DD2552 - Seminars on Theoretical Computer Science, Programming Languages and Formal Methods, Seminar 9

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Last Seminar and Today

Last seminar:

- Markov Decision Processes (MDPs)

Today:

- note on tool support for MDPs
- probabilistic/stochastic priced/weighted *timed* automata

MDPs and tools

- PRISM supports numerical analysis of MDPs
- MDPs can be simulated in PRISM, but nondetermism resolved uniformly at random
- SMC for MDPs implemented in PLASMA-Lab (https://project.inria.fr/plasma-lab/)
- PLASMA-Lab supports PRISM language

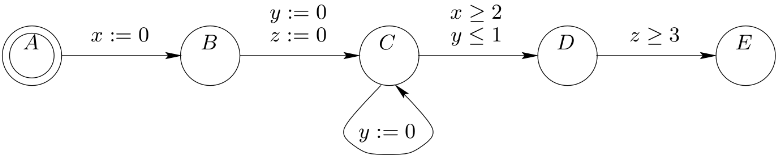
Timed Automata (Alur and Dill, 1994)

With each ω-automaton we associate a finite set of (real-valued) clocks.

A clock can be set to zero simultaneously with any transition of the automaton. At any instant, the reading of a clock equals the time elapsed since the last time it was set.

With each transition we associate an enabling condition which compares the current values of the clocks with time constants.

Timed Automata Example



interpretation:

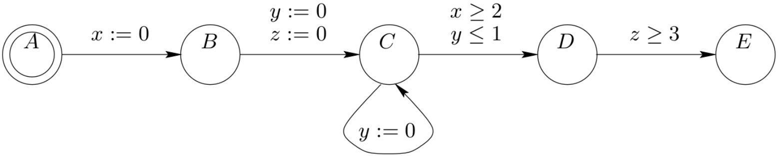
- A,B,C,D are tasks
- all tasks except C performed once
- C performed at most once
- edges are task dependencies

Timed Automata Paths and Traces

- in paths, we record clock states, transitions, and durations
- paths must respect constraints and clock reset
- traces (path prefixes) can show *reachability* of locations/tasks

$$(A,0,0,0) \xrightarrow{\tau}\xrightarrow{\epsilon(1)} (B,1,1,1) \xrightarrow{\tau}\xrightarrow{\epsilon(1)} (C,2,1,1) \xrightarrow{\tau}\xrightarrow{\epsilon(2)} (D,4,3,3) \xrightarrow{\tau} (E,4,3,3)$$

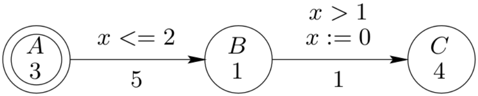
Timed Automata Problems



- is there a schedule to perform all tasks (reach E)?
- what is the optimal schedule (minimum time) to reach E?
- both problems are decidable

Priced/Weighted Timed Automata (WTA/PTA)

- introduce "prices" (numbers) for both locations and edges
- the price of a transition gives the cost for taking it
- the price on a location specifies the cost per time-unit for staying in that location



Priced Timed Automata Problems

- each trace is associated with a cost (sum of prices)
- problem: what is the minimum cost to reach location l?
- decidable for *linear* prices

WCTL

- a logic for properties of PTAs
- extension of logic TCTL for TAs

$$\phi ::= \top \mid a \mid \neg\phi \mid \phi \wedge \phi \mid E(\phi\, U_{\leq c}\, \phi) \mid A(\phi\, U_{\leq c}\, \phi)$$
$$c \in Z^{\geq 0}$$

- intuition for E: exists a path satisifying $\phi\, U \phi'$, where cost at state satisfying ϕ' is $\leq c$.
- intuition for A: all (infinite) paths satisfy $\phi\, U \phi'$, and cost at state in each path satisfying ϕ' is $\leq c$.

WCTL Model Checking Hardness

Definition

A *stopwatch cost* is a cost for a location l s.t. $cost(l) \subseteq \{0, 1\}$.

Theorem

Model checking WCTL on PTAs with three clocks and one stopwatch cost is undecidable.

Networks of Priced Timed Automata (David et al., 2011)

- abstract definitions over set of clocks X
- a clock can be assigned a valuation mapping (positive real)
- a clock can be assigned a rate (non-negative integer)
- partition PTA actions into inputs and outputs
- locations have rate vectors

Network Setup

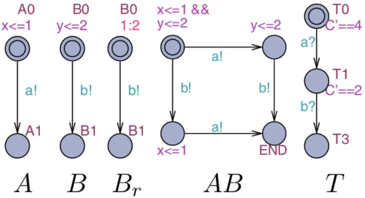
Define composition of n PTAs:

$$\mathcal{A}_1 | \ldots | \mathcal{A}_n$$

We are interested in *interaction* between PTAs:

- output of one becomes input of another
- need notion of *composability*

NTPA Example



Probabilistic Semantics of NTPAs

- delays determined by probability distributions
- inputs determined by probability distributions
- define PWCTL logic