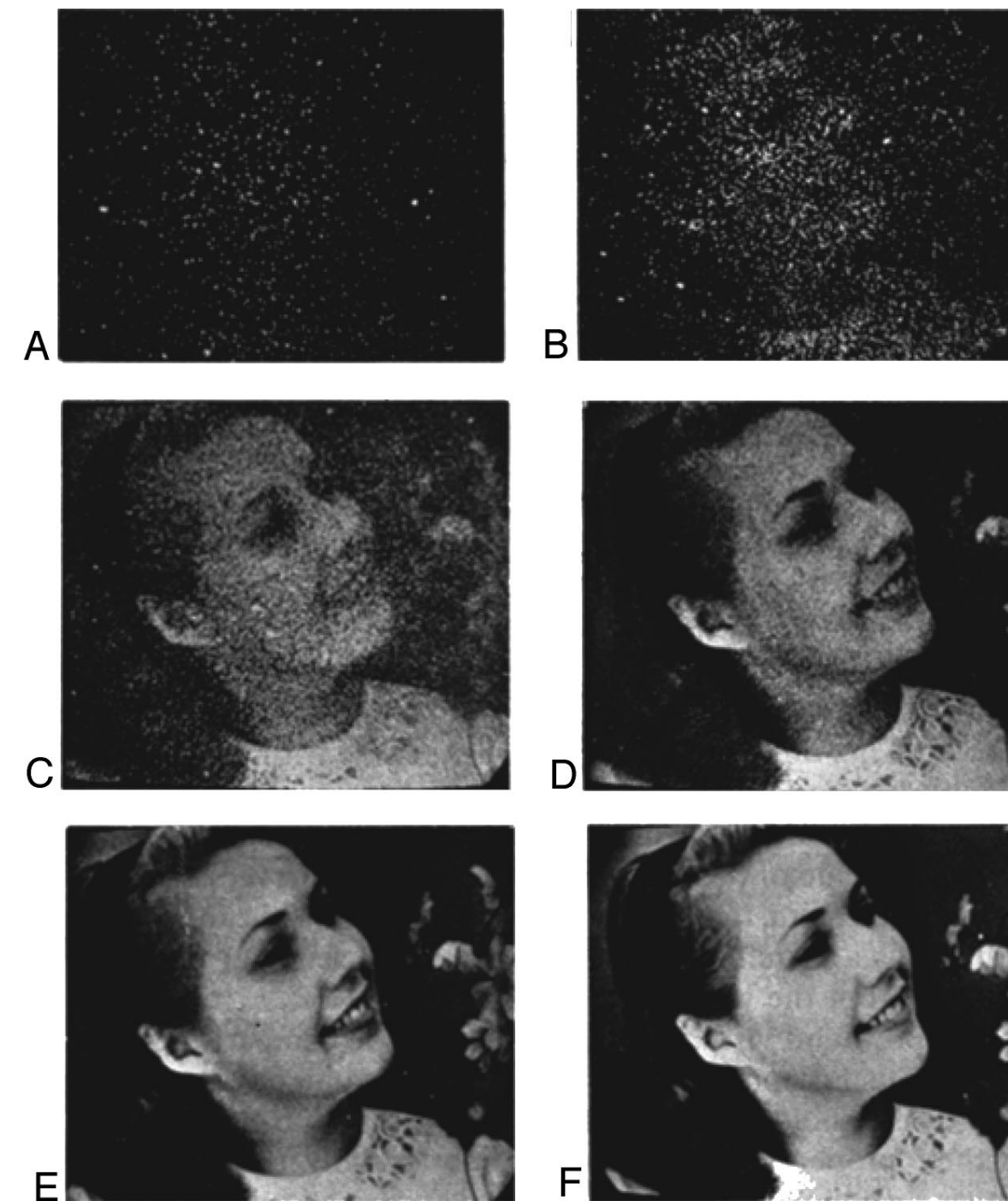


Fig. 1. Picture used by Rose,⁴ of woman with flowers, to demonstrate the maximum amount of information that can be represented with varying numbers of photons. A, 3×10^3 ; B, 1.2×10^4 ; C, 9.3×10^5 ; D, 7.6×10^5 ; E, 3.6×10^5 ; F, 2.8×10^7 . Each photon is represented as a discrete visible speck. The inherent statistical fluctuations in photon density limit one's ability to detect or identify features in the original scene.

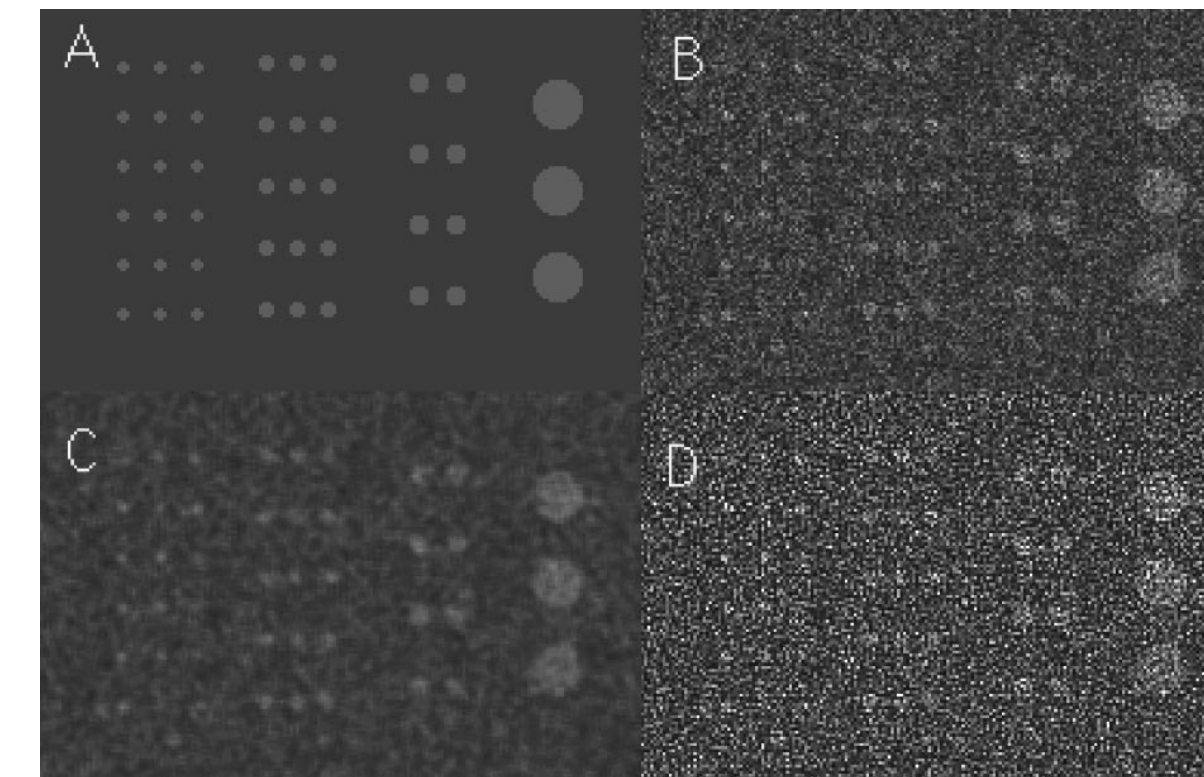


Fig. 4. Demonstration that pixel SNR (SNR_p) is not a good primary measure of image quality. SNR_p is defined as the ratio of peak signal amplitude and noise standard deviation, σ_p , per pixel. This figure also shows that signal appearance is highly variable at low SNR, as is expected, given the statistical nature of the images. A, Signal array of disks (diameters of 4, 5, 6, 16 pixels) with amplitude of 24 gray levels. B, Same signal array with added zero-mean white noise (σ_p of 24 gray levels), so $\text{SNR}_p = 1.0$ for all the signals. The Rose model and the ideal observer SNR's are 3.5, 4.6, 5.7, and 14.4. C, After smoothing of portion B. The low-pass-filtered noise has σ_p of 8.2 gray levels, so $\text{SNR}_p = 2.9$ for all the signals. D, After edge enhancement of portion B, resulting in a σ_p value of 39.6 gray levels and a SNR_p value of 0.6. The filtering would have no effect on ideal observer SNR. The Rose model is not a valid method for calculating SNR with filtered images.

SDNR and dose

$$\text{SDNR} = \frac{N_1 - N_2}{\sqrt{N_1} + \sqrt{N_2}} \propto \sqrt{N}$$

$$\text{dose} \propto N$$

For att förbättra SDNR med en faktor 2, allt annat oförändrat, måste man öka dosen med en faktor

1. 1
2. 2
3. 3
4. 4
5. Det går ej att säga

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For att förbättra SDNR med en faktor 2, allt annat oförändrat, måste man öka dosen med en faktor 4

1. 1

2. 2

3. 3

4. 4

5. Det går ej att säga

Övningar:
att göras innan nästa
föreläsning!

Övning

Antag att fotoner med energin 100 keV ska gå genom en filter gjord av 5 mm Al.

1. Rita en figur av detta.
2. Beteckna de nödvändiga storheterna
3. Bestäm linear attenuation coefficient för 5 mm Al (tag hjälp av NIST tabeller)
4. Bestäm attenueringsfaktor i detta fall
5. Bestäm andel primary photons genom Al-filter
6. Bestäm andel attenuated photons i Al-filter

Fråga Mamo om du behöver hjälp!

