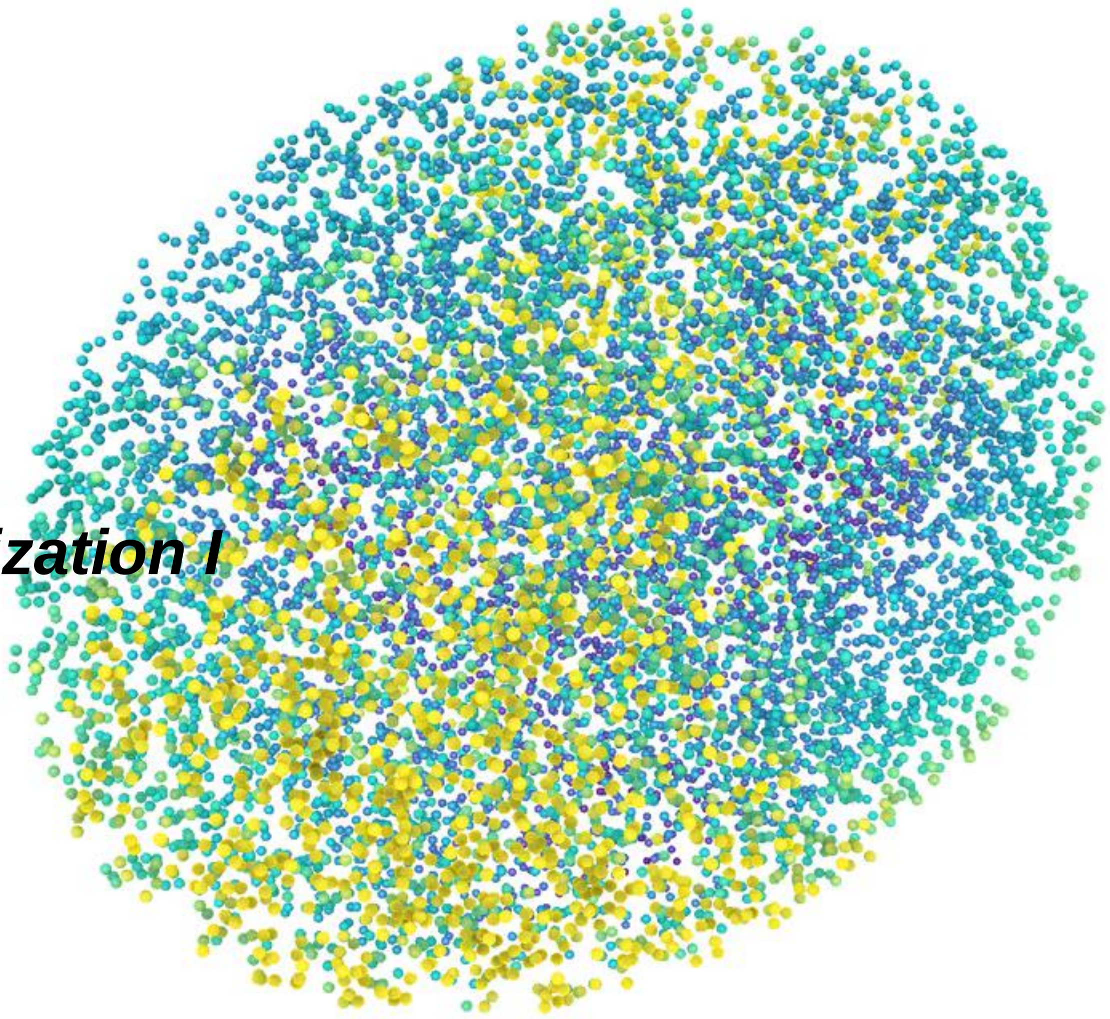
Visualization, DD2257

Prof. Dr. Tino Weinkauf

Geometry-based Vector Field Visualization I

steady vector fields

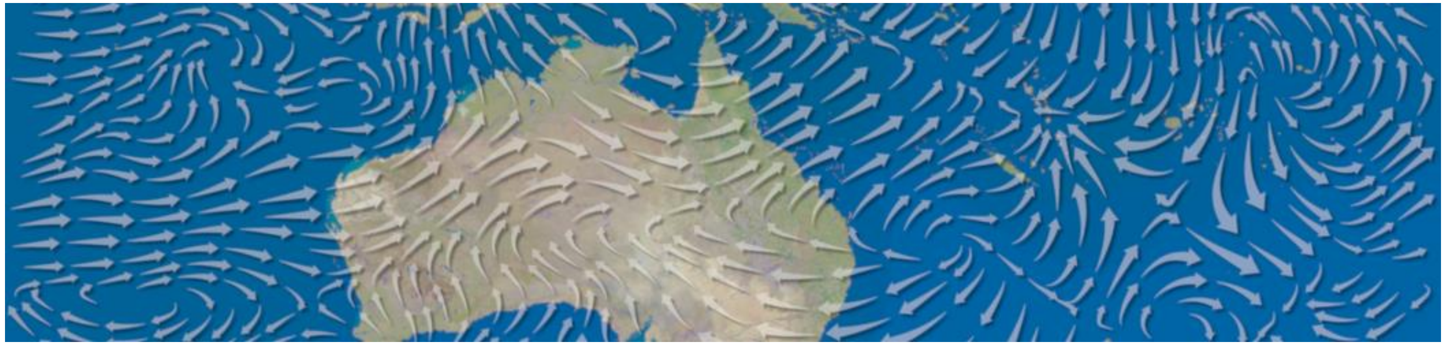


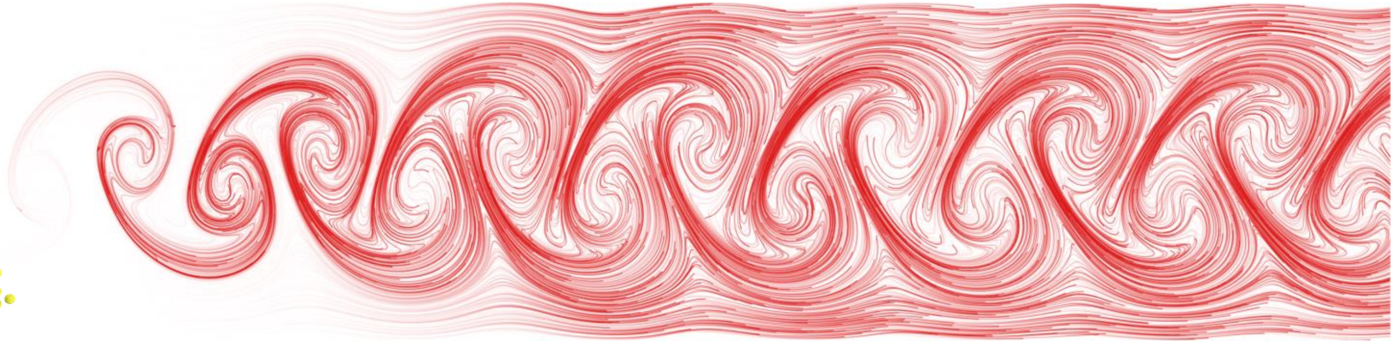
Geometry-based Vector Field Visualization

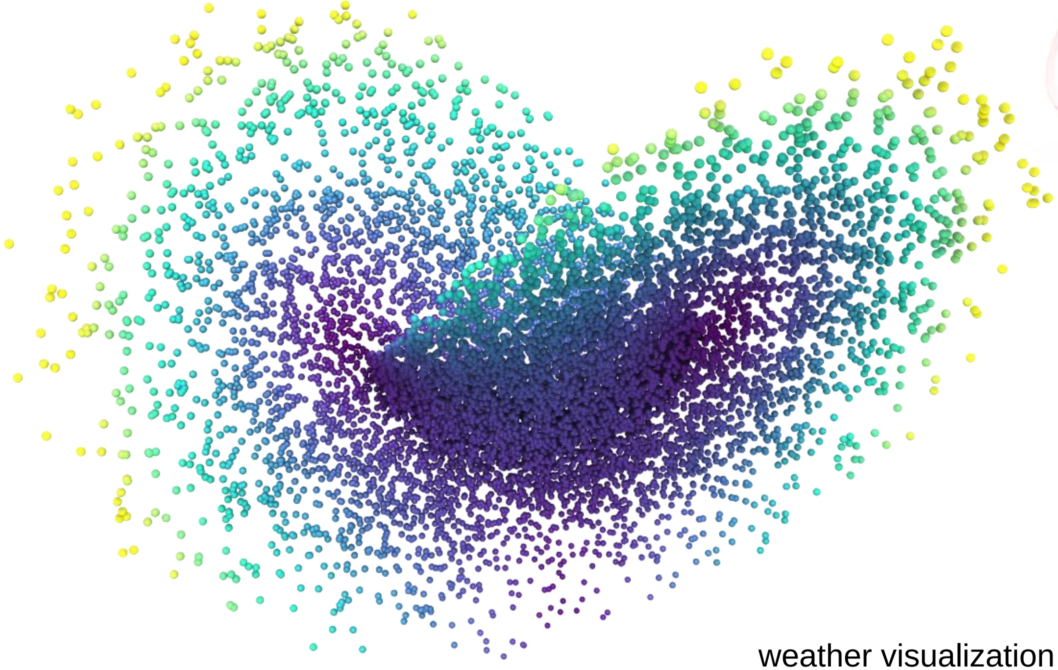
geometric objects

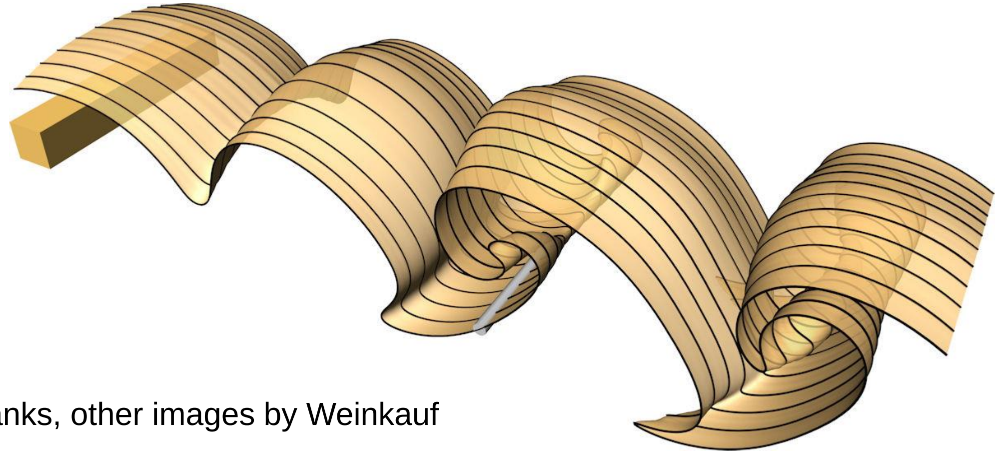
shape is directly related to the flow

points, lines, surfaces, …



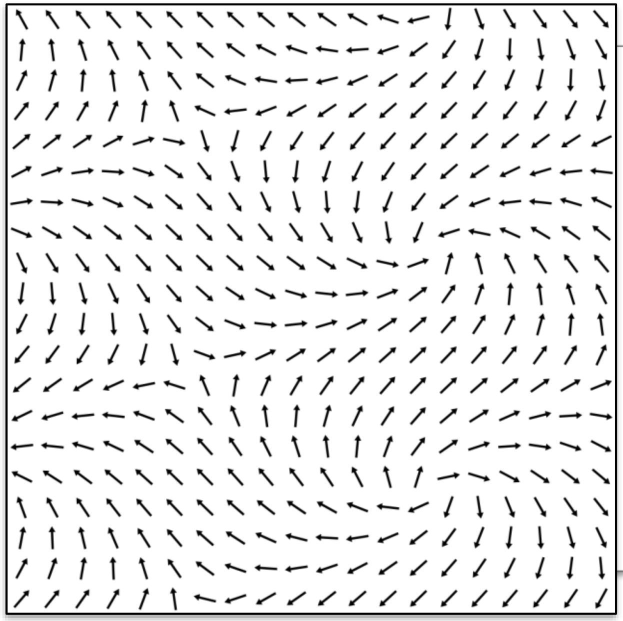






weather visualization by Turk & Banks, other images by Weinkauf

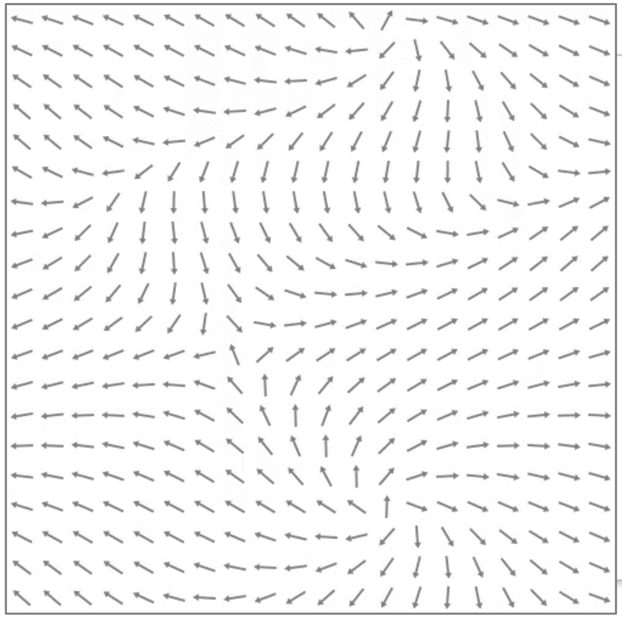
$\mathbf{v}\colon \mathrm{E}^n \to \mathbb{R}^n$



Steady Vector Fields

- tangent curves / stream lines
- stream surfaces

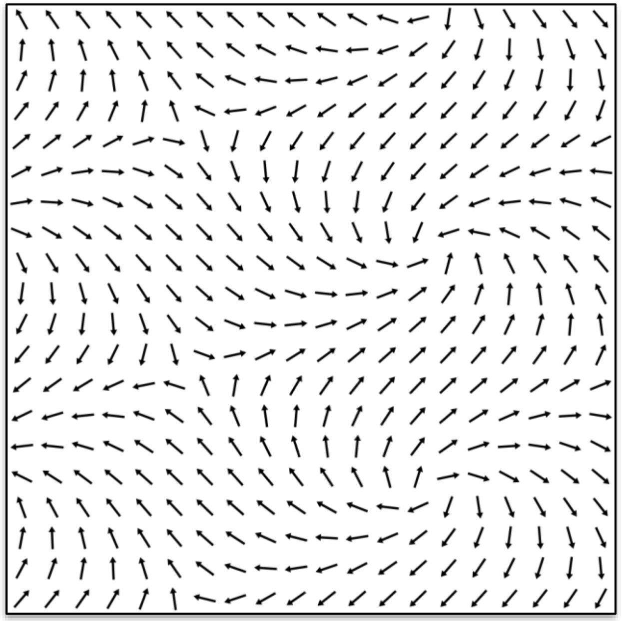
$\mathbf{v}\colon \mathrm{E}^{n+1} \to \mathbb{R}^n$



Unsteady Vector Fields

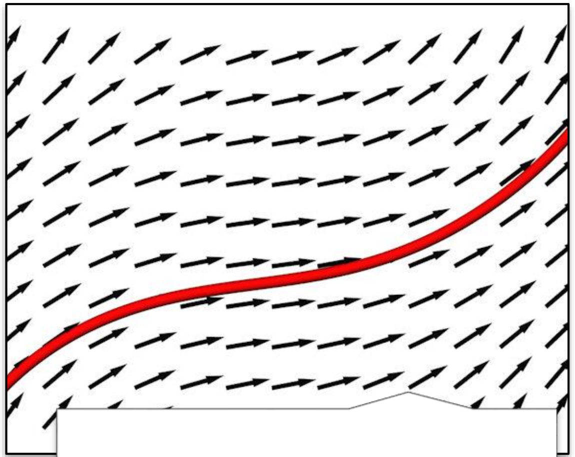
- path/streak/time lines
- path/streak/time surfaces

$\mathbf{v}\colon \mathrm{E}^n \to \mathbb{R}^n$

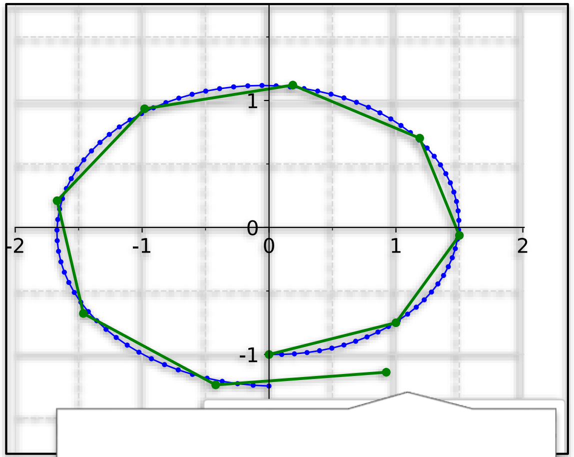


Steady Vector Fields

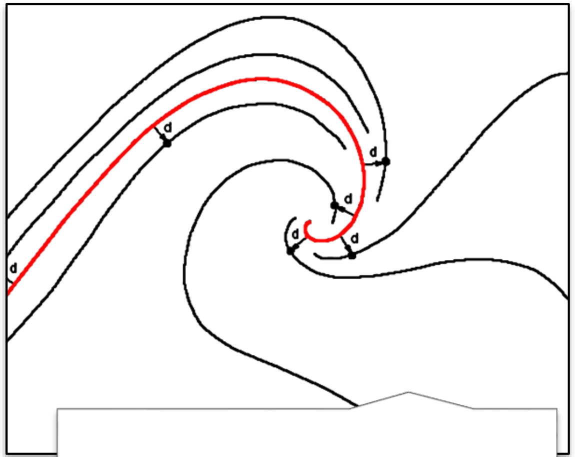
- tangent curves / stream lines
- stream surfaces



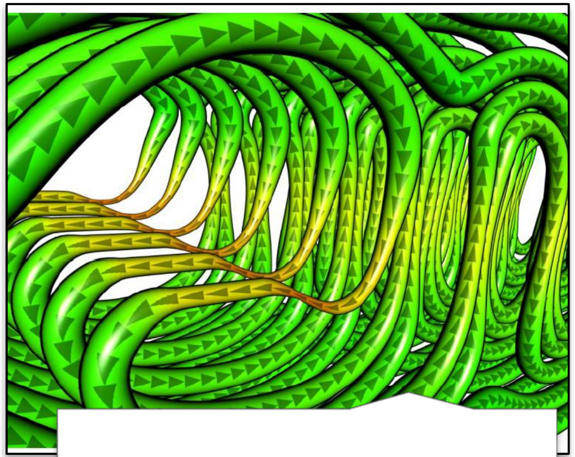
Definition



Integration



Seeding



3D impression

very important

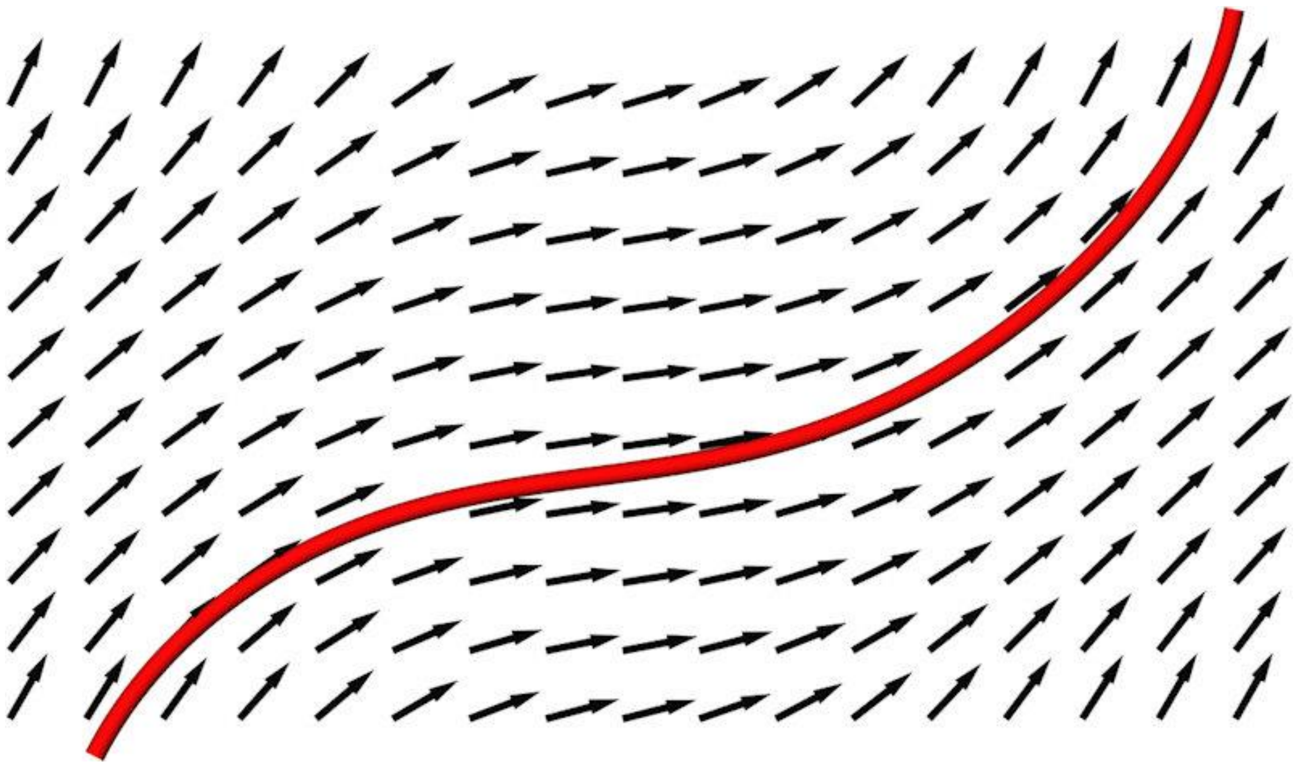
Tangent Curves

A **tangent curve** $\mathbf{x}(\tau)$ of the vector field $\mathbf{v}$ is a curve in $E^{2/3}$ with the property:

$$\frac{d}{d\tau}\mathbf{x}(\tau) = \dot{\mathbf{x}}(\tau) = \mathbf{v}(\mathbf{x}(\tau))$$

tangent vectors of $\mathbf{x}(\tau)$

Interpretation: path of a massless particle in a flow described by $\mathbf{v}$



very important

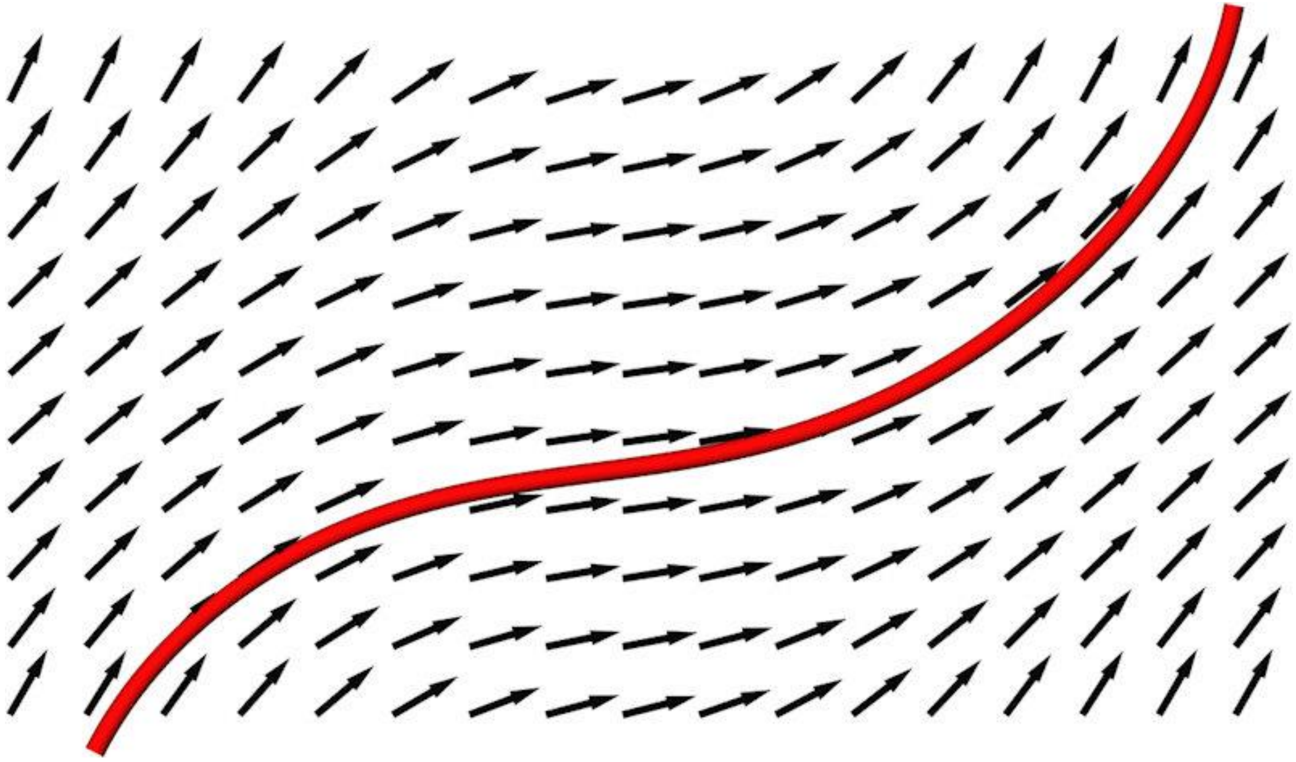
Properties of Tangent Curves

Tangent curves do not intersect each other

Given a point in the vector field **v**, there is one and only one tangent curve through it

A parametric description of tangent curves is usually not possible.

but it is possible for linear vector fields.

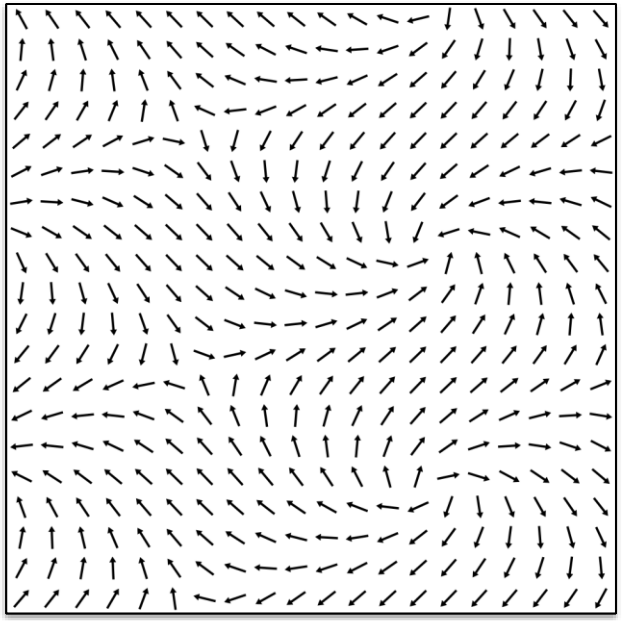


Tangent Curve versus Stream Line

Terminology:

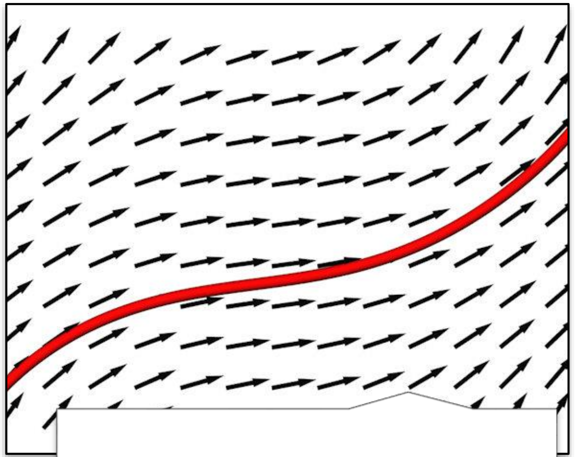
- ***stream lines*** is an alternative name for tangent curves
- often used
- comes from fluid dynamics and has specific meaning there

$\mathbf{v}\colon \mathrm{E}^n \to \mathbb{R}^n$

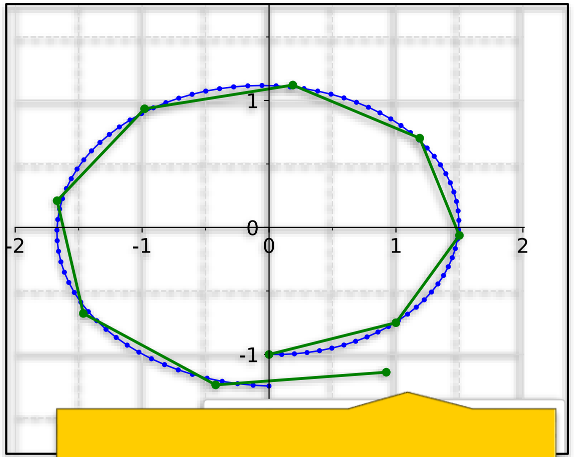


Steady Vector Fields

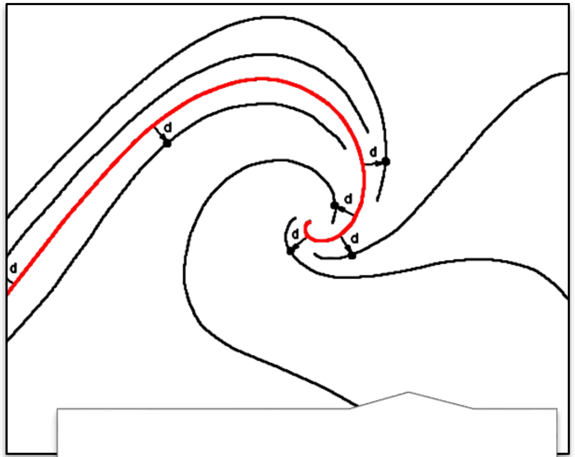
- tangent curves / stream lines
- stream surfaces



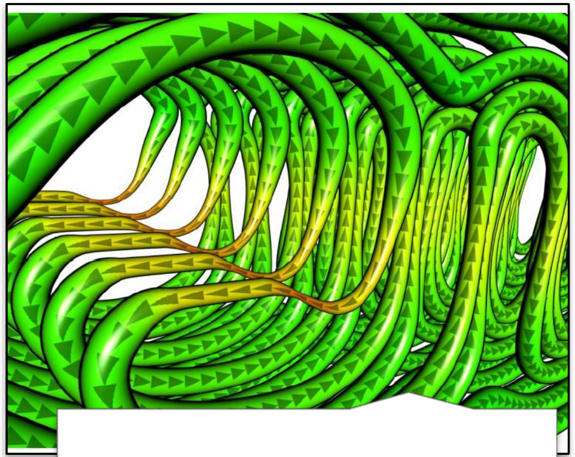
Definition



Integration



Seeding



3D impression

Computation of Tangent Curves

- numerical integration
- no closed form solution
- Exceptions:
  - linear vector fields (2D/3D): tangent curves can be formulated as parametric exponential curves
  - 2D quadratic Bezier curves (parabolas) are a subclass of tangent curves of 2D linear vector fields
  - 3D non-planar cubic curves are a subclass of tangent curves of 3D linear vector fields

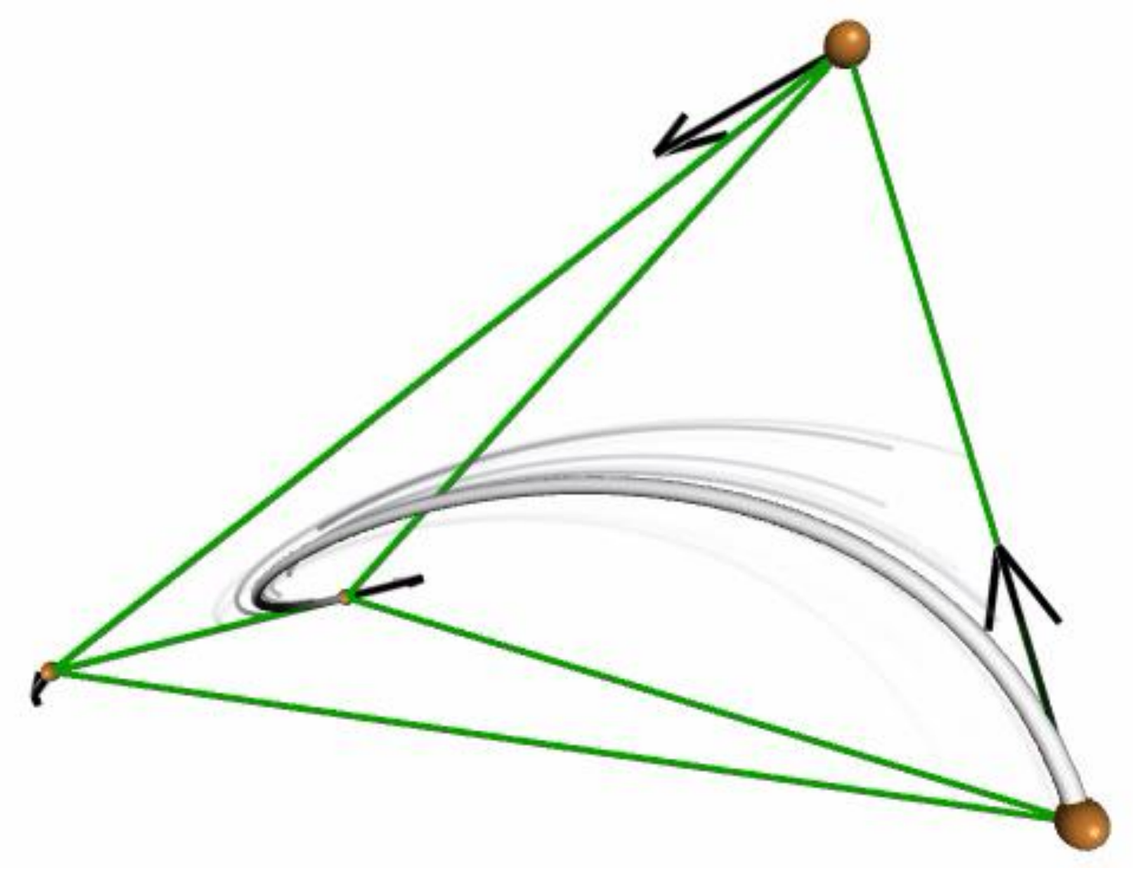


image from Weinkauf & Theisel, Topological Construction and Visualization of Higher Order 3D Vector Fields Eurographics 2004

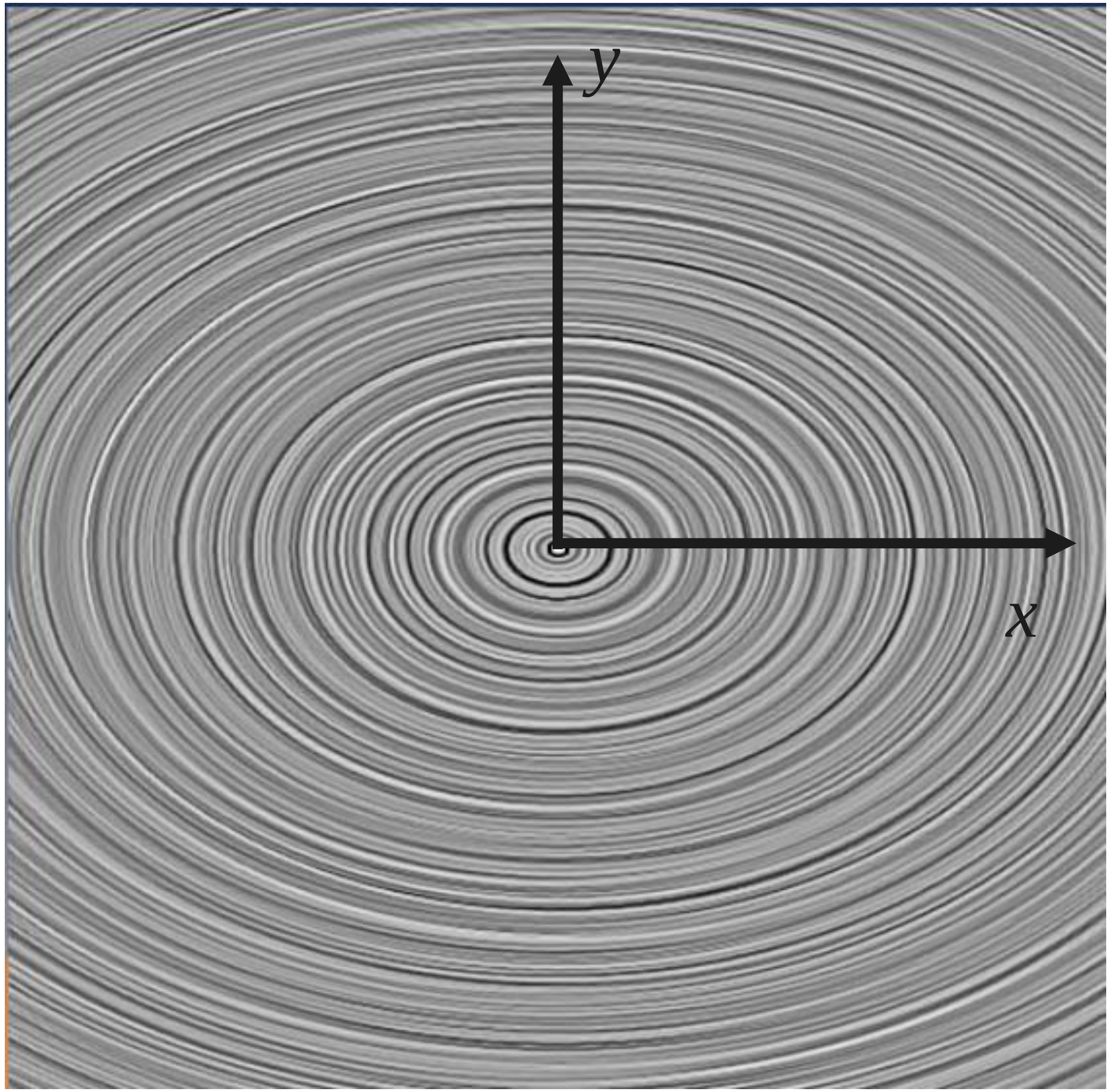
Numerical Integration of Tangent Curves

approximate tangent curve
using polyline $\mathbf{x}_i$

test case:

$$\mathbf{v}(x, y) = \begin{pmatrix} -y \\ x/2 \end{pmatrix}$$

exact solution: ellipses
starting integration from $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$



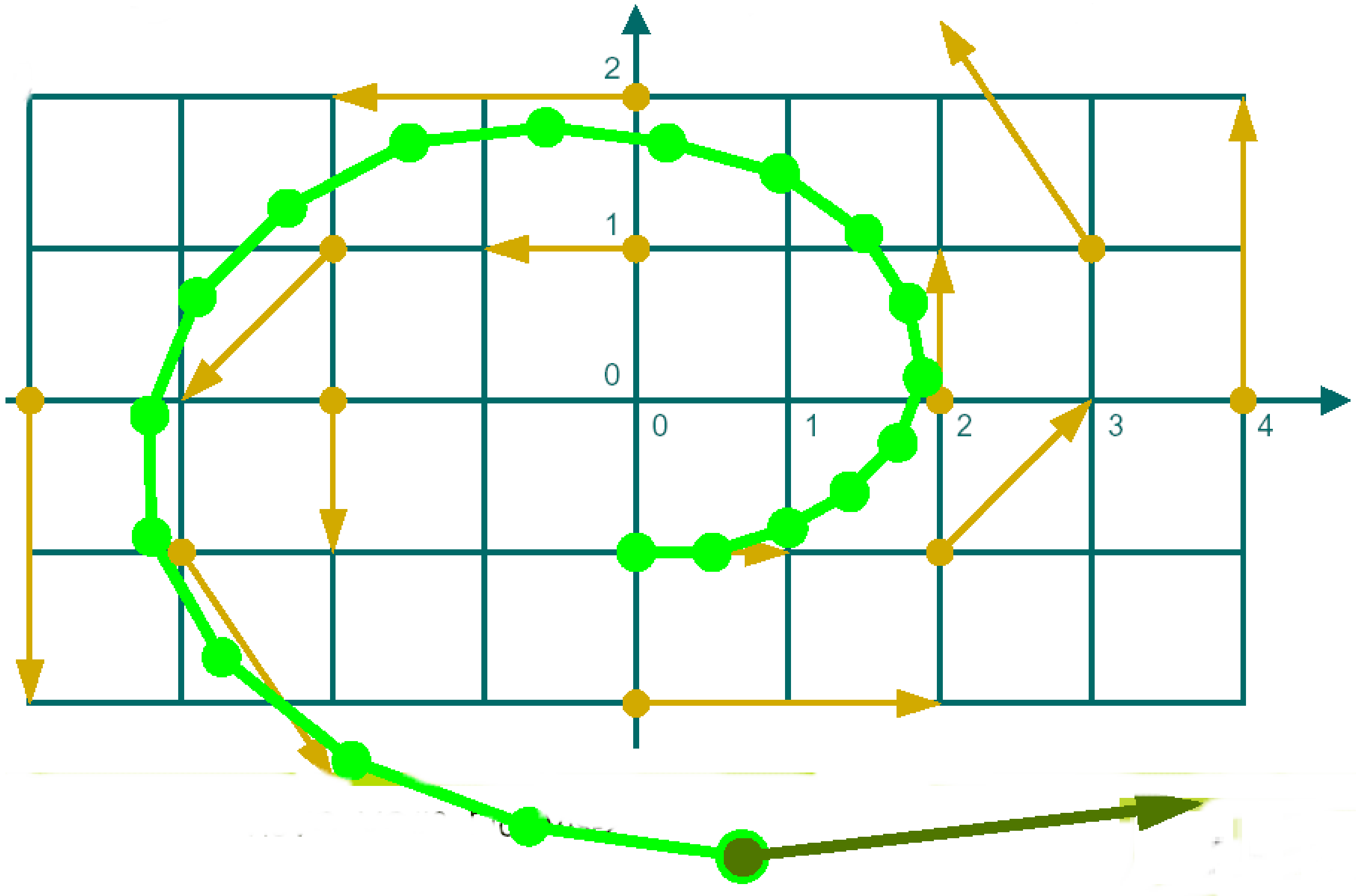
Euler Integration of Tangent Curves

$$\mathbf{x}_{i+1} = \mathbf{x}_i + s\mathbf{v}(\mathbf{x}_i)$$

s: step size

large numerical error

error proportional to s^2

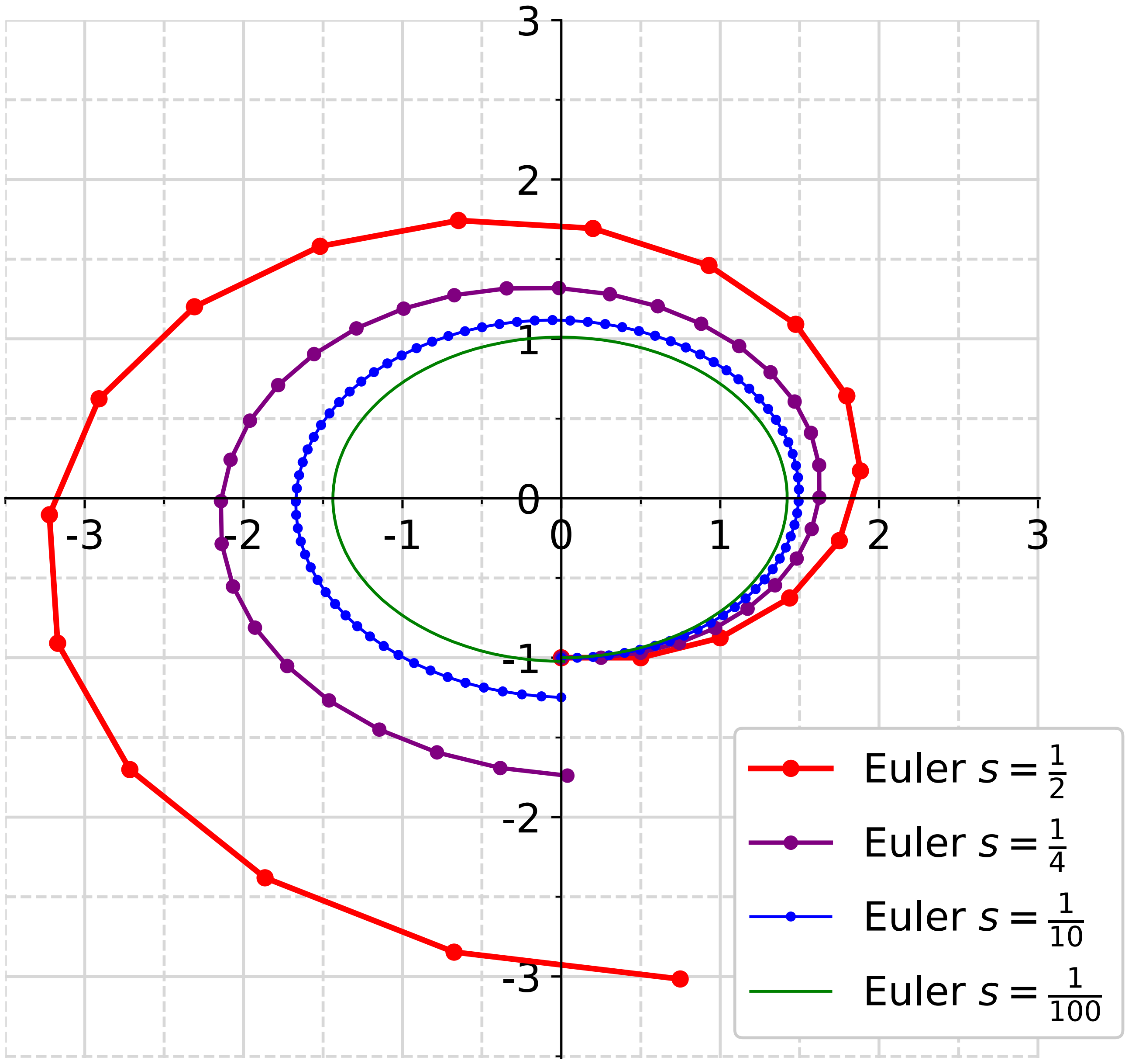


Euler Integration

low accuracy

needs many tiny steps to achieve acceptable results

In the example, the green curve requires 890 steps of step size 1/100 to come somewhat close to the starting point.



Second order Runge Kutta integration (RK2)

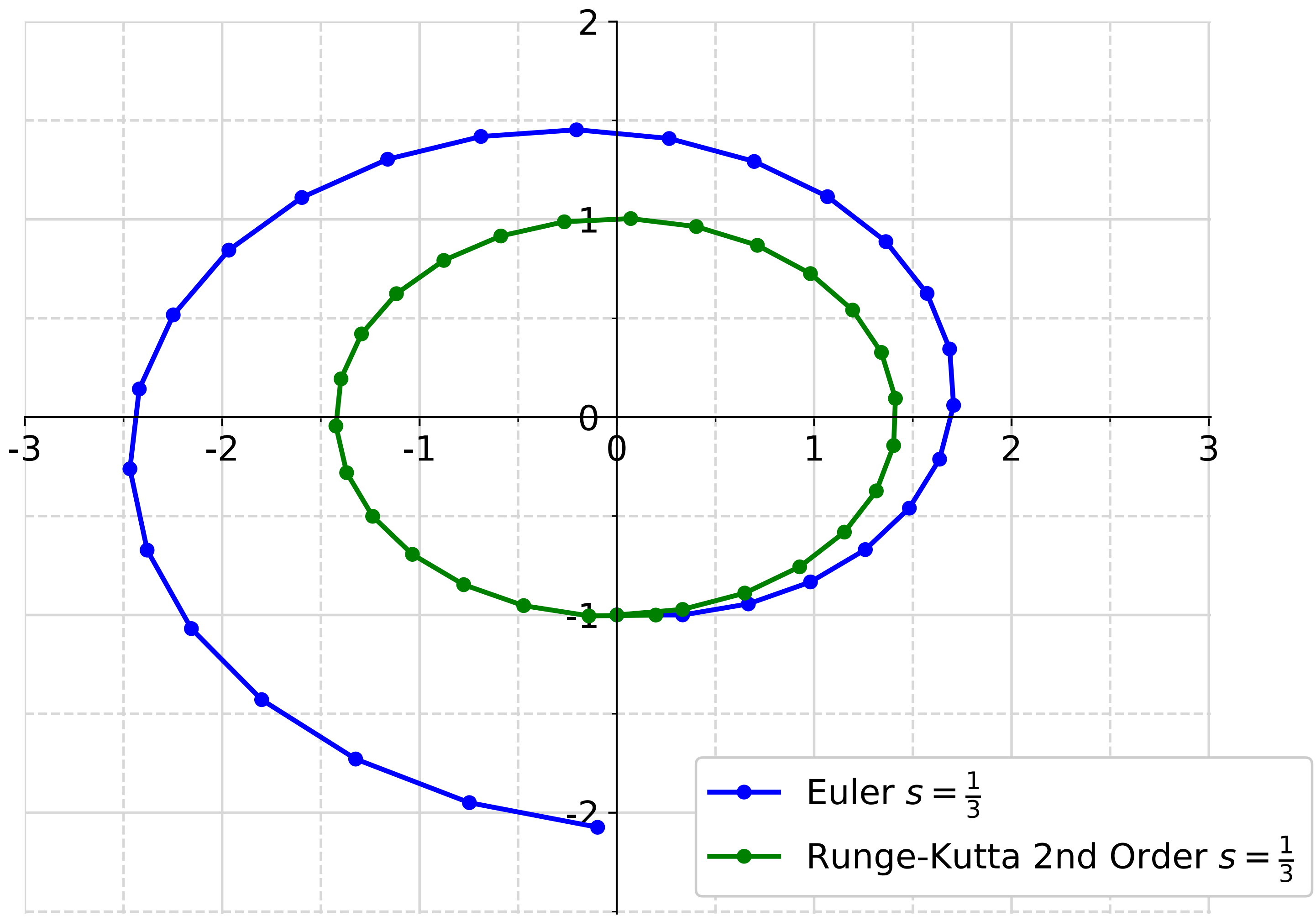
$$\mathbf{x}_{i+1} = \mathbf{x}_i + s\mathbf{v}(\mathbf{x}_i + \frac{s}{2}\mathbf{v}(\mathbf{x}_i))$$

procedure:

- go half a step forward
- evaluate vector there
- use at starting point

also called RK2

better than Euler integration



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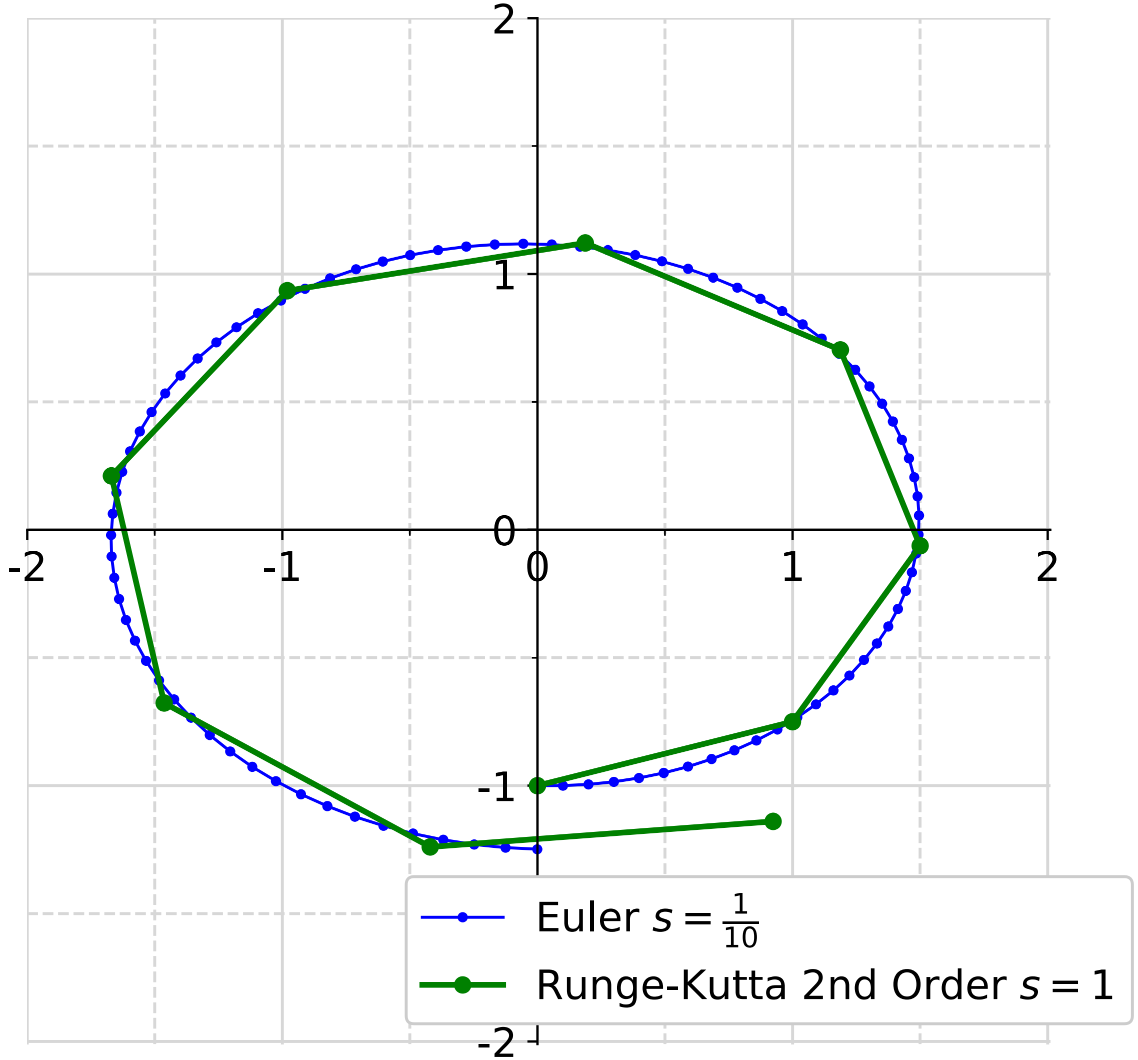


also called RK2

better than Euler integration

Fourth order Runge Kutta integration (RK4)

a step is a convex combination of 4 vectors

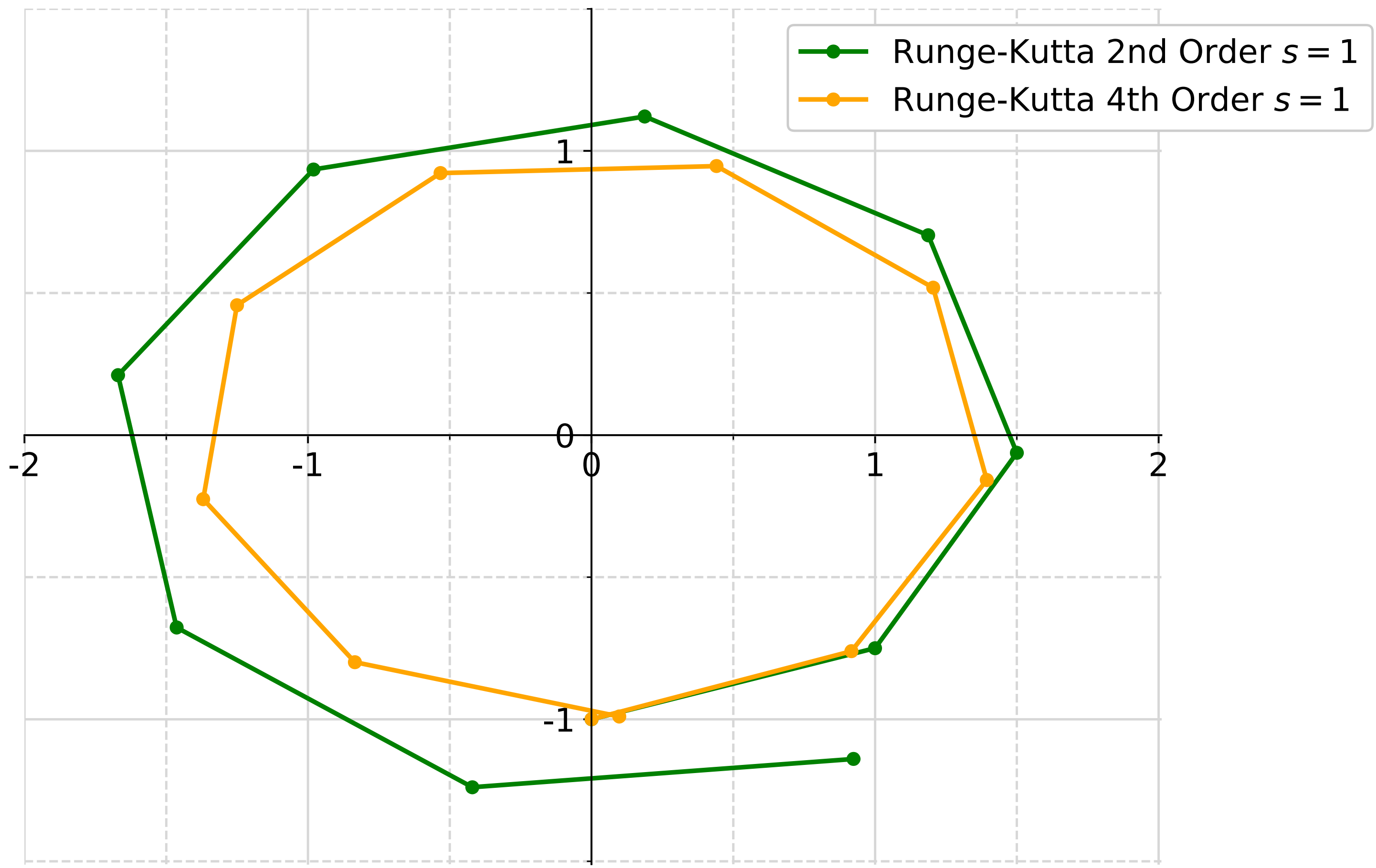
better than RK2

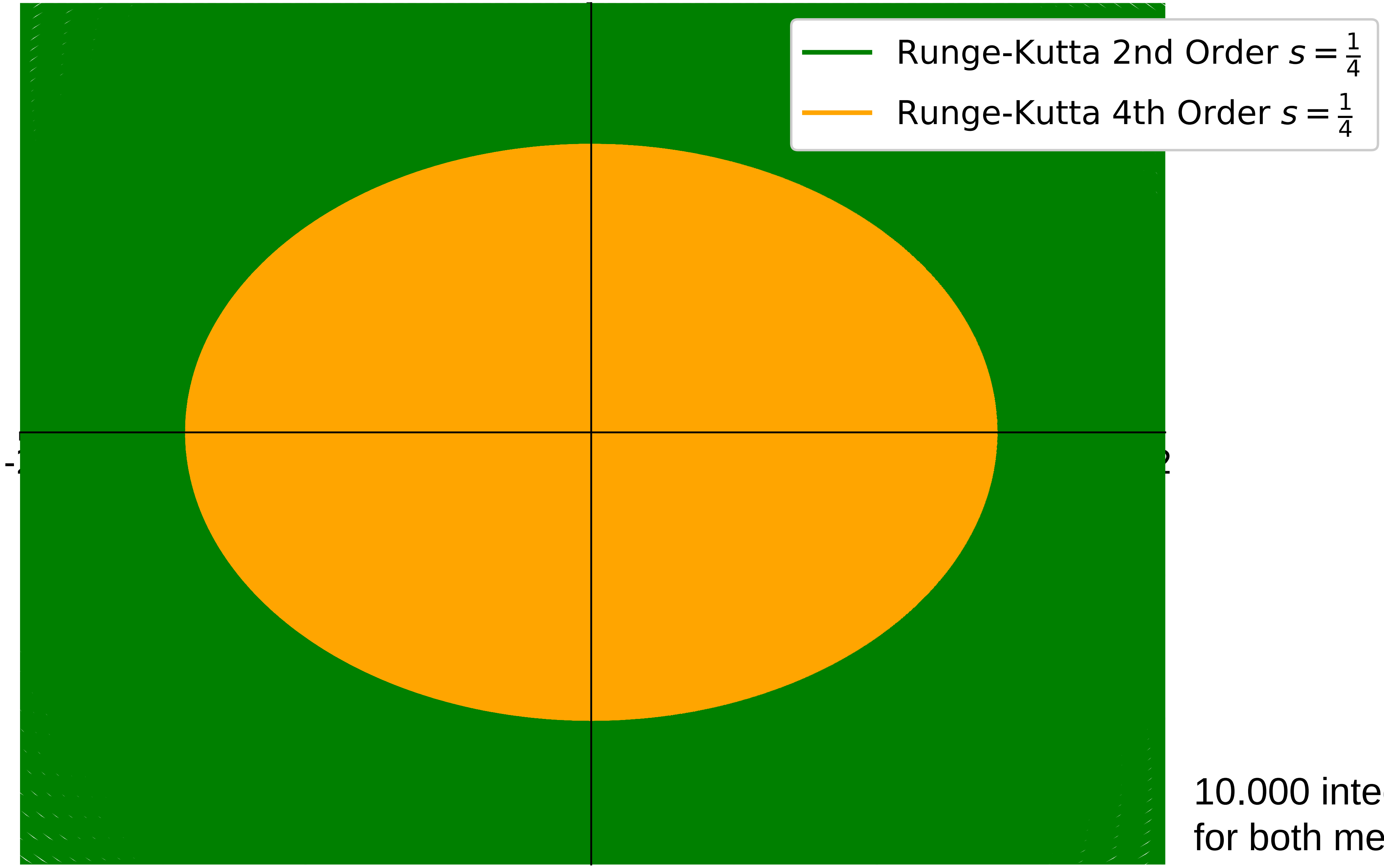
error proportional to s^5

$$\mathbf{x}_{i+1} = \mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_i) = \mathbf{x}_i + s \cdot \left(\frac{\mathbf{v}_1}{6} + \frac{\mathbf{v}_2}{3} + \frac{\mathbf{v}_3}{3} + \frac{\mathbf{v}_4}{6} \right)$$

with $\mathbf{v}_1 = \mathbf{v}(\mathbf{x}_i) \quad , \quad \mathbf{v}_2 = \mathbf{v}\left(\mathbf{x}_i + \frac{s}{2} \mathbf{v}_1 \right)$

$$\mathbf{v}_3 = \mathbf{v}\left(\mathbf{x}_i + \frac{s}{2} \mathbf{v}_2 \right) \quad , \quad \mathbf{v}_4 = \mathbf{v}\left(\mathbf{x}_i + s\, \mathbf{v}_3 \right)$$





10.000 integration steps
for both methods

Adaptive Step Size for RK4: RK4(3)

- In areas of rather straight tangent curves, the step size can be larger than in areas of highly curved tangent curves.
- do RK3 ➔ difference between RK4 and RK3 gives estimation for step size

$$\mathbf{f}_{\mathrm{RK3}}(\mathbf{x}_i) = \mathbf{x}_i + s \cdot \left(\frac{\mathbf{v}_1}{6} + \frac{\mathbf{v}_2}{3} + \frac{\mathbf{v}_3}{3} + \frac{\mathbf{v}(\mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_i))}{6} \right)$$

gives $\Delta = \mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_i) - \mathbf{f}_{\mathrm{RK3}}(\mathbf{x}_i) = \frac{\mathrm{s}}{6}(\mathbf{v}_4 - \mathbf{v}(\mathbf{f}_{\mathrm{RK4}}(\mathbf{x}_i)))$

- Define error tolerance t. Then the optimal step size is:

$$s^* = s \cdot \sqrt[5]{\rho \frac{t}{\Delta}}$$

ρ>1: safety factor

- Procedure:
  - ask integrator to compute Δ with step size s
  - If $\Delta > t$, then repeat current step with step size s^*
  - otherwise proceed and take $s = \min(s^*, s_{\max})$ for next step

Choice of Integrator

Depends on data/application!

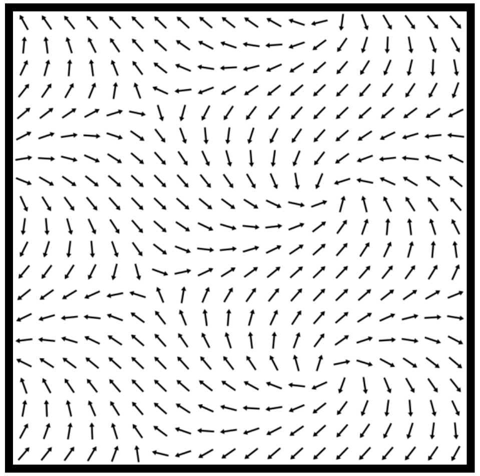
Rules of thumb:

- never use Euler!
- RK4 or RK4(3) is safe.

