Visualization, DD2257

Prof. Dr. Tino Weinkauf

Geometry-based Vector Field Visualization II

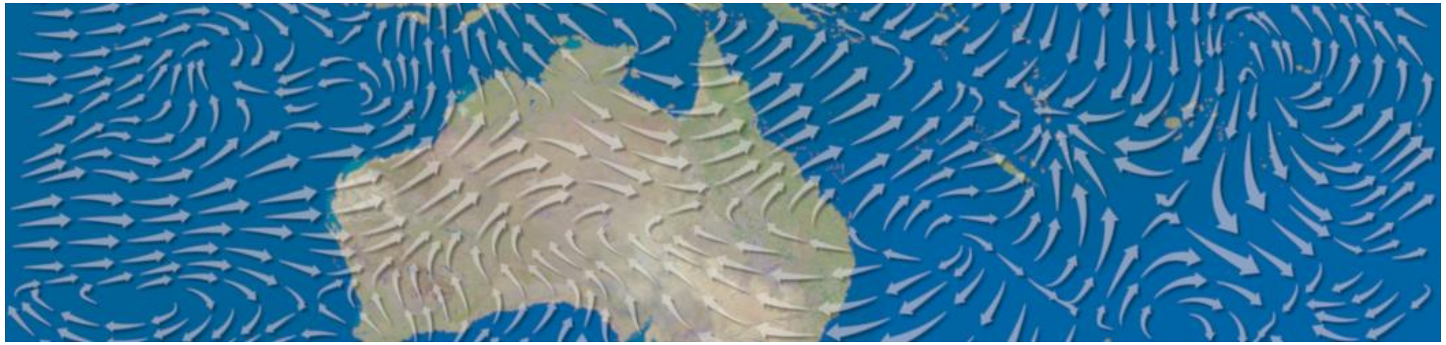
unsteady vector fields

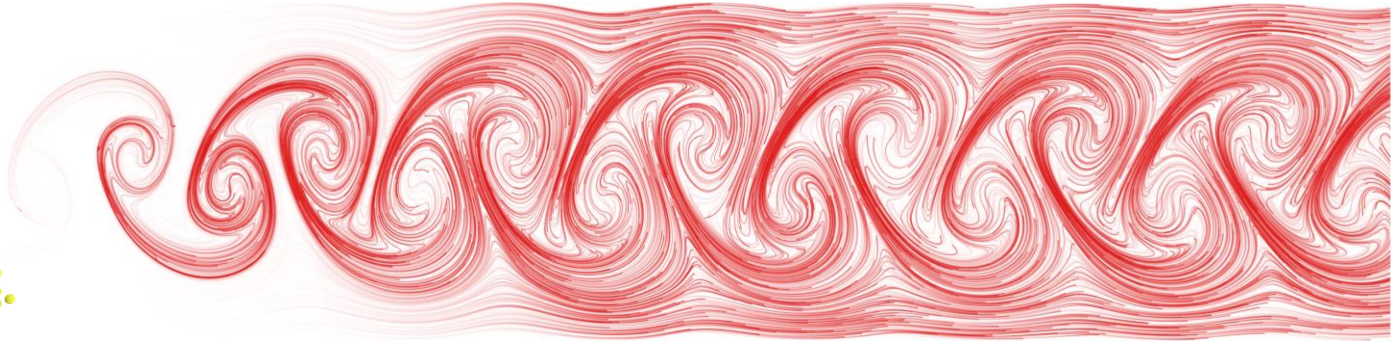
Geometry-based Vector Field Visualization

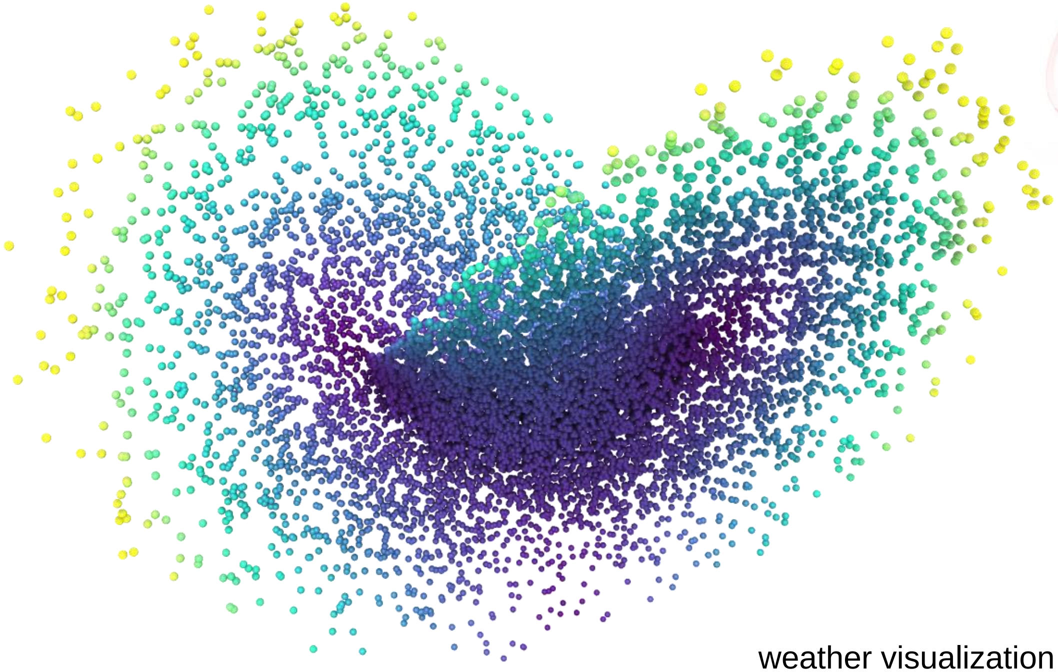
geometric objects

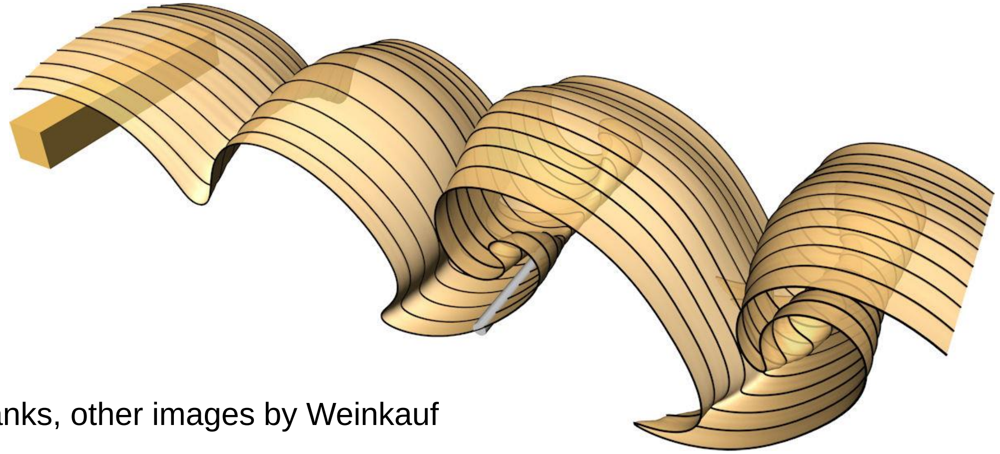
shape is directly related to the flow

points, lines, surfaces, …



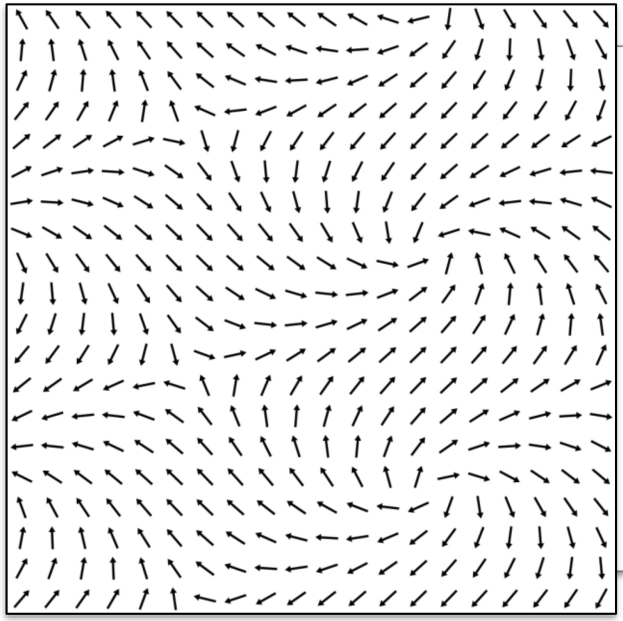






weather visualization by Turk & Banks, other images by Weinkauf

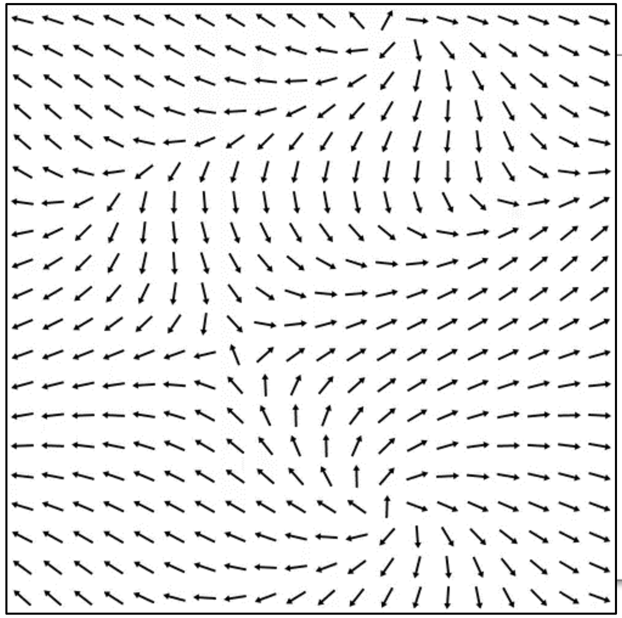
$\mathbf{v}: \mathrm{E}^n \to \mathbb{R}^n$



Steady Vector Fields

- tangent curves / stream lines
- stream surfaces

$\mathbf{v}: \mathrm{E}^{n+1} \to \mathbb{R}^n$

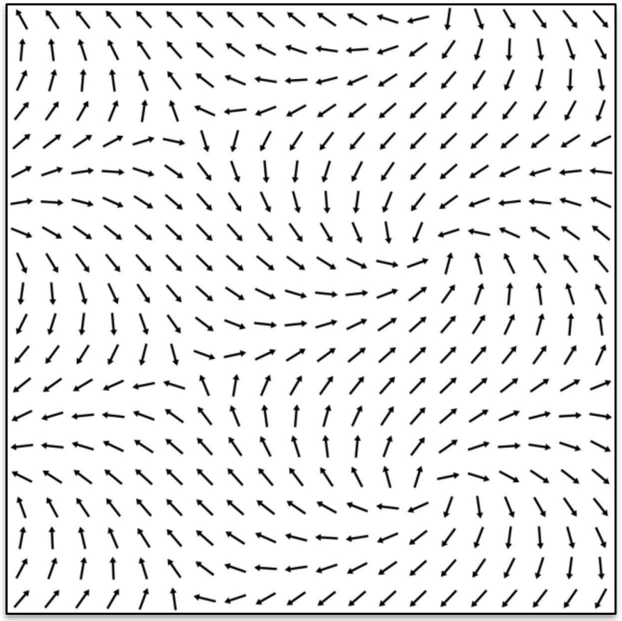


Unsteady Vector Fields

- path/streak/time lines
- path/streak/time surfaces

$\mathbf{v}\colon \mathrm{E}^n \to \mathbb{R}^n$

steady vector field



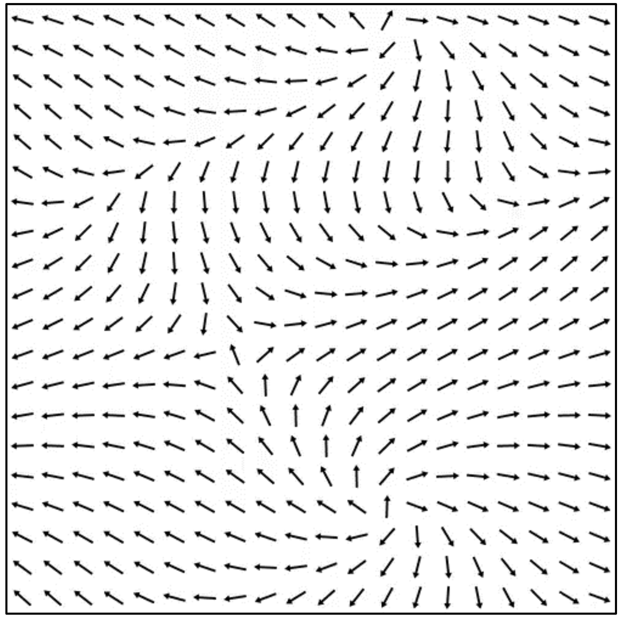
$$\mathbf{v}(x,y) = \begin{pmatrix} u(x,y) \\ v(x,y) \end{pmatrix}$$

$$\mathbf{v}(x,y) = \begin{pmatrix} \cos(x+2y) + x\cos(2\pi) \\ \sin(x-2y) + y\sin(3\pi) \end{pmatrix}$$ example

$\mathbf{v}\colon \mathrm{E}^{n+1} \to \mathbb{R}^n$

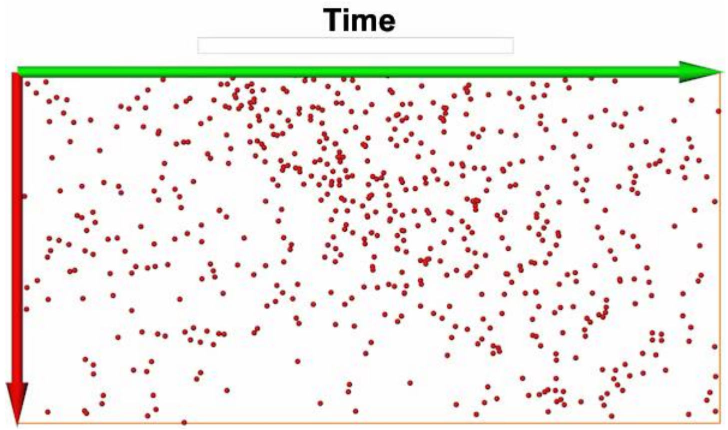
unsteady vector field

time-dependent vf

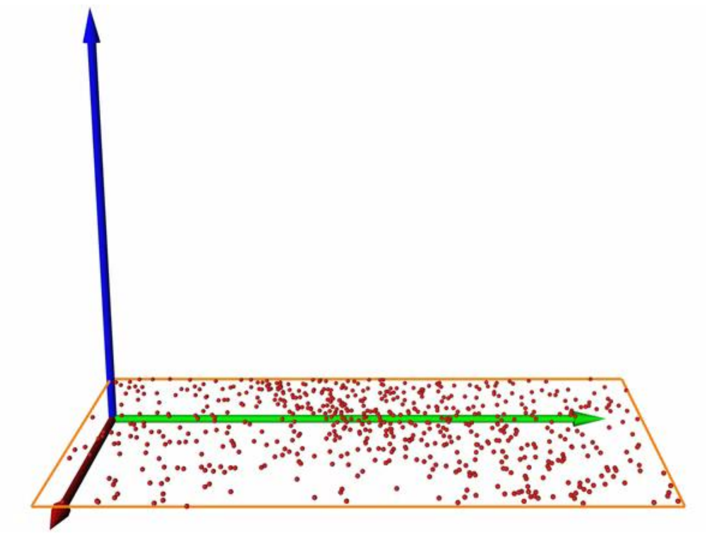


$$\mathbf{v}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \end{pmatrix}$$

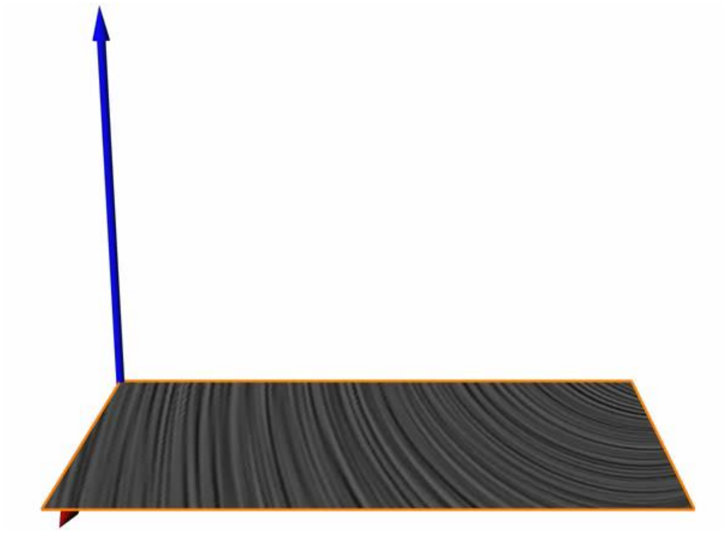
$$\mathbf{v}(x,y,t) = \begin{pmatrix} \cos(x+2y) + x\cos(2\pi t) \\ \sin(x-2y) + y\sin(\pi + 2\pi t) \end{pmatrix}$$ example



2D time-dependent vector field
particle visualization



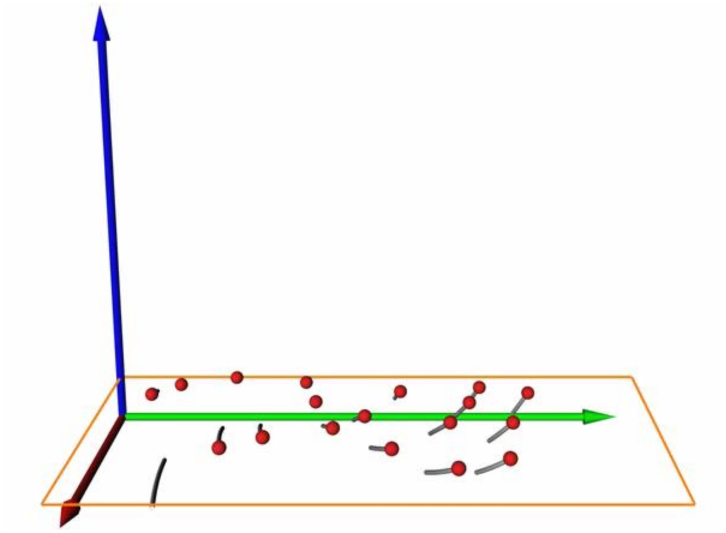
2D time-dependent vector field
particle visualization
space-time diagram



stream lines

curve tangential to the vector field
in each point for a **fixed time**

describes motion of a massless particle
in a **steady** flow field



path lines

curve tangential to the vector field
in each point **over time**

describes motion of a massless particle
in an **unsteady** flow field

very important

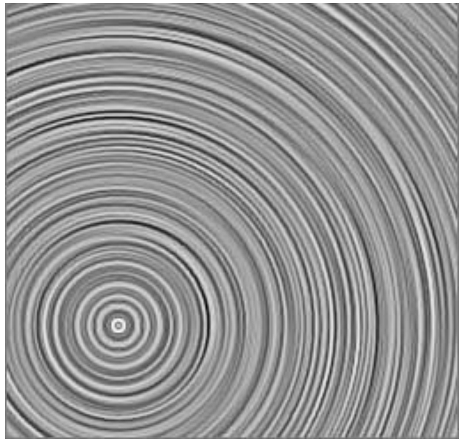
steady vector field

$$\mathbf{v}(x, y)$$

$$\frac{d}{d\tau}\mathbf{x}(\tau) = \mathbf{v}(\mathbf{x}(\tau))$$
$$\text{with } \mathbf{x}(0) = \mathbf{x}_0$$

stream lines

$$\mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix}$$



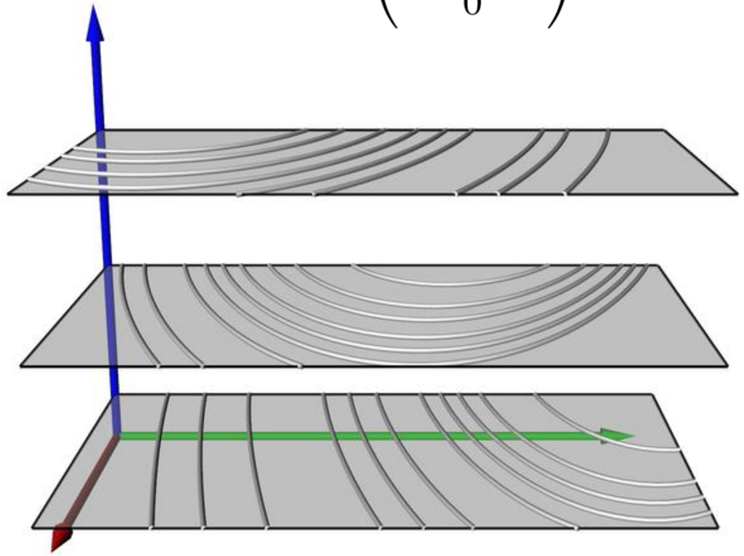
unsteady vector field

$$\mathbf{v}(x, y, t)$$

$$\frac{d}{d\tau}\mathbf{x}(\tau) = \mathbf{v}(\mathbf{x}(\tau), t_0)$$
$$\text{with } \mathbf{x}(0) = \mathbf{x}_0$$

stream lines

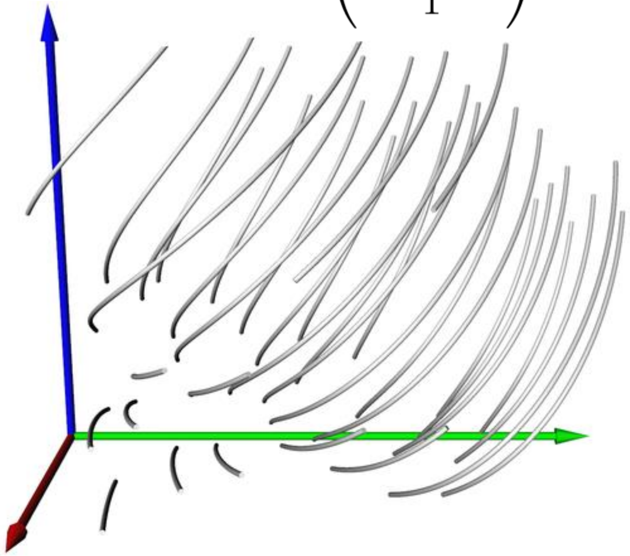
$$\mathbf{v}(x, y, t) = \mathbf{s}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \\ 0 \end{pmatrix}$$



$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{v}(\mathbf{x}(t), t)$$
$$\text{with } \mathbf{x}(t_0) = \mathbf{x}_0$$

path lines

$$\mathbf{v}(x, y, t) = \mathbf{p}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \\ 1 \end{pmatrix}$$



very important

	steady vector field $\mathbf{v}(x,y)$	unsteady vector field $\mathbf{v}(x,y,t)$	
	stream lines	stream lines	path lines
2D	$\mathbf{v}(x,y) = \begin{pmatrix} u(x,y) \\ v(x,y) \end{pmatrix}$	$\mathbf{s}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \\ 0 \end{pmatrix}$	$\mathbf{p}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \\ 1 \end{pmatrix}$
3D	$\mathbf{v}(x,y,z) = \begin{pmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{pmatrix}$	$\mathbf{s}(x,y,z,t) = \begin{pmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \\ 0 \end{pmatrix}$	$\mathbf{p}(x,y,z,t) = \begin{pmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \\ 1 \end{pmatrix}$

*n*D unsteady vector field
➔ *(n+1)*D steady vector field

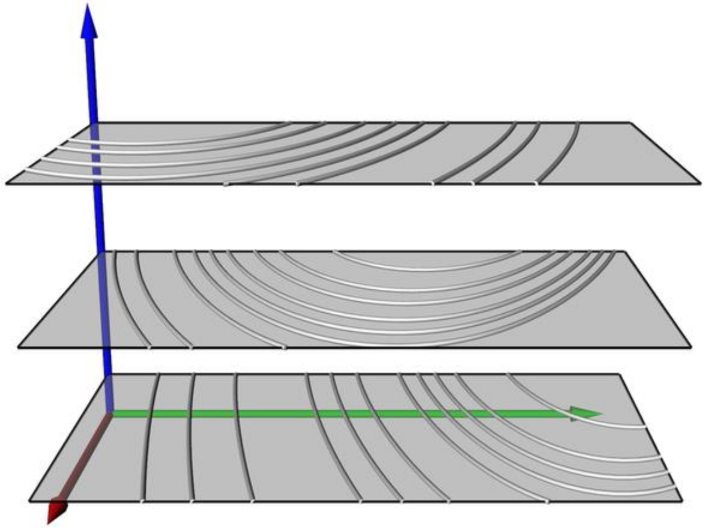
very important

Characteristic Curves

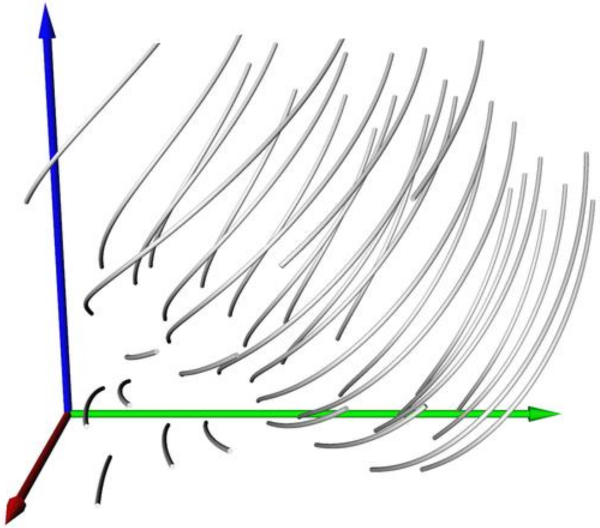
concept of tangent curves → concept of ***characteristic curves***

stream lines
path lines
streak lines
time lines

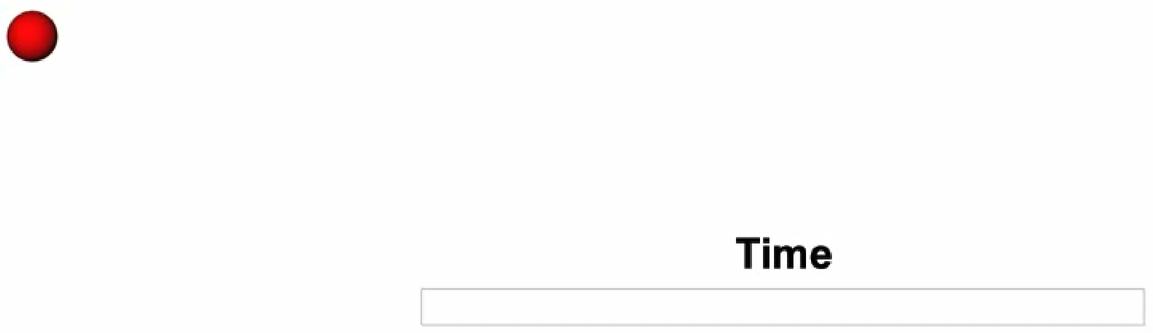
Based on tangent curves,
we define 4 types of ***characteristic curves*** of a vector field.



stream lines

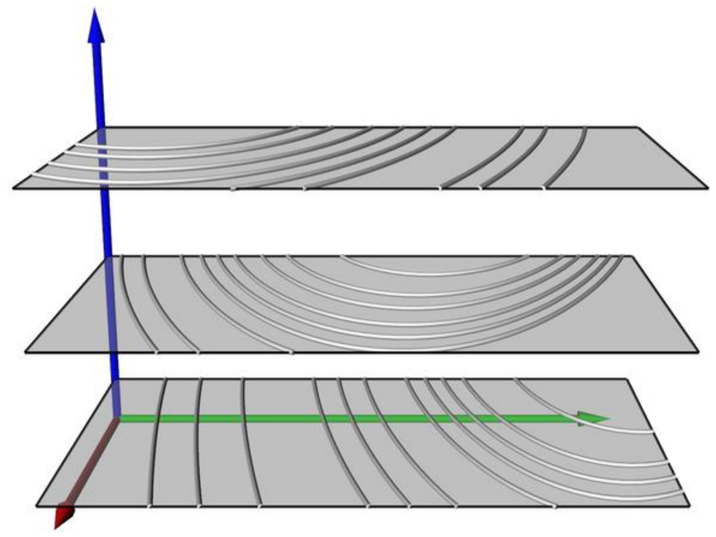


path lines

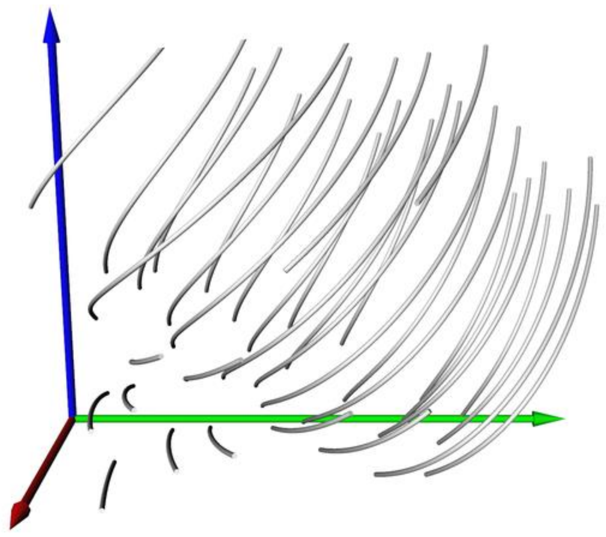


streak line

location of all particles set out at a fixed point at different times

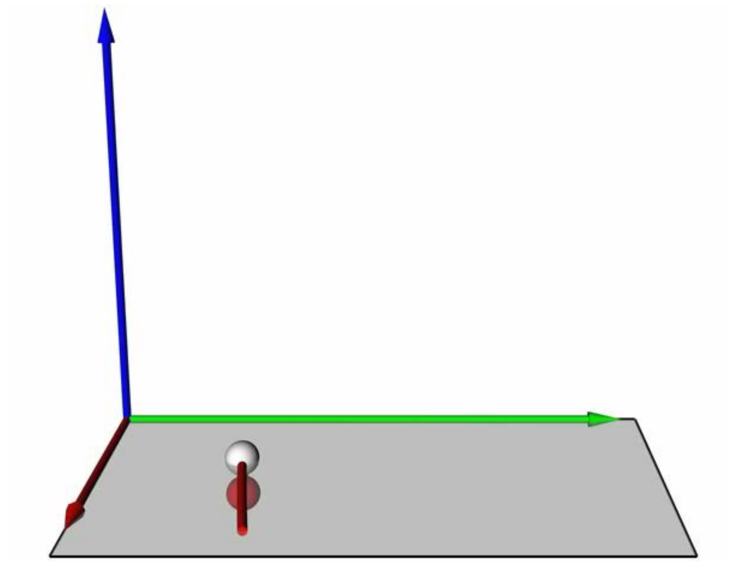


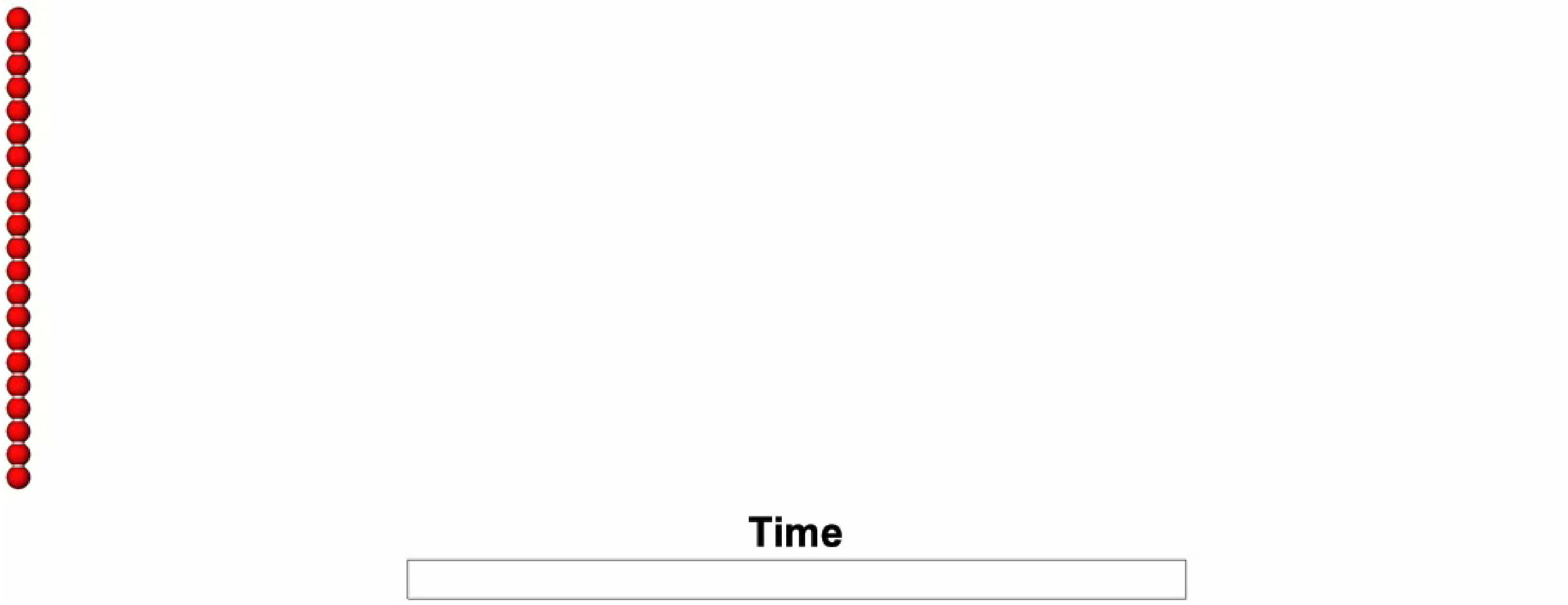
stream lines



path lines

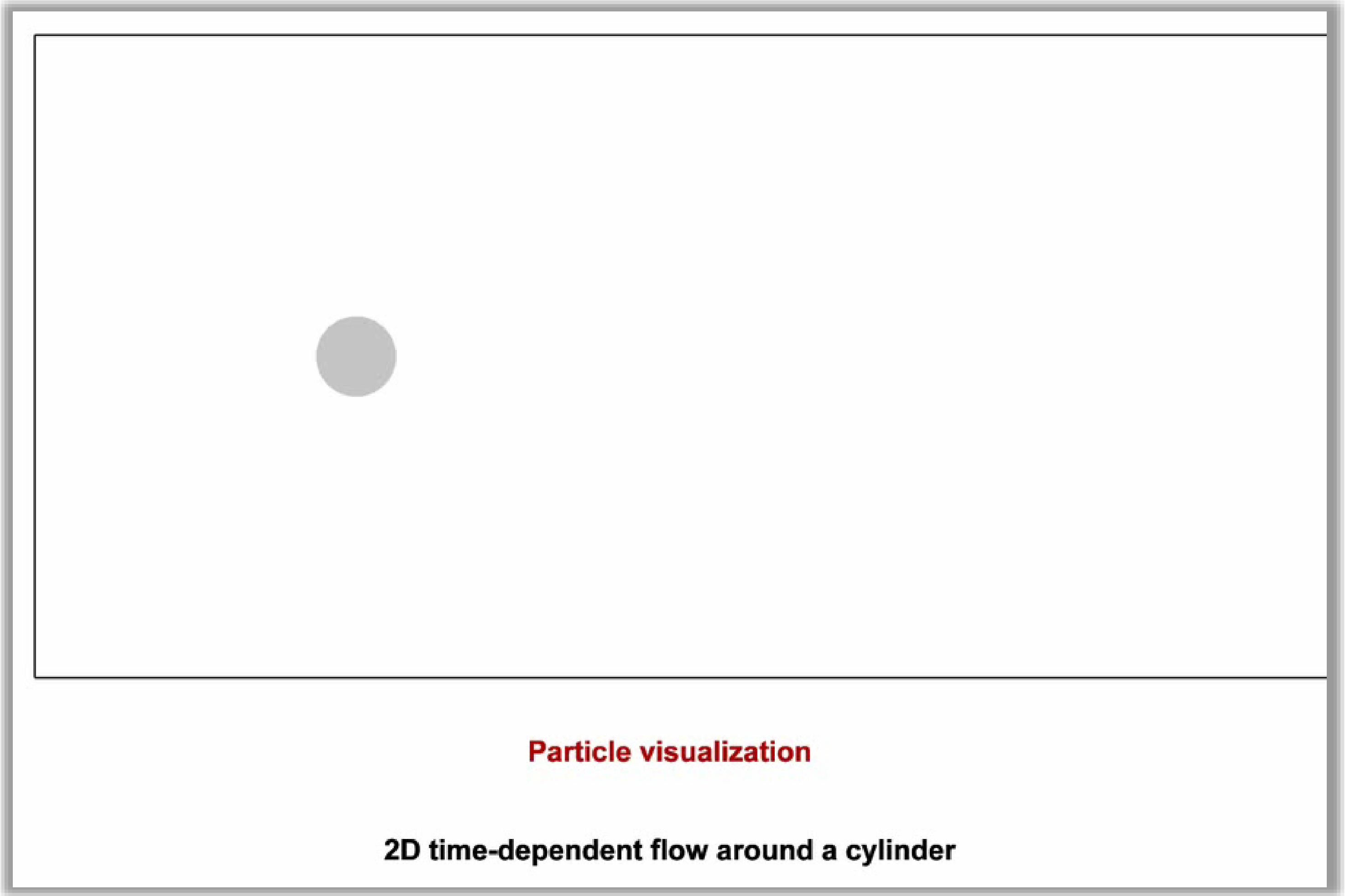
streak lines





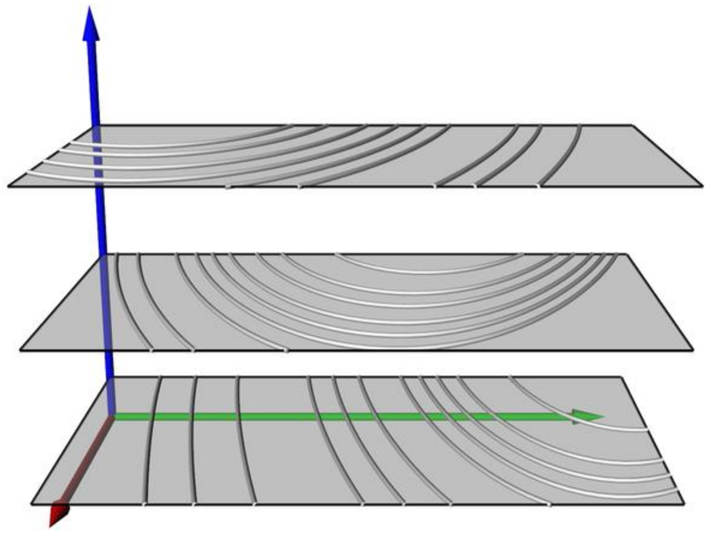
time line

location of all particles set out on a certain line at a fixed time

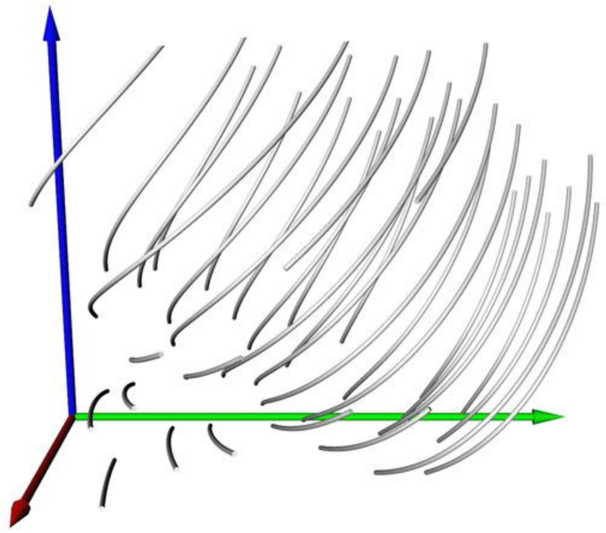


time line

location of all particles set out on a certain line at a fixed time

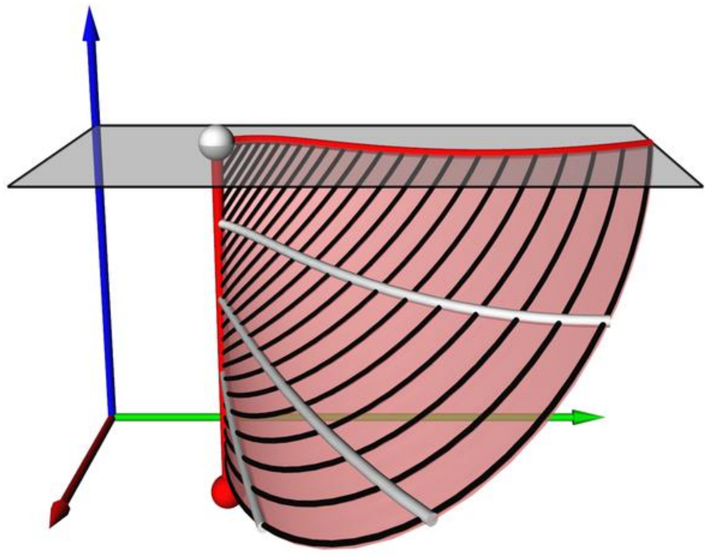


stream lines

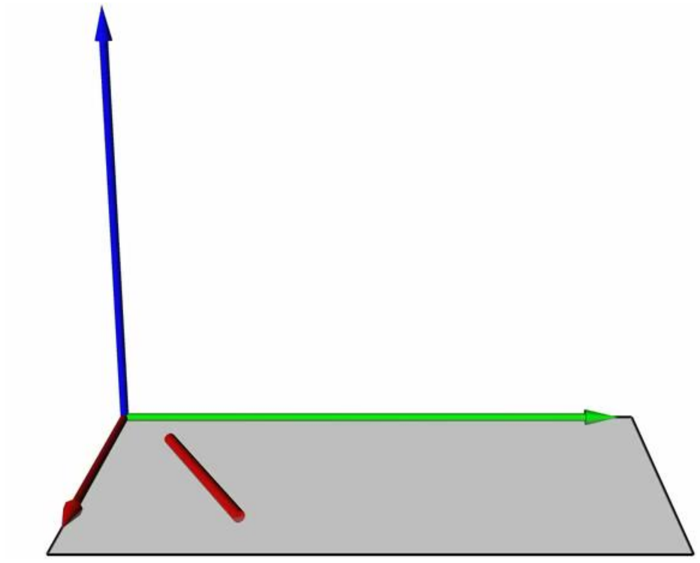


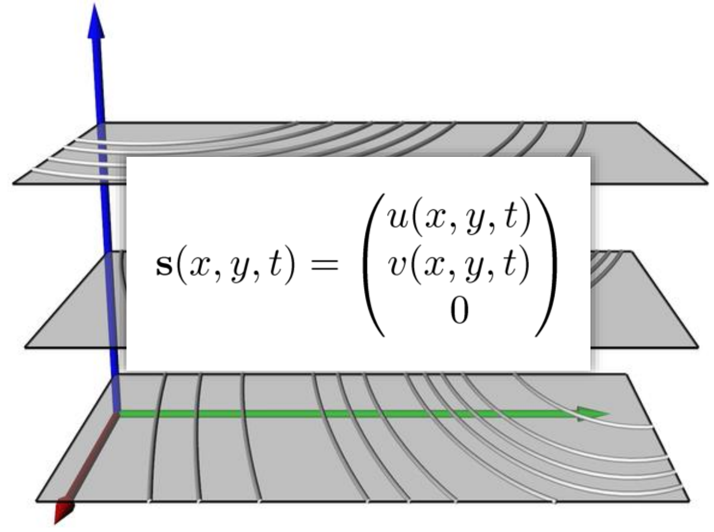
path lines

streak lines

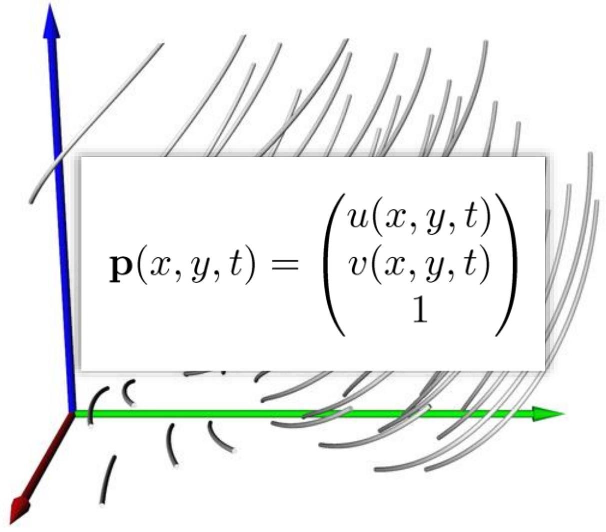


time lines



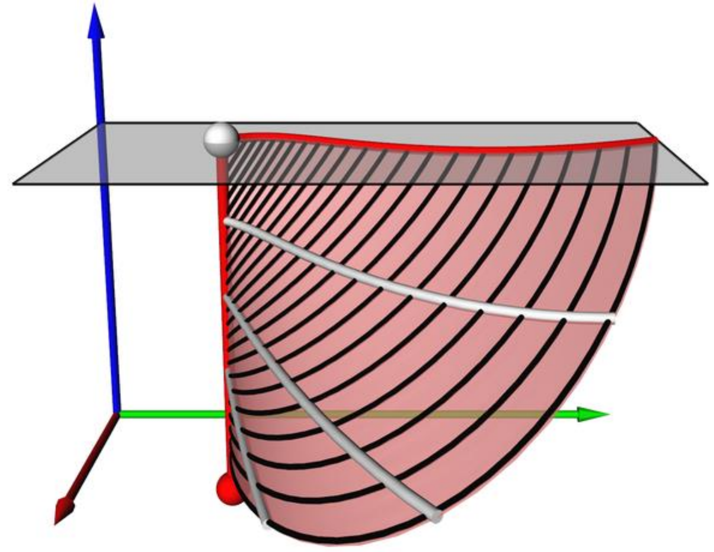


stream lines

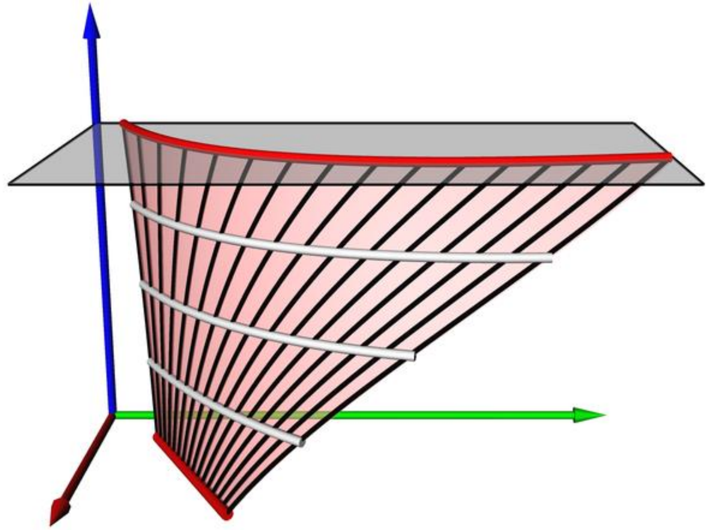


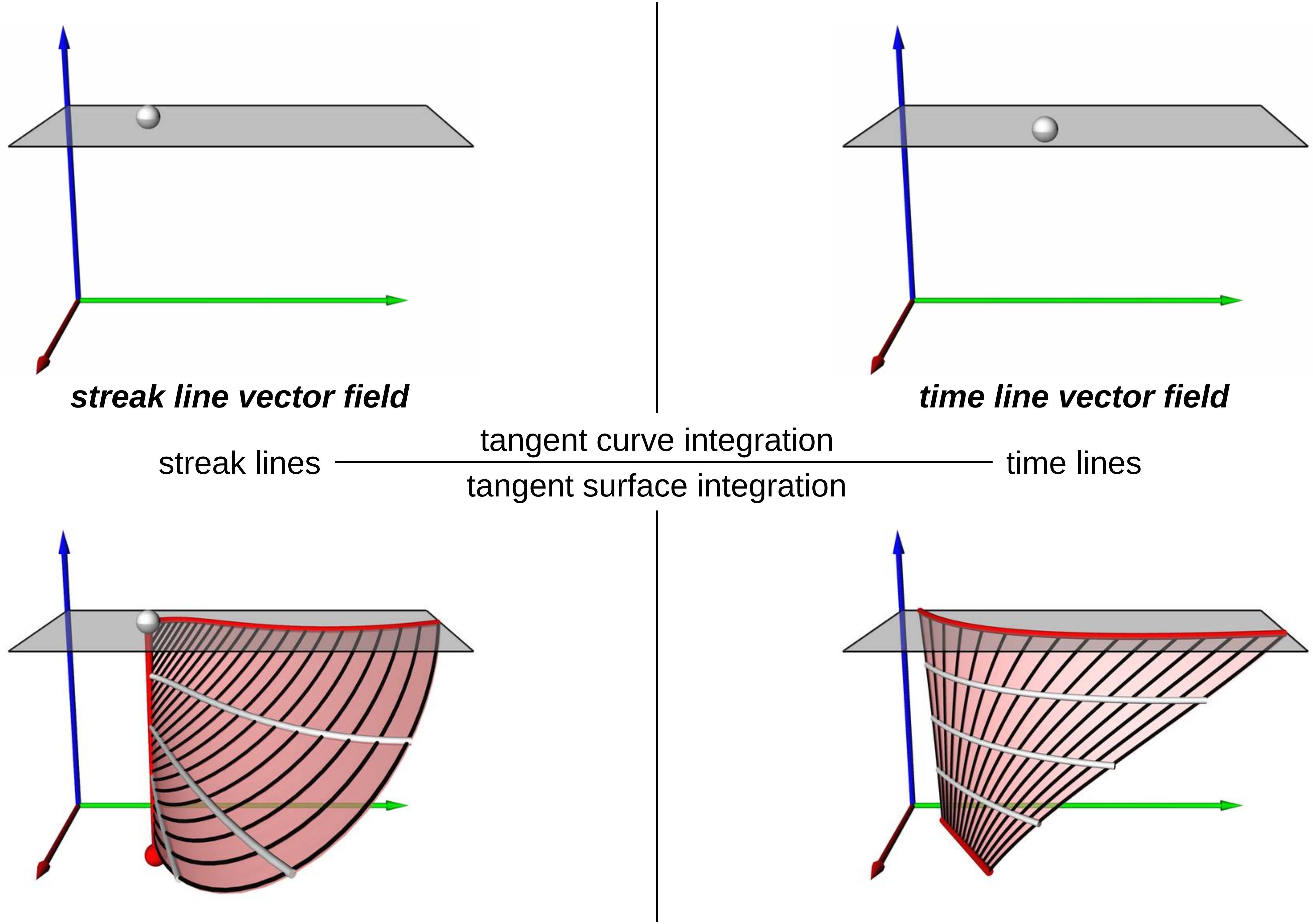
path lines

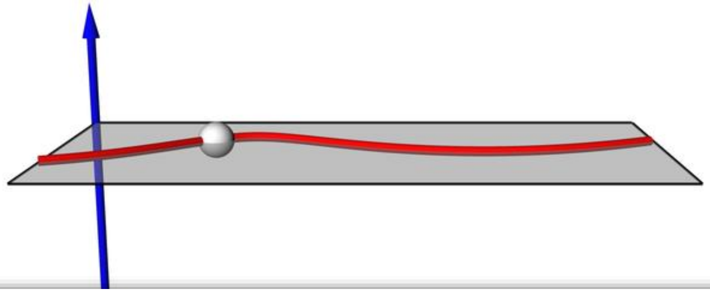
streak lines



time lines

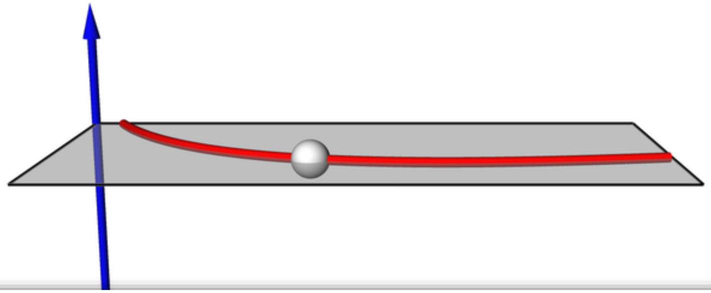



streak line vector field
time line vector field
streak lines
tangent curve integration
tangent surface integration
time lines



$$\bar{\bar{\mathbf{q}}}(\mathbf{x},t,\tau) = \begin{pmatrix} (\nabla\phi_t^{\tau}(\mathbf{x}))^{-1}\cdot\frac{\partial\phi_t^{\tau}(\mathbf{x})}{\partial t} + \mathbf{v}(\mathbf{x},t) \\ 0 \\ -1 \end{pmatrix}$$

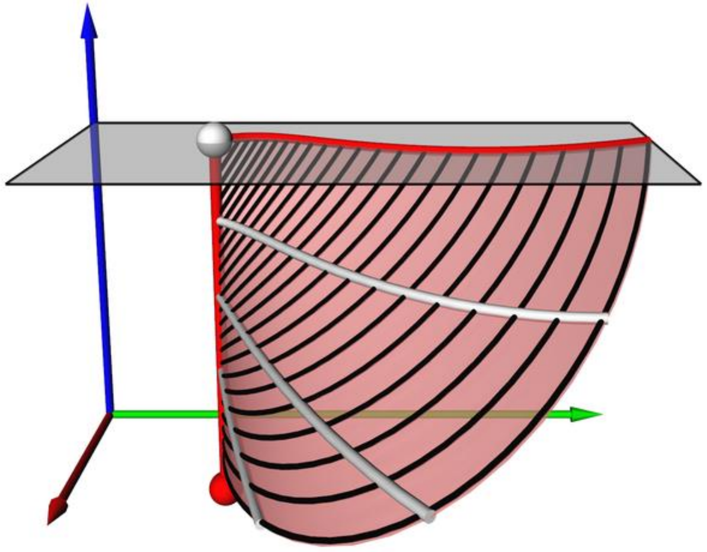
streak line vector field

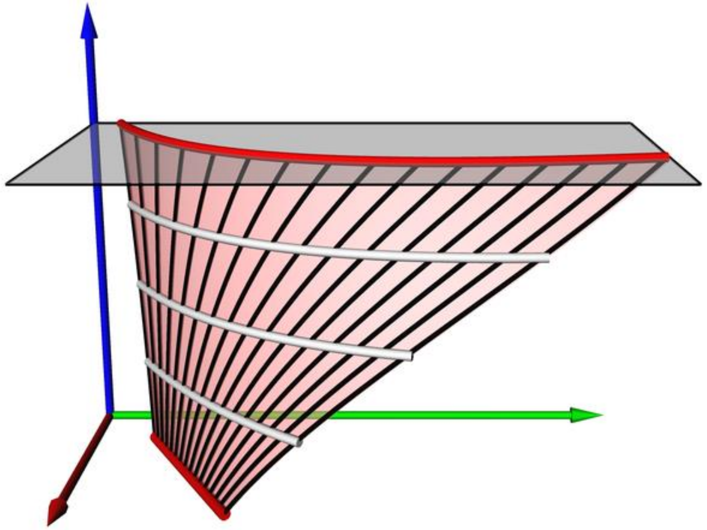


$$\bar{\bar{\mathbf{q}}}(\mathbf{x},t,\tau) = \begin{pmatrix} (\nabla\phi)^{-1}\cdot\mathbf{c} \\ 0 \\ 0 \end{pmatrix}$$

time line vector field

streak lines —— tangent curve integration / tangent surface integration —— time lines





Weinkauf and Theisel, IEEE Visualization 2010 & Eurographics 2012

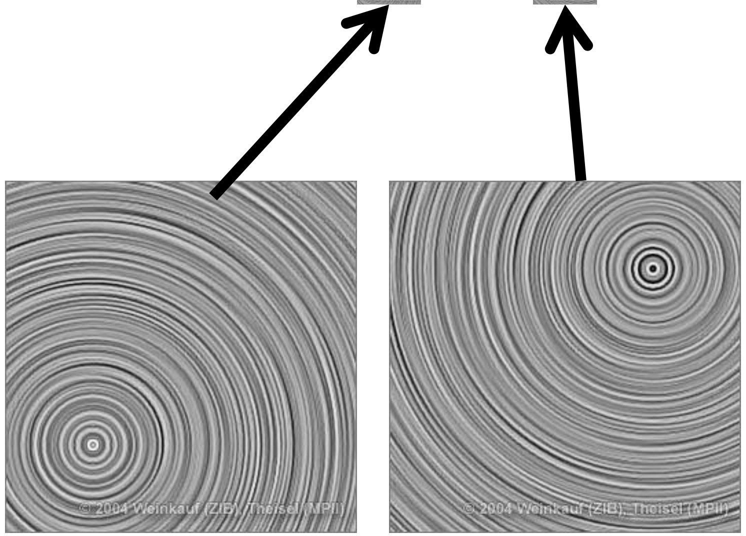
very important

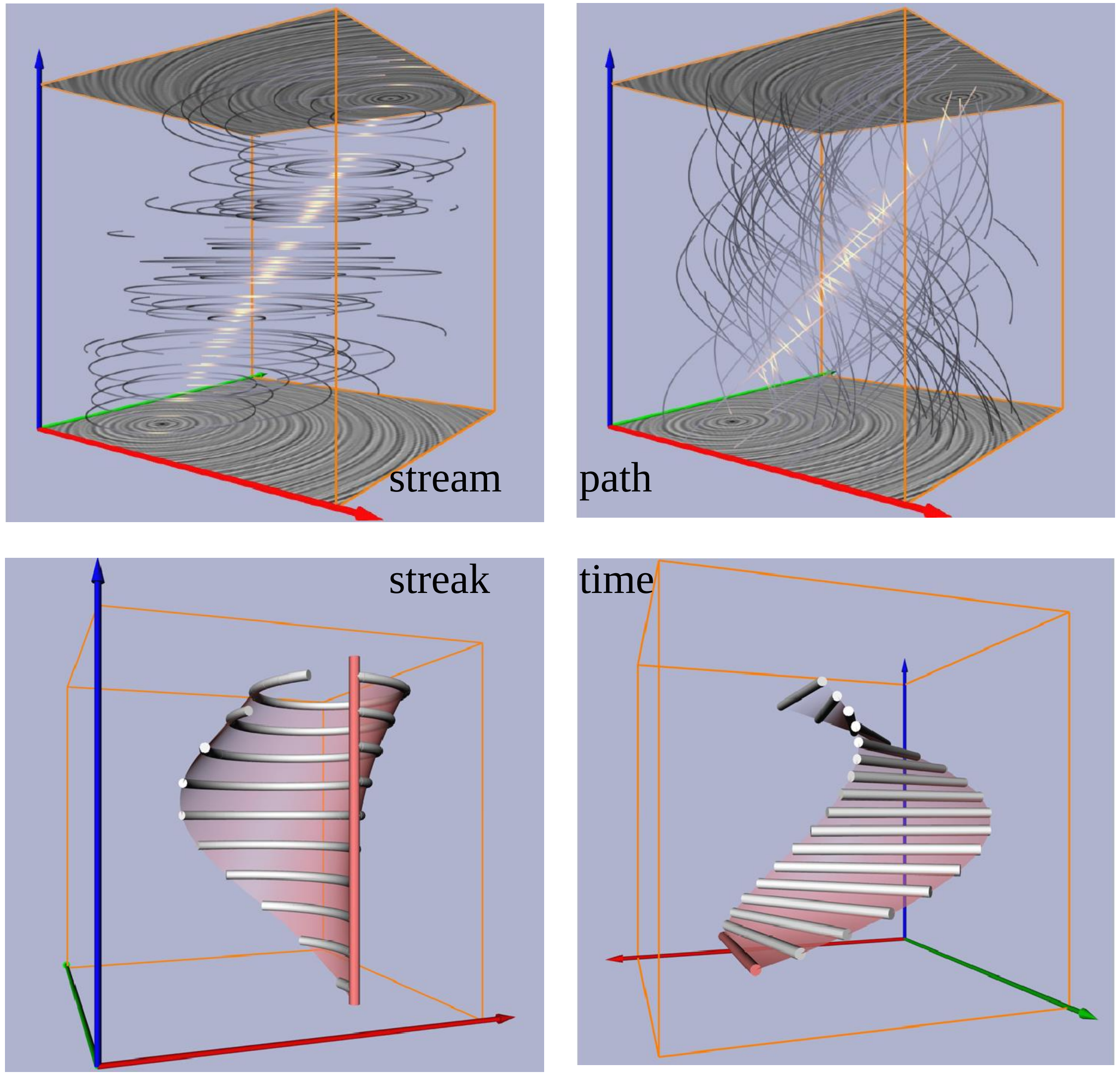
Characteristic Curves

- **Stream line:** curve tangential to the vector field in each point for a fixed time
  - describes the motion of a massless particle in a steady flow field
- **Path line:** curve tangential to the vector field in each point over time
  - describes the motion of a particle over time in an unsteady flow field
- **Streak line:** location of all particles set out at a fixed point at different times
- **Time line:** location of all particles set out on a certain line at a fixed time

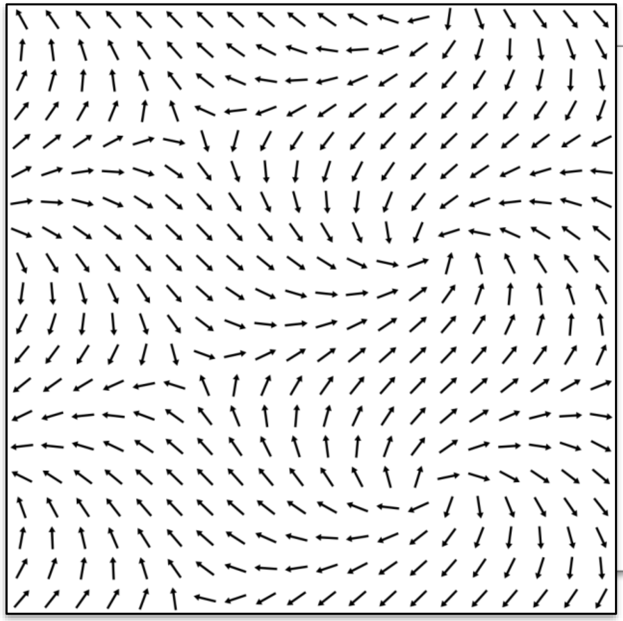
Example:

$\mathbf{v}(\mathbf{x},t) = (1-t)$  $+\ t$ 





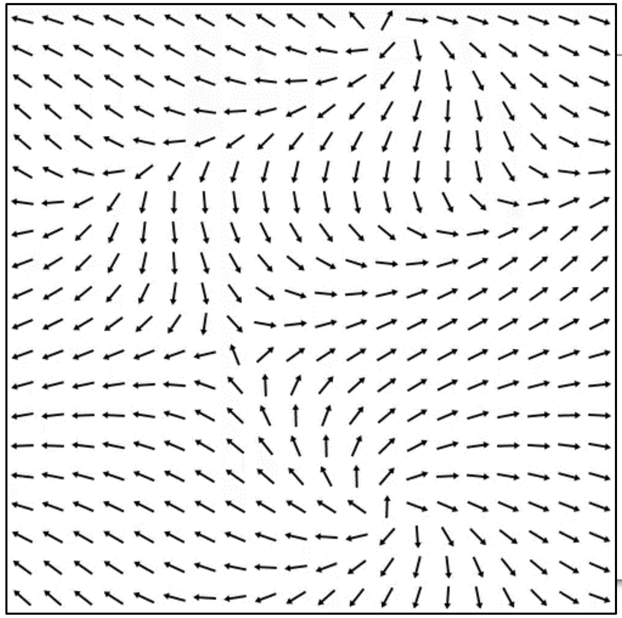
$\mathbf{v}: \mathrm{E}^n \rightarrow \mathbb{R}^n$



Steady Vector Fields

- stream, path, and streak lines coincide

$\mathbf{v}: \mathrm{E}^{n+1} \rightarrow \mathbb{R}^n$



Unsteady Vector Fields

- stream, path, and streak lines are different

very important

Properties of Characteristic Curves

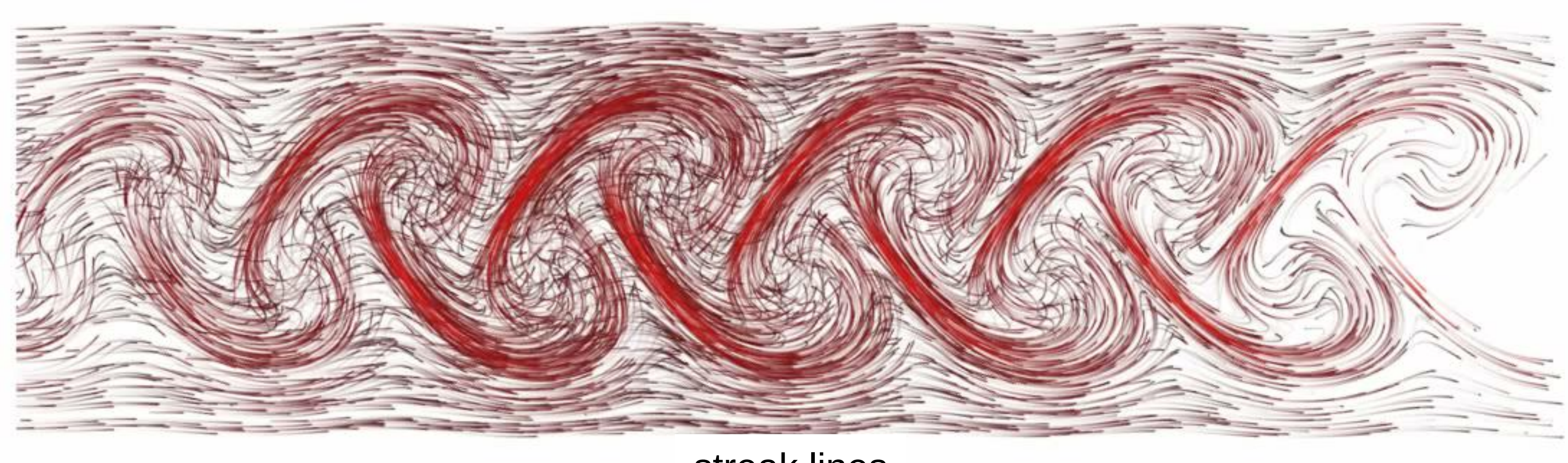
- **Stream and Path lines:**
  - Through all non-critical points (**x**,t) in space-time there is exactly one stream/path line passing through it.
- **Path lines:**
  - Path lines intersect in space.
- **Streak lines:**
  - Many streak lines through every spatio-temporal point.
- **Time lines:**
  - Depends on seeding. If seeding lines do not intersect, then time lines do not intersect either.
- (see tabular summary on the next page, same content)

very important

Properties of Characteristic Curves

	do not intersect each other		one and only one curve through a point in the domain		integration
	space	space-time	space	space-time	
stream line	✓	✓	✓	✓	line
path line	✗	✓	✗ (>1)	✓	line
streak line	✗	✗	✗ (>1)	✗ (>1)	surface **
time line	✓ *	✓ *	✓ *	✓ *	surface **

* If seeding lines do not intersect, then time lines do not intersect either.
** Line-type integration requires derived vector field, which incurs high computational costs and only pays off when many curves must be integrated.

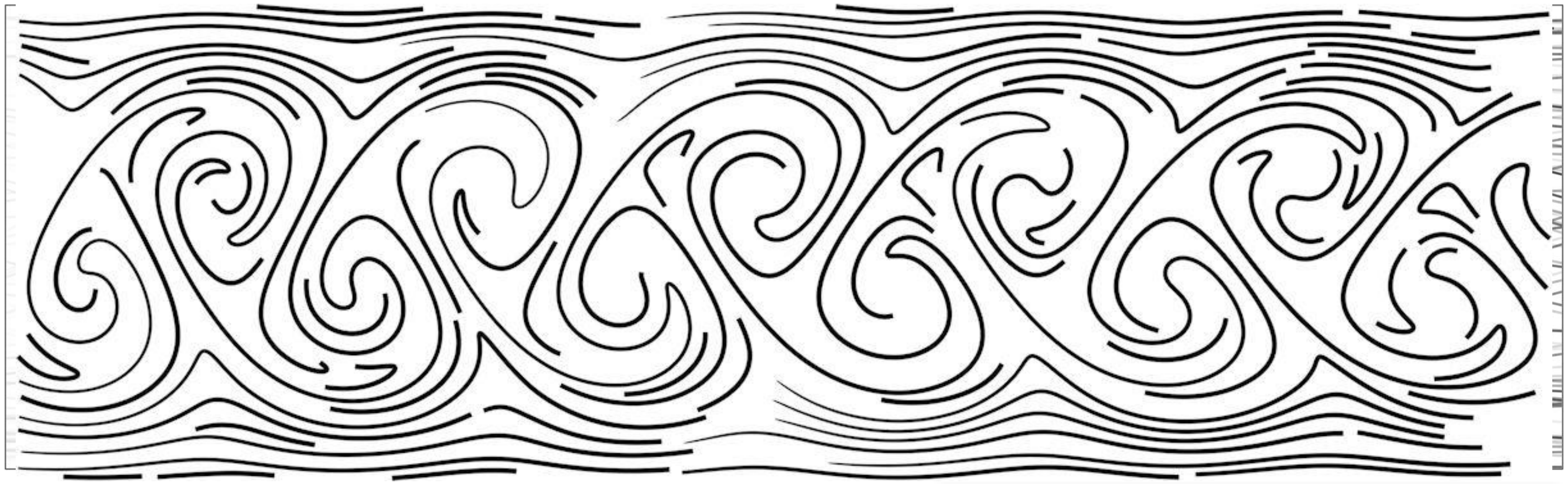


streak lines

2D time-dependent flow behind a cylinder

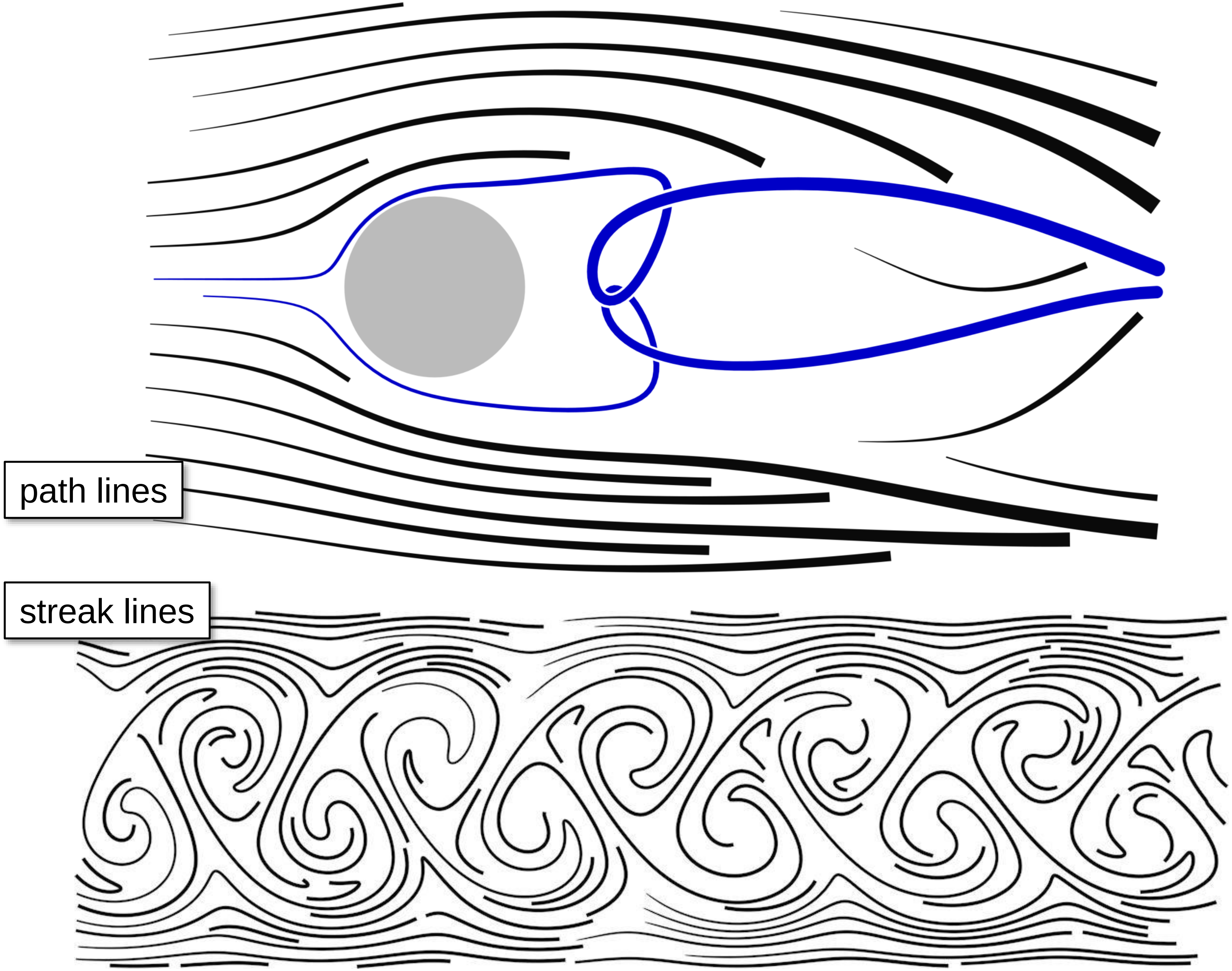
streak lines without intersection

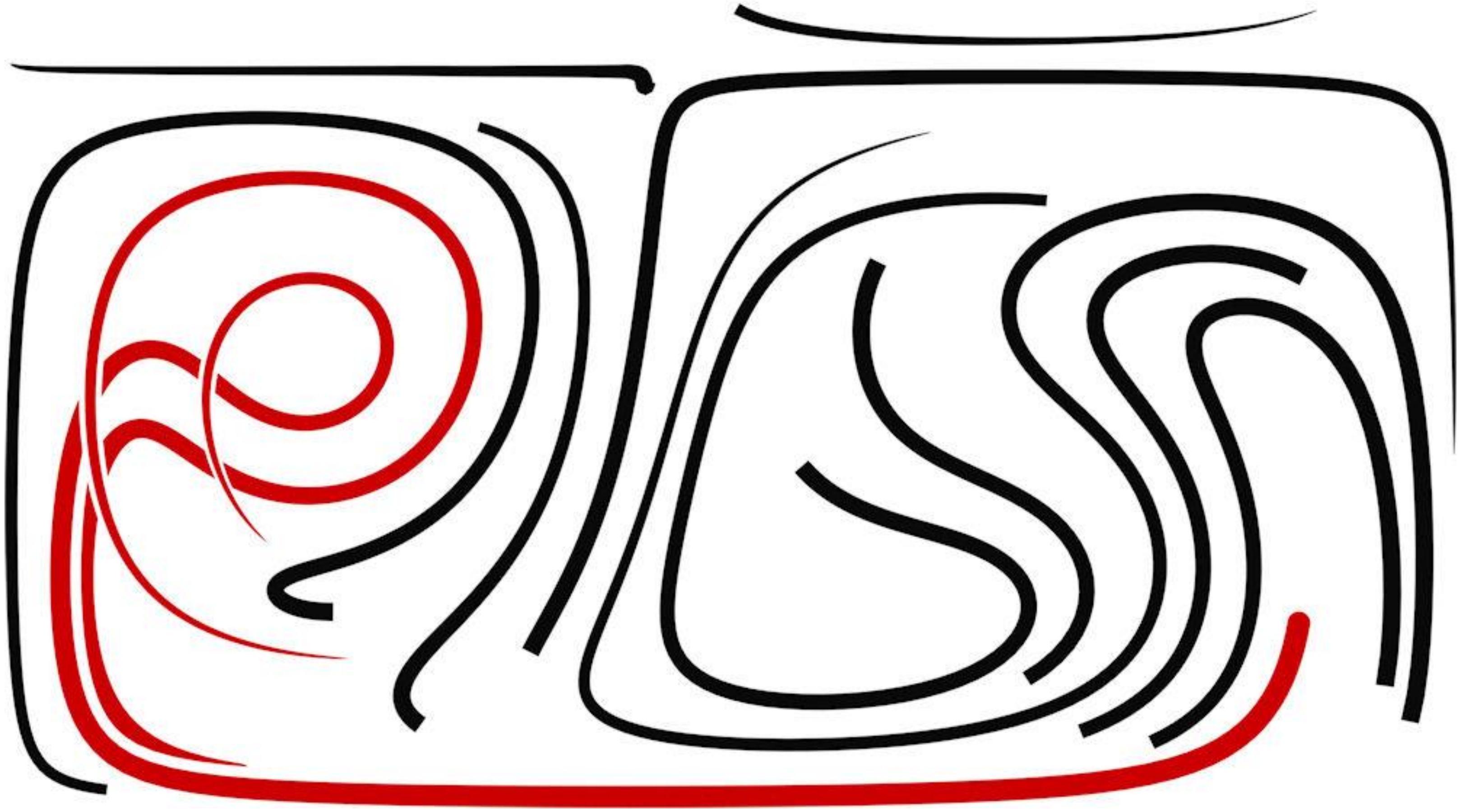
path lines



Weinkauf, Theisel, Sorkine, TopoInVis 2011

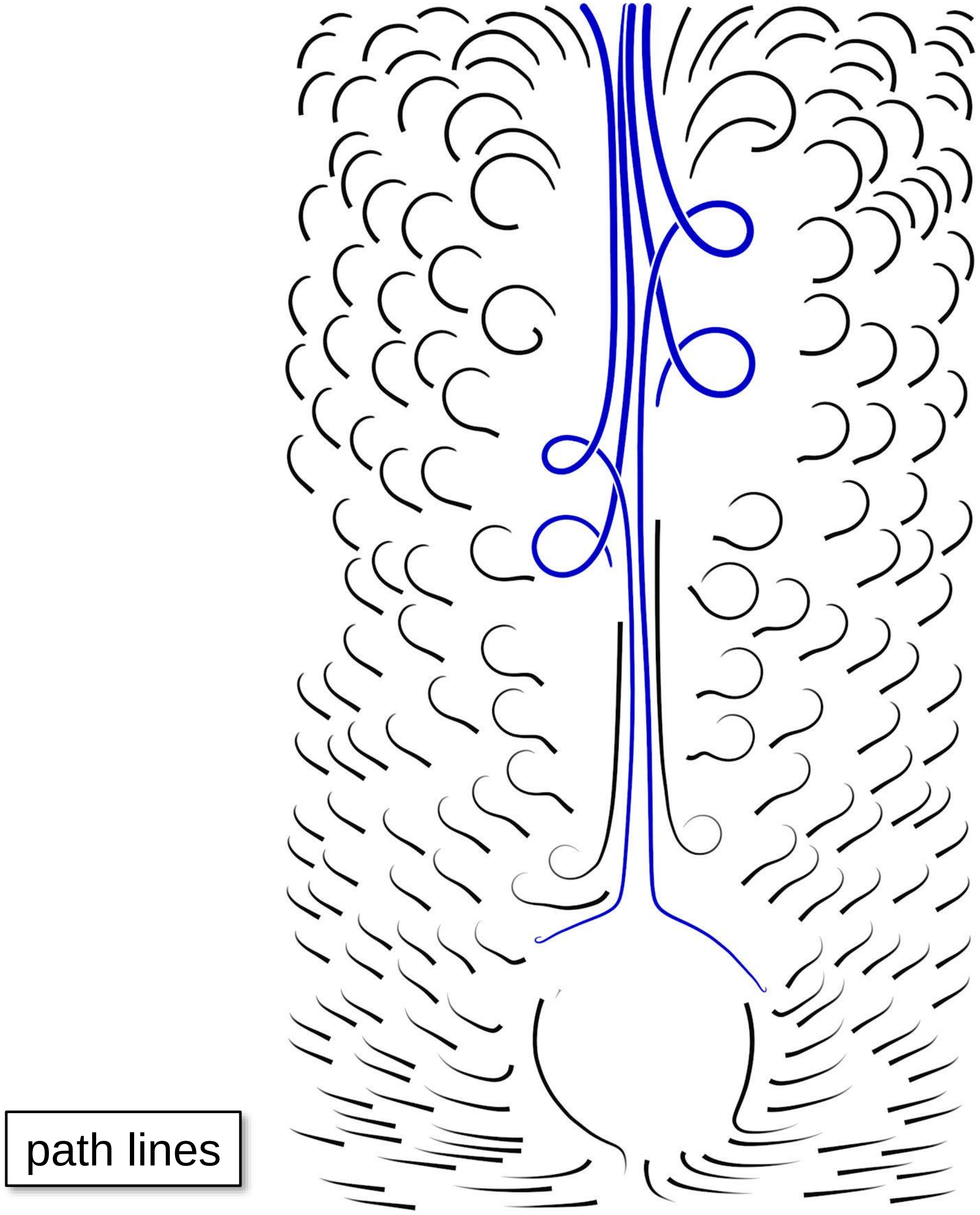
Flow behind a cylinder

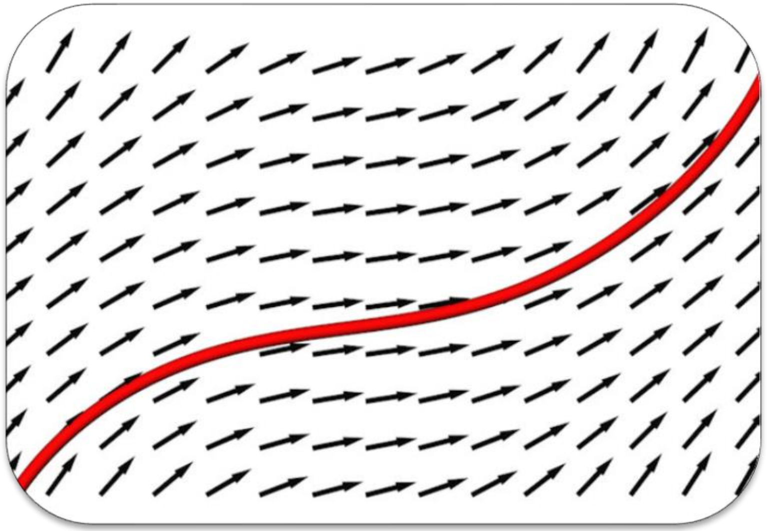




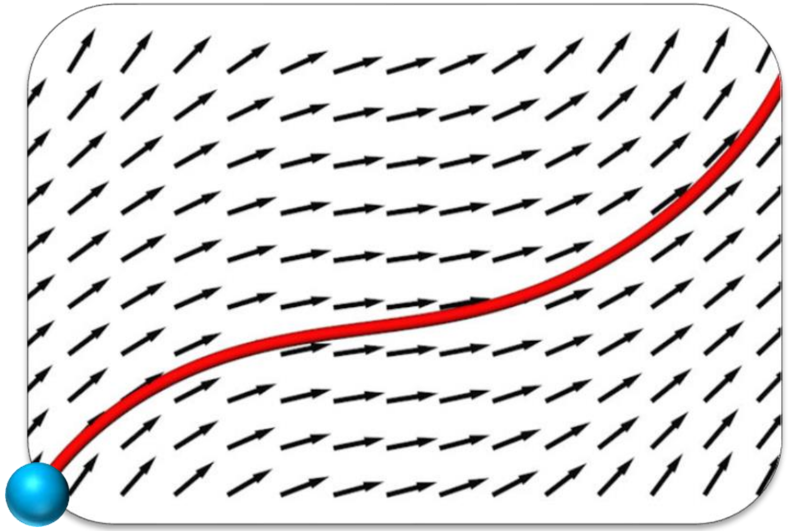
streak lines

Heated cylinder

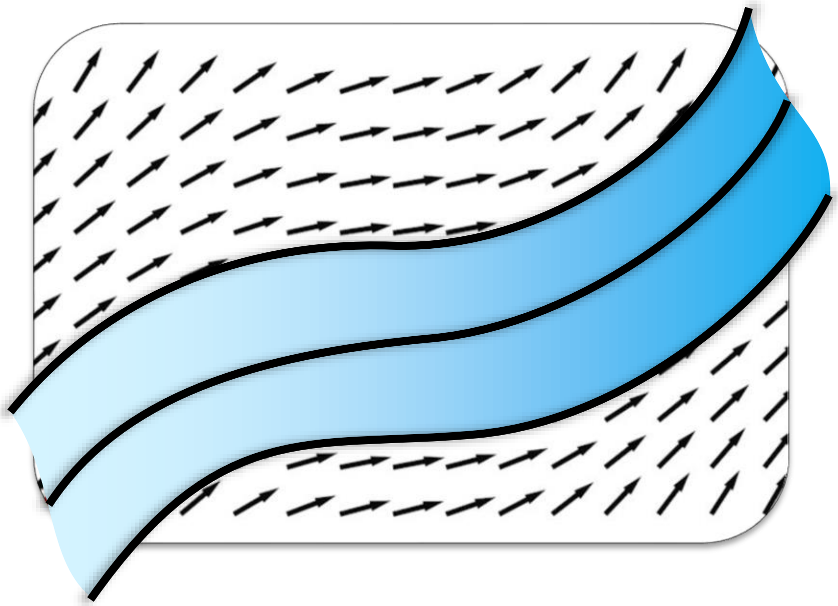




characteristic curves
integral curves
integral lines



particles



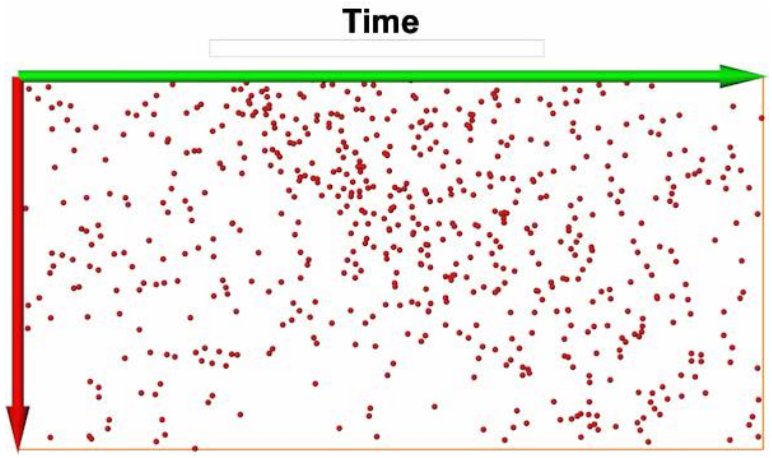
integral
surfaces

Particle Tracer

similar to steady case

integrate path lines

only show current integration step as a circle / sphere



particle visualization from the beginning of the lecture

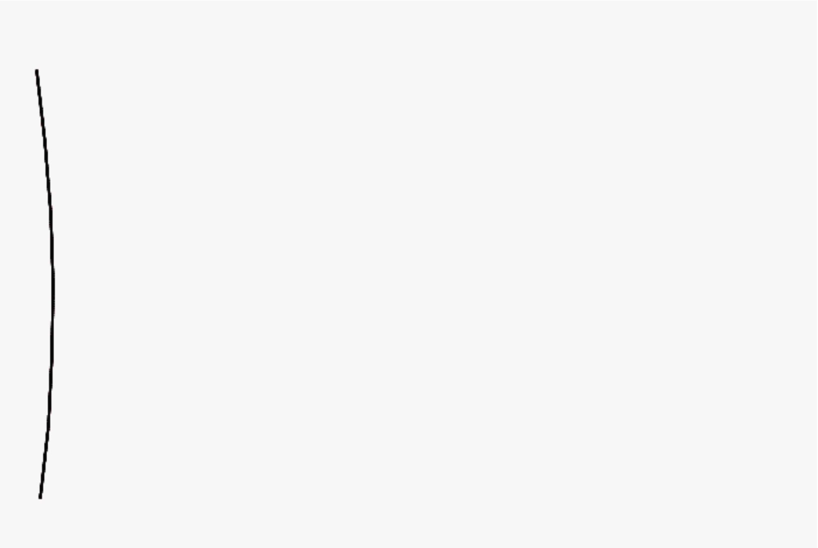
flow behind a cylinder developing the *von Kármán vortex street*

very important

steady vector field

$$\mathbf{v}(\mathbf{x})$$

stream surface



General stream surfaces
[Hultquist, Vis 1992]

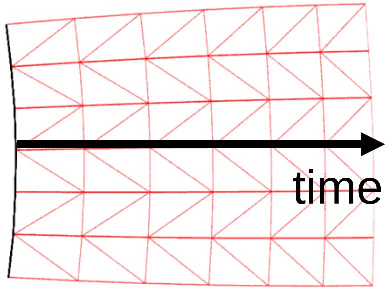
Topological considerations
[Stalling, Phd thesis, 1998]

High accuracy
[Garth et al., Vis 2008]
[Schulze et al., SGP 2012]

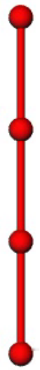
unsteady vector field

$$\mathbf{v}(\mathbf{x}, t)$$

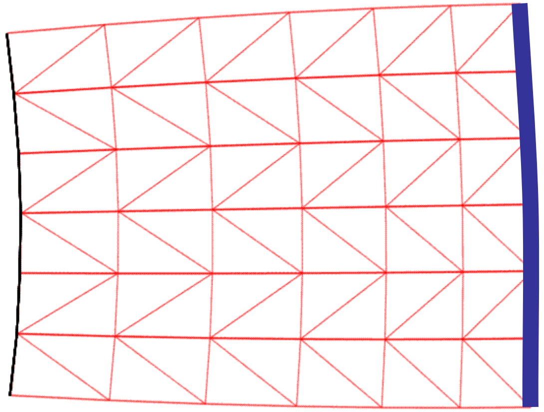
path surface



streak surface



very important



Adaptive insertion/removal of points at **front line**

- Euclidean distance
- Angle
- …

stream & path surfaces

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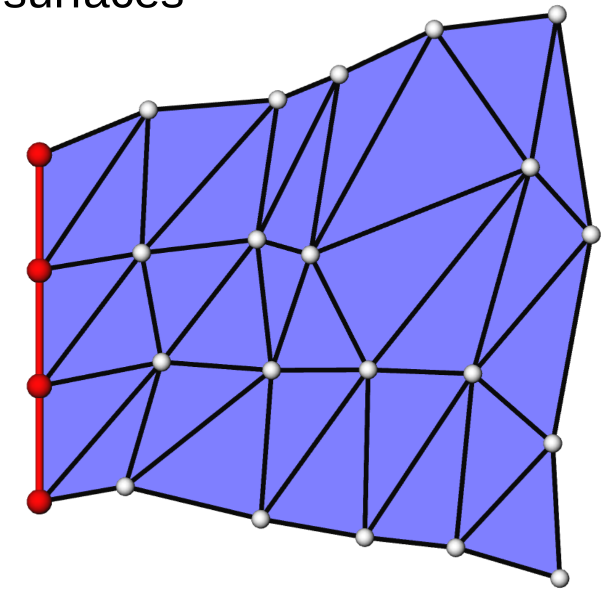
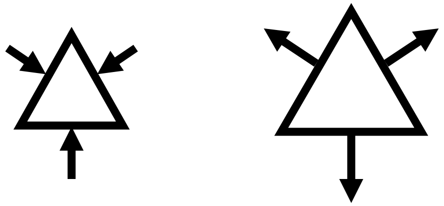
streak & time surfaces





Complete remeshing in every time step is necessary to stay within resolution constraints video from Krishnan, Garth, Joy, Time and Streak Surfaces for Flow Visualization in Large Time-Varying Data Sets, IEEE Vis 2009

very important

- Integrating a stream/path/streak/time line/surface means to look at the following subspaces of a 3D unsteady flow (which itself is 4D):

	…line	**…surface**
Stream…	1D (line)	2D (surface)
Path…	1D (line)	2D (surface)
Streak…	2D (surface)	3D (volume)
Time…	2D (surface)	3D (volume)

smoke abstractions

artistic photographs from W. Brennan

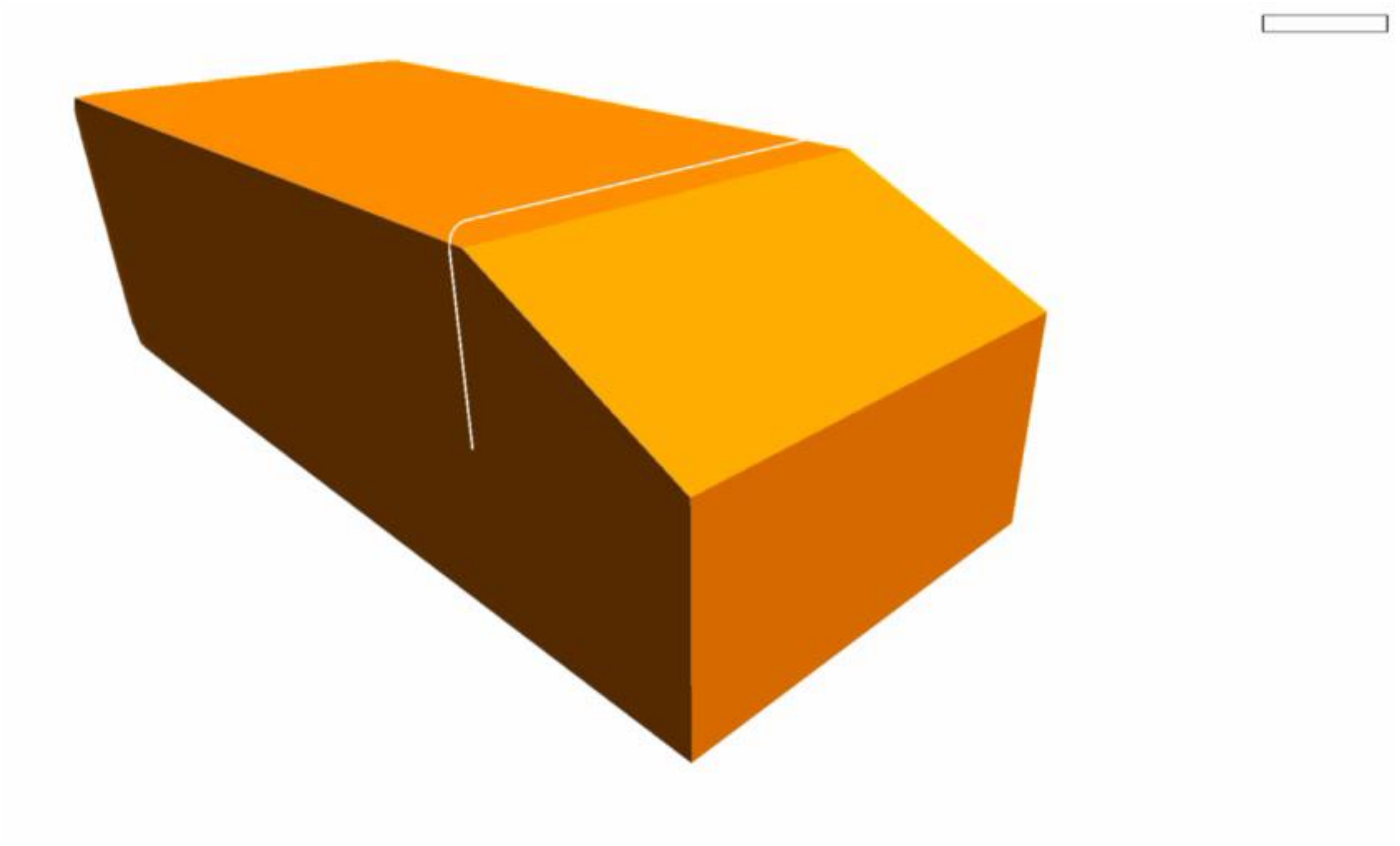
streak surface behind Ahmed body

rendered with smoke appearance

von Funck, Weinkauf, Theisel, Seidel
Smoke Surfaces: An Interactive Flow Visualization Technique Inspired by Real-World Flow Experiments
IEEE Vis 2008

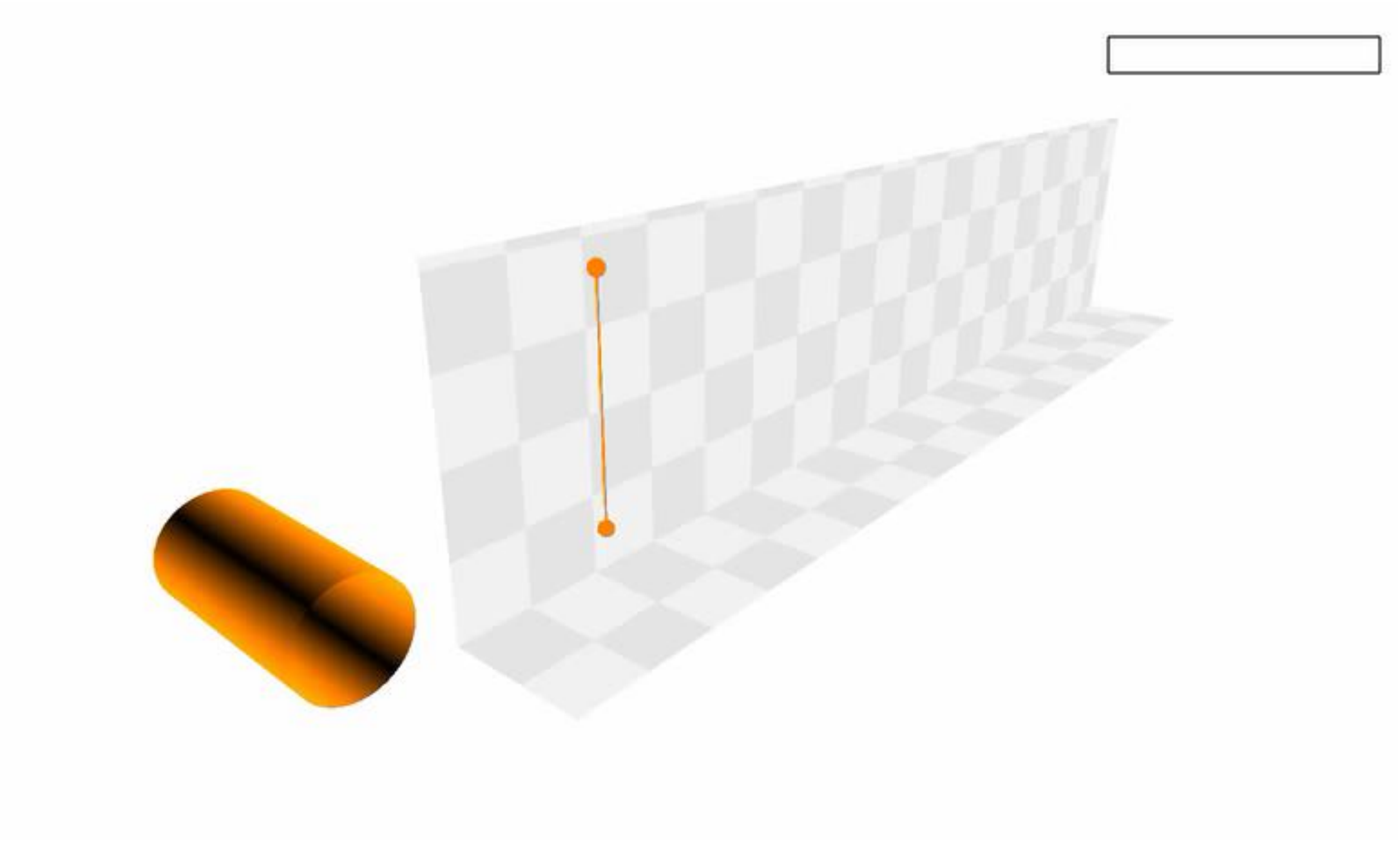
data from Erik Wassen, TU Berlin

Streak Surface around Ahmed Body



video from von Funck, Weinkauf, Theisel, Seidel, Smoke Surfaces, IEEE Vis 2008; data from Erik Wassen, TU Berlin

Interactive Seeding and Streak/Time Surfaces behind Cylinder



video from von Funck, Weinkauf, Theisel, Seidel, Smoke Surfaces, IEEE Vis 2008; data from Bernd Noack, TU Berlin

IRIS:
Illustrative Rendering of Integral Surfaces

Mathias Hummel, Christoph Garth,
Hans Hagen, Bernd Hamann, Kenneth I. Joy

Technische Universität Kaiserslautern

IDAV
UC Davis

Summary

- Integration in unsteady vector fields
  - autonomous ODE
- Characteristic Curves
  - stream lines
  - path lines
  - streak lines
  - time lines
- Integral Surfaces
  - adaptive resolution
  - rendering