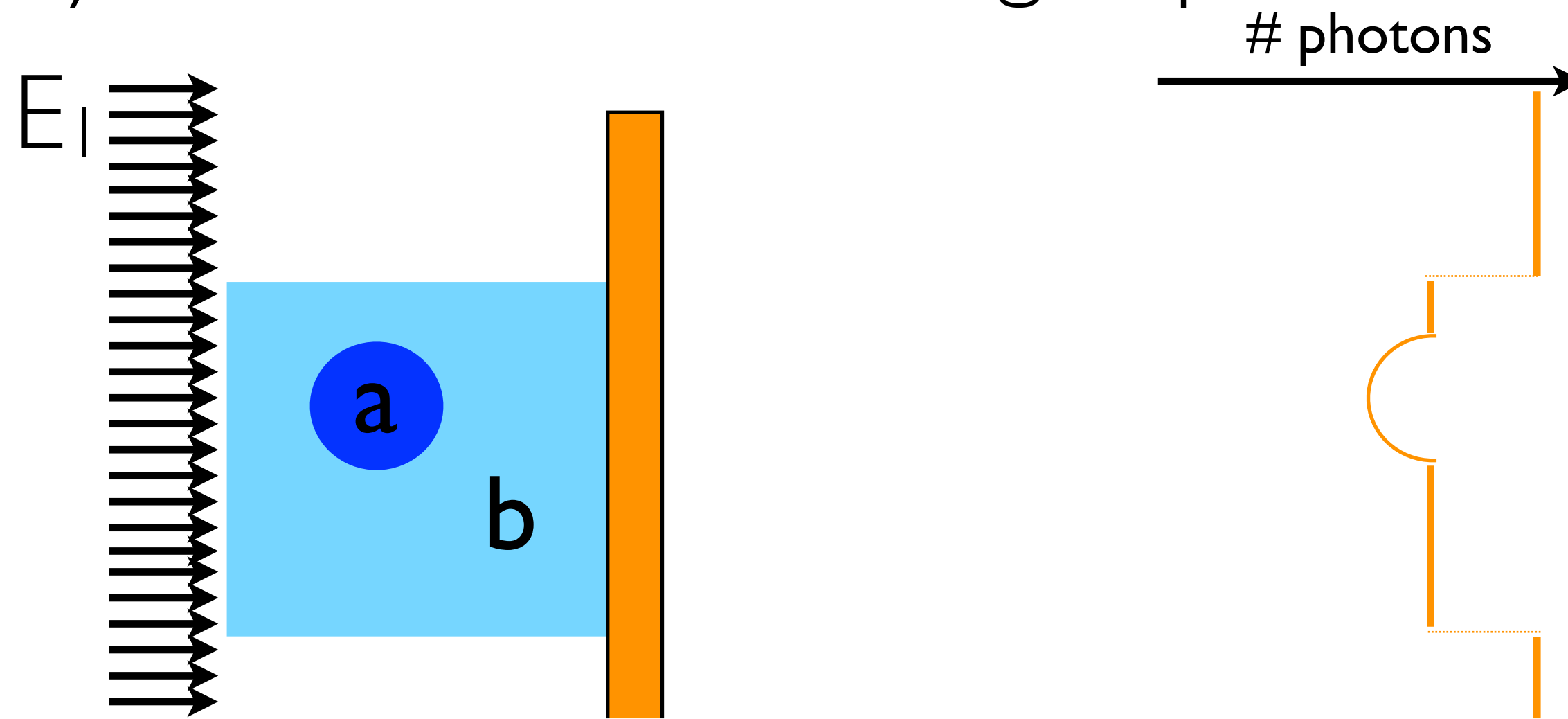
Q: the linear attenuation coefficient of lead for photons of energy of 500 keV is:

- 1. 0,1614
- 2. 0,1614 cm²/g
- 3. 1,1832
- 4. 1,1832 cm⁻¹

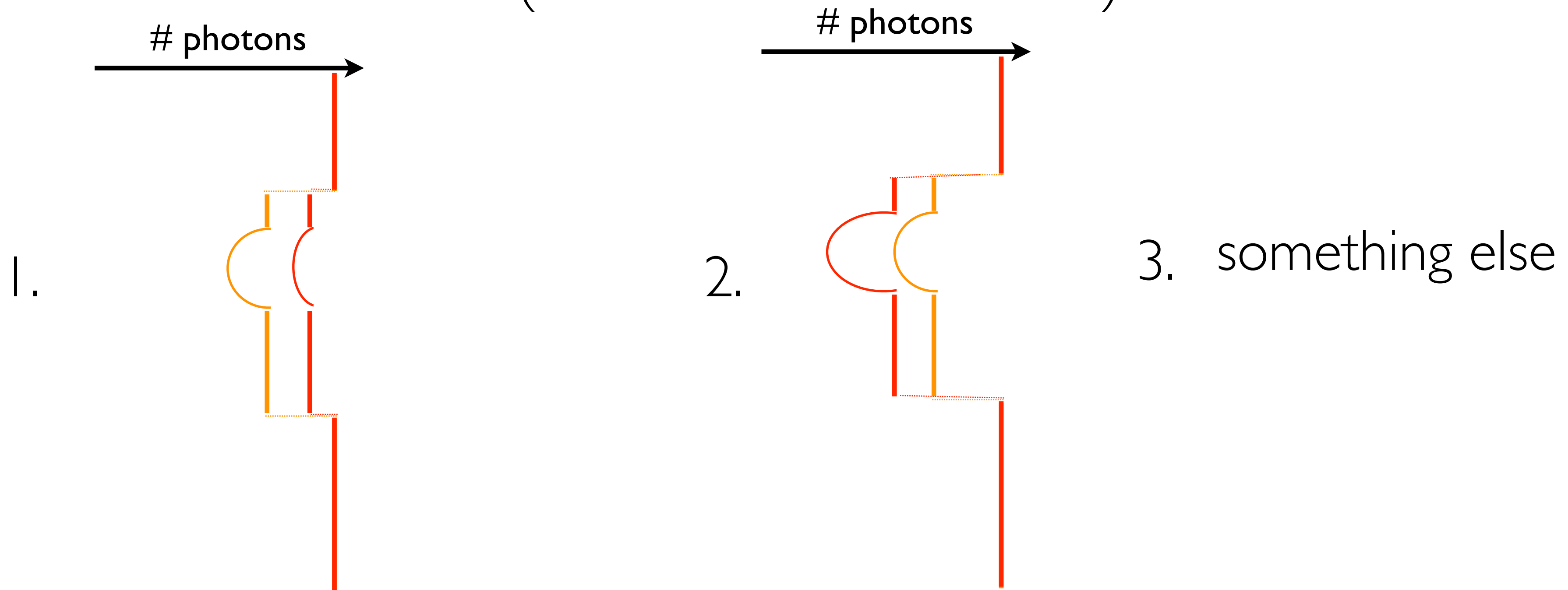
Q: what is the attenuation factor of 1 mm of lead for 500 keV photons?

Q: what is the lead thickness needed in order to attenuate 90% of incoming 500 keV photons?

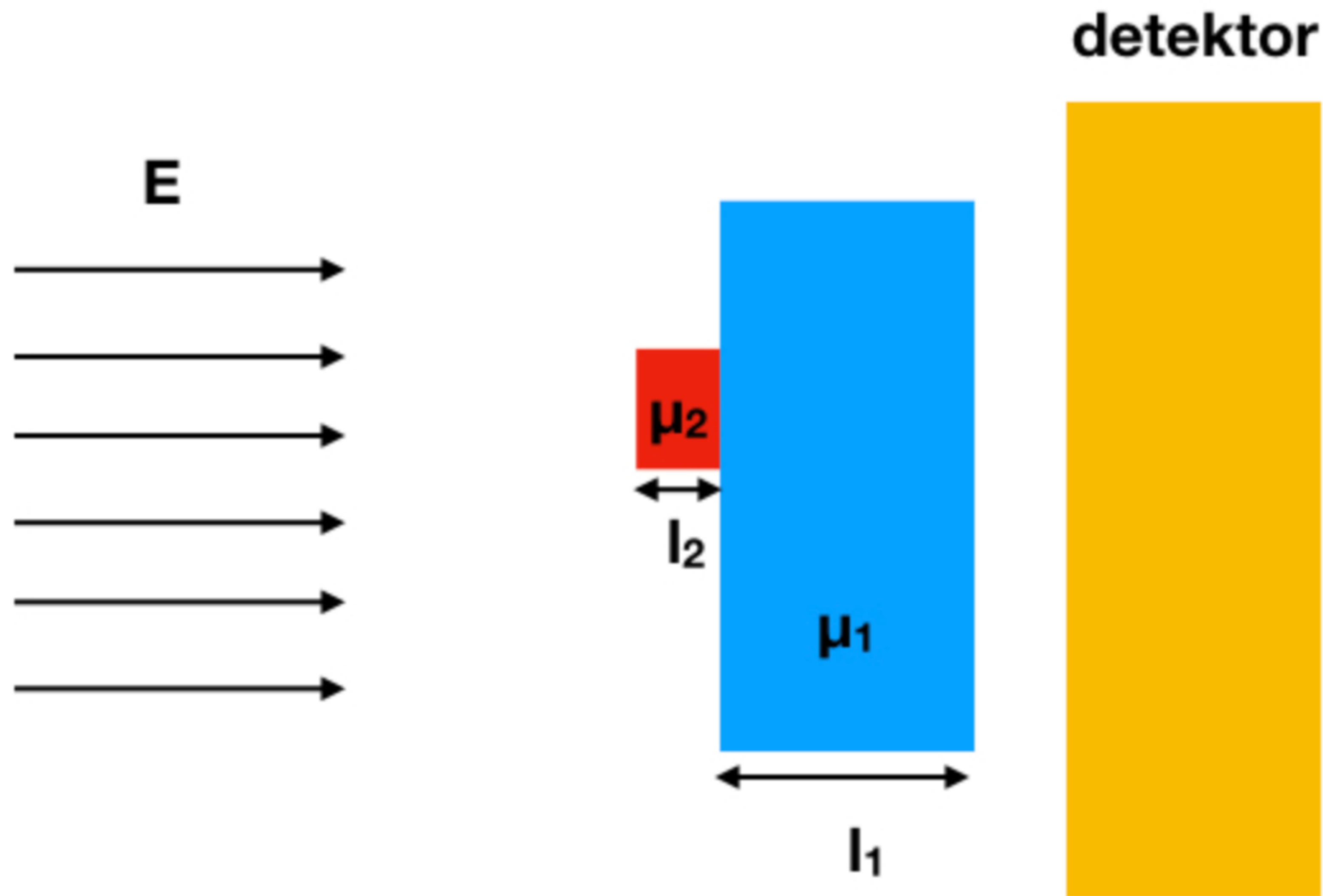
suppose you make the following experiment:

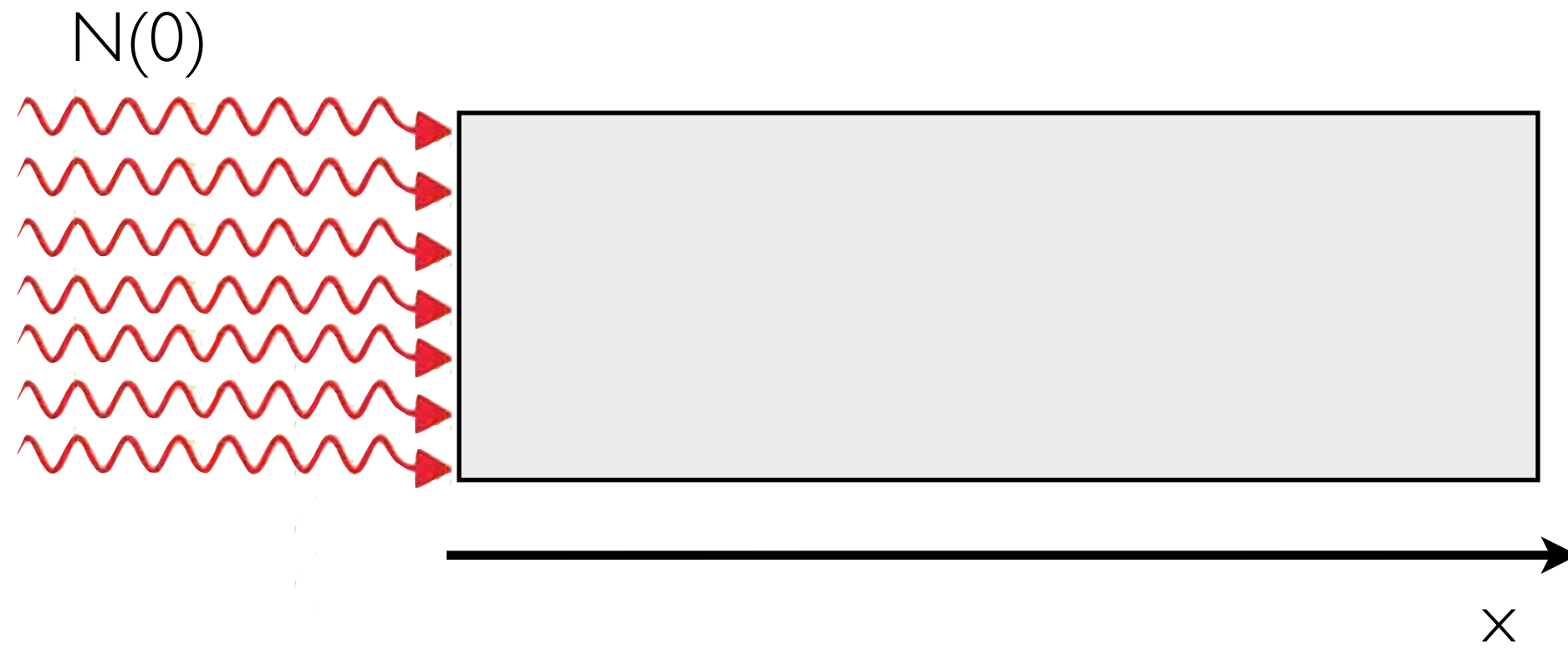


Now you increase the energy of the incoming photons.
What will be the result? (red line is the new result)

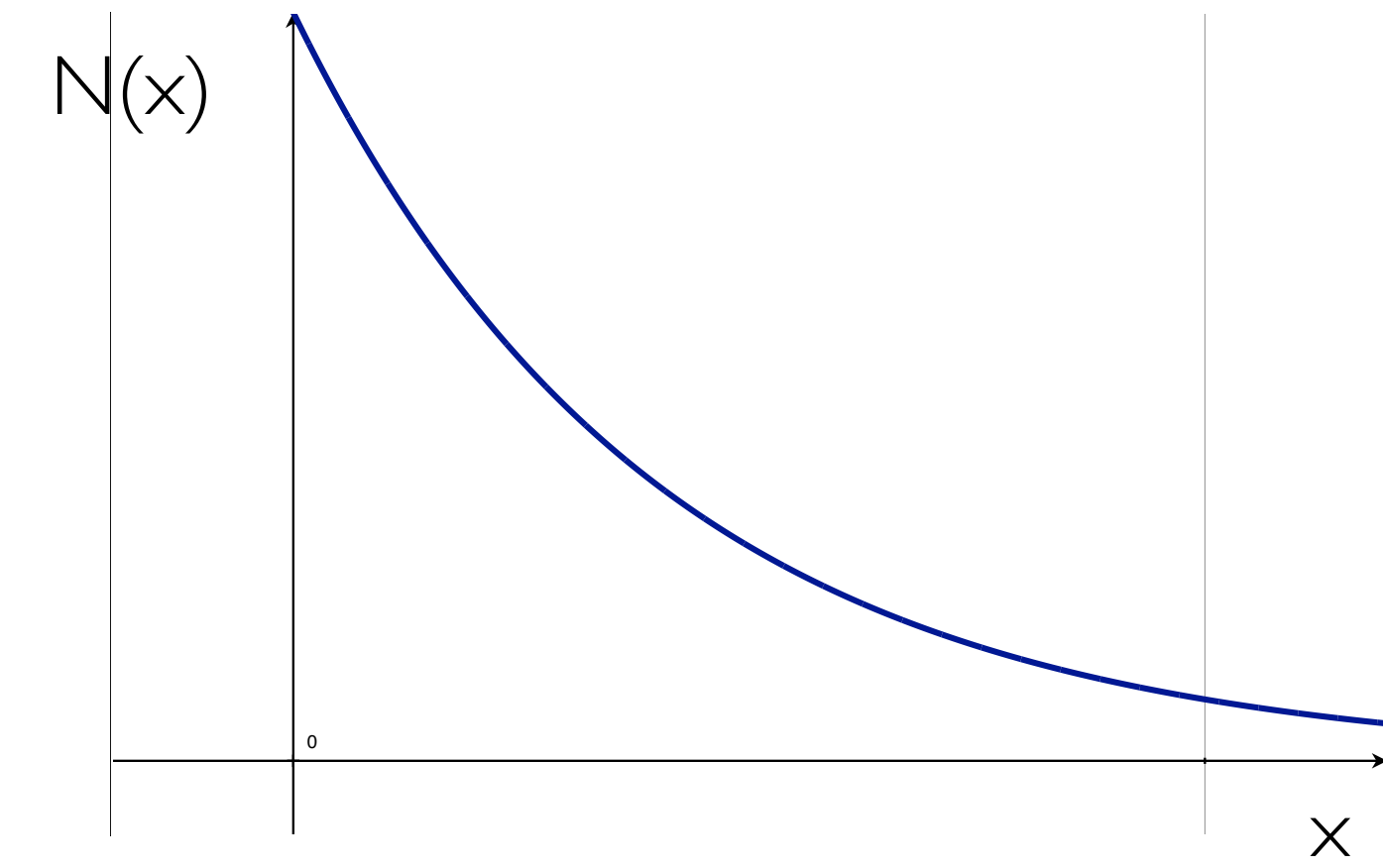


Bestäm objektkontrast med avseende på det röda objektet i situationen i figuren nedan

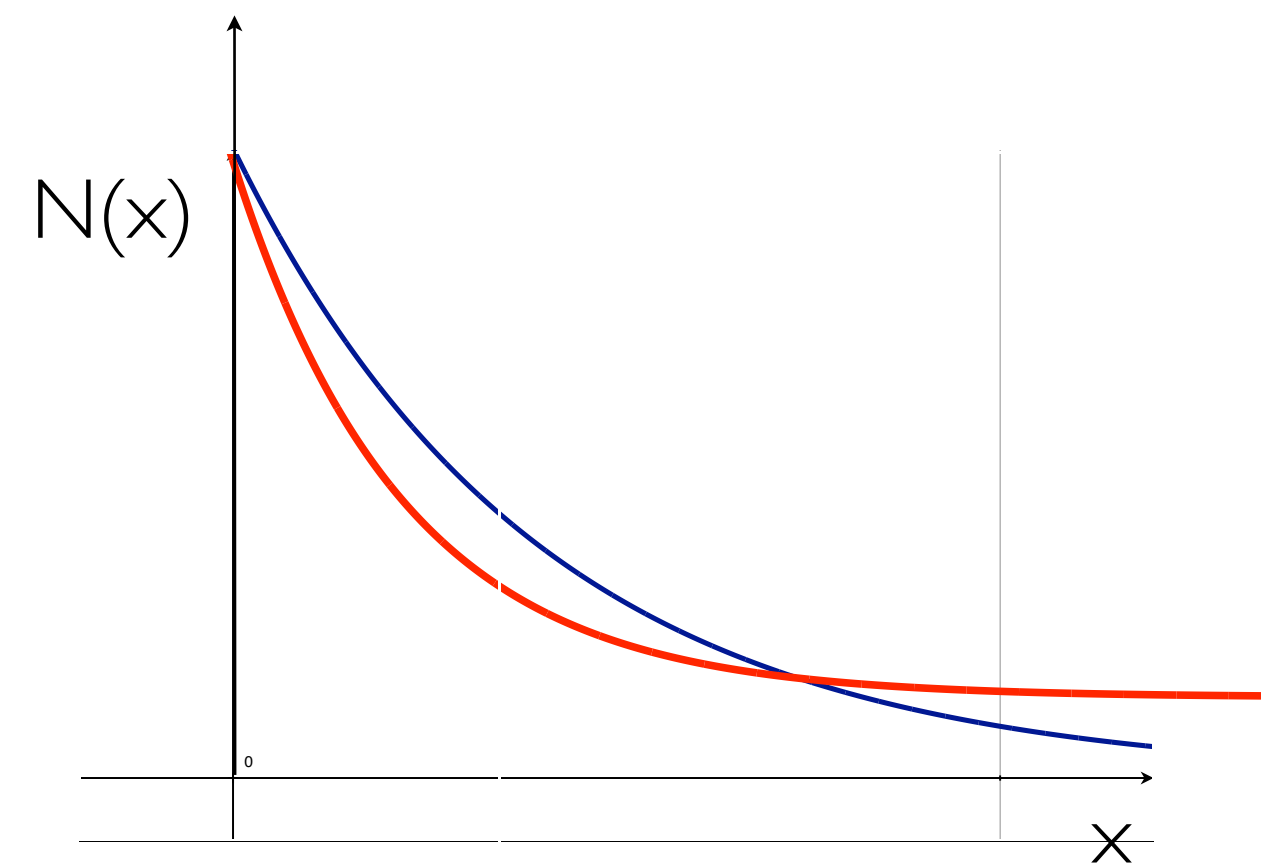




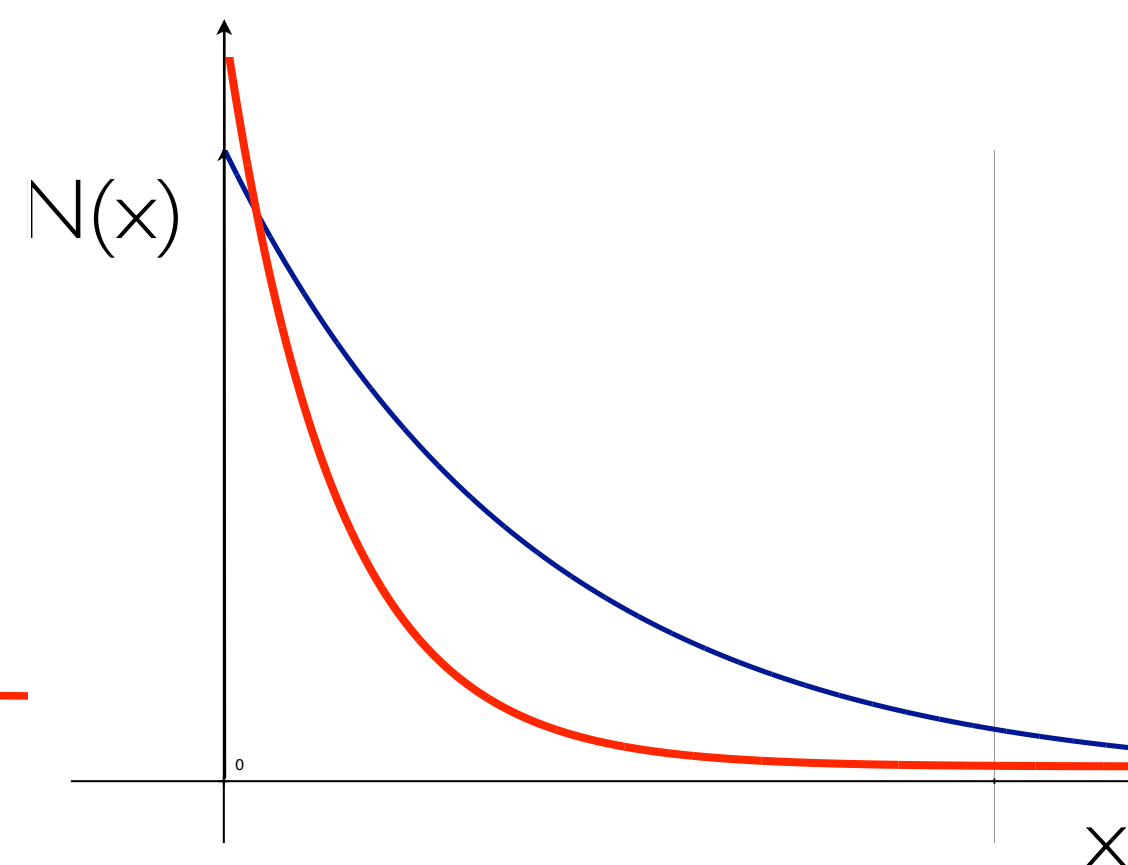
$N(x)$ looks like:



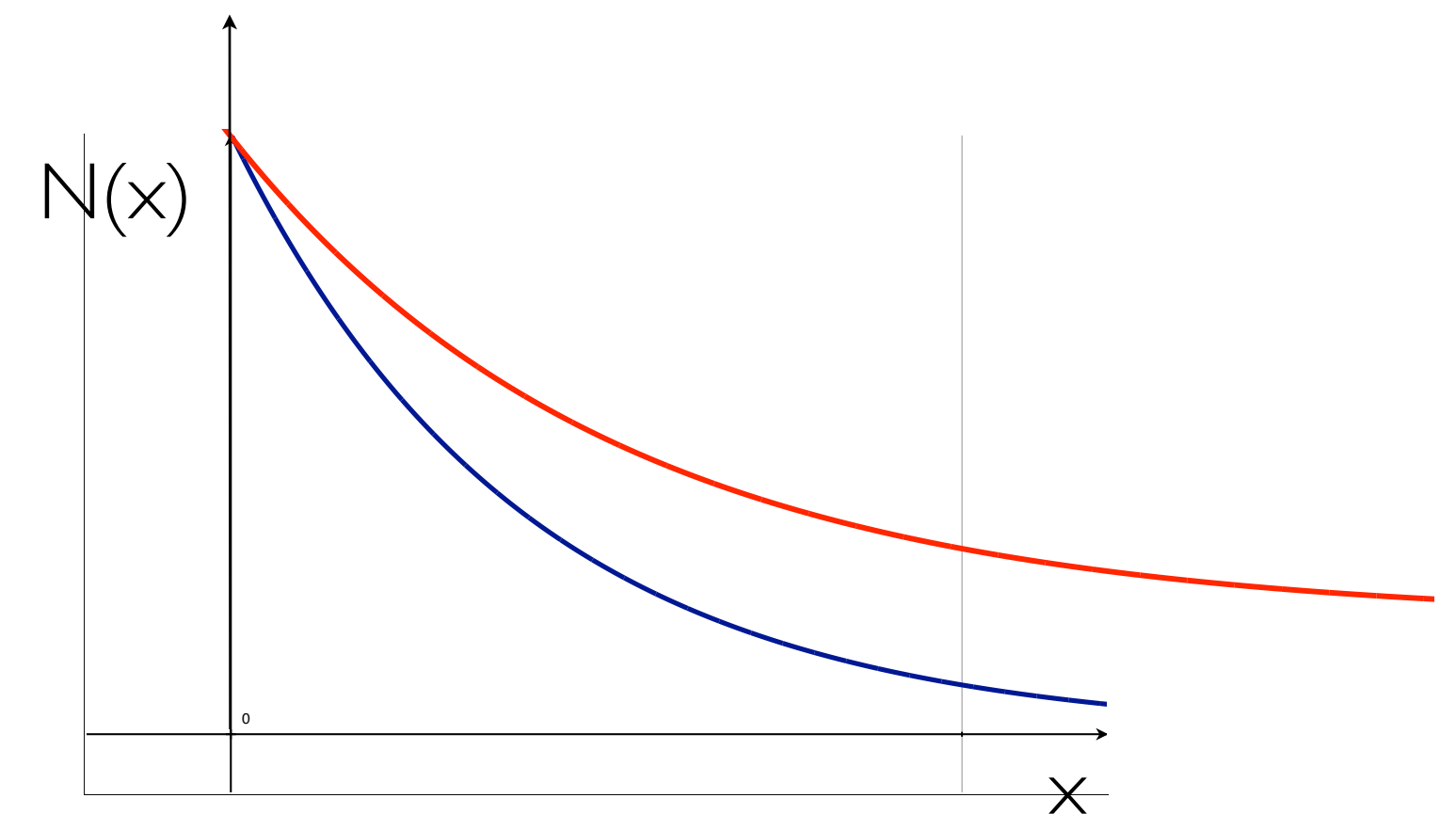
Now the energy of the incoming photons is increased. $N(x)$ looks like:



1



2



3

När en stråle fotoner passerar en viss tjocklek av en viss material så attenueras strålens intensitet till 50% av sitt ursprungliga värde. Vad händer om tjockleken fördubblas?

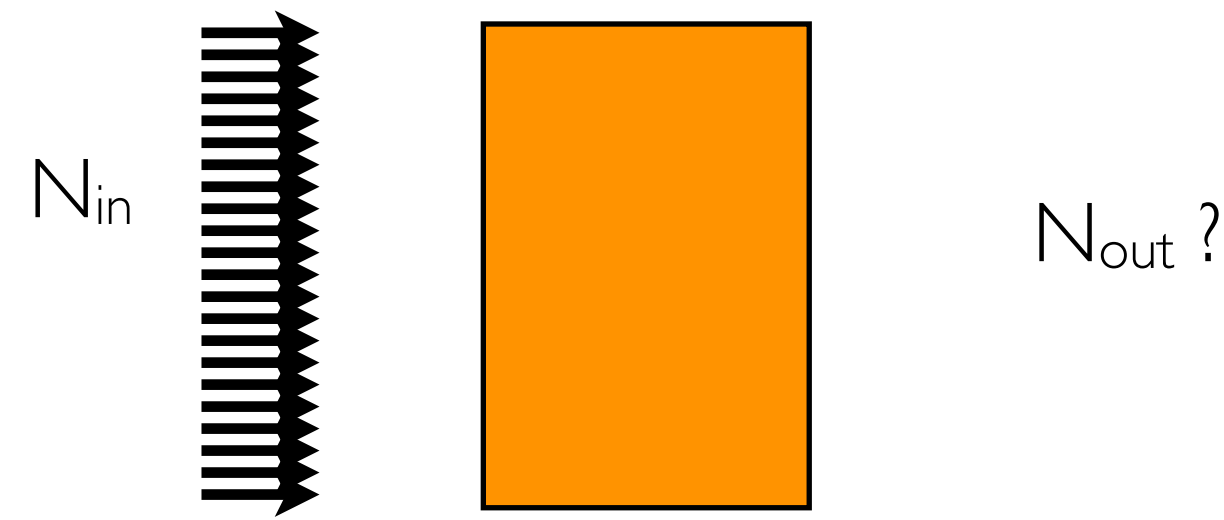
1. attenueringsfaktor fördubblas
2. attenueringsfaktor halveras
3. attenueringsfaktor kvadreras
4. attenueringsfaktor blir 0
5. något annat

När en stråle fotoner passerar en viss tjocklek av en viss material så attenueras strålens intensitet till 50% av sitt ursprungliga värde. Vad händer om man använder en material av samma men med dubbelt så hög densitet?

1. attenueringsfaktor fördubblas
2. attenueringsfaktor halveras
3. attenueringsfaktor kvadreras
4. attenueringsfaktor blir 0
5. något annat

När en stråle fotoner passerar en viss tjocklek av en viss material så attenueras strålens intensitet till 30% av sitt ursprungliga värde. Vad händer om man skickar genom samma filter en stråle fotoner som har dubbelt så hög energi som den ursprungliga strålen?

1. attenueringsfaktor fördubblas
2. attenueringsfaktor halveras
3. attenueringsfaktor kvadreras
4. aattenueringsfaktor blir 0
5. något annat



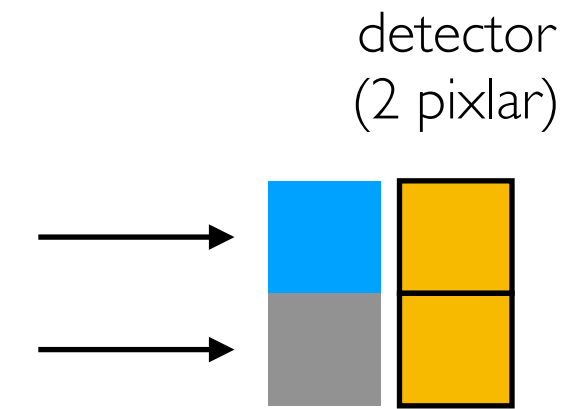
What do you think N_{out} is dependent on?

1. target material
2. target area perpendicular to the flux
3. target thickness
4. N_{in}
5. energy of incoming photons
6. temperature
7. time over which the incoming photons are delivered
8. quality of the detector
9. none of the above, N_{out} is a stochastic variable

Övning:

Bestäm objektkontrast samt SDNR i följande fall:

1. Vatten och aluminium, 1 cm tjocka, samma antal fotoner in, N_0 , med $E = 60$ keV. Bestäm detta för $N_0 = 10, 10^3, 10^5$.
Presentera ditt svar som en 2x3 tabell (objektkontrast och SDNR vs antal fotoner).
2. Samma som ovan, men vatten och aluminium bitar läggs på 0,3 mm bly.
3. Skriv ned det du har lärt dig genom övningen om objektkontrast samt SDNR.



Roligare övning:

Man vill bestämma om ett block post-it lappar innehåller 1000 eller 1001 post-it genom planröntgen. Antag att man använder en energi vid vilken pappers linear attenuation coefficient är $0,1 \text{ cm}^{-1}$ och att källan som används strålar 10^5 fotoner/s mot blocket. Bestäm minsta möjliga tiden som krävs. (Du måste göra en uppskattning på tjockleken på en post it).

(Dokumentet kontrastochSDNR.pdf i CANVAS kan vara av hjälp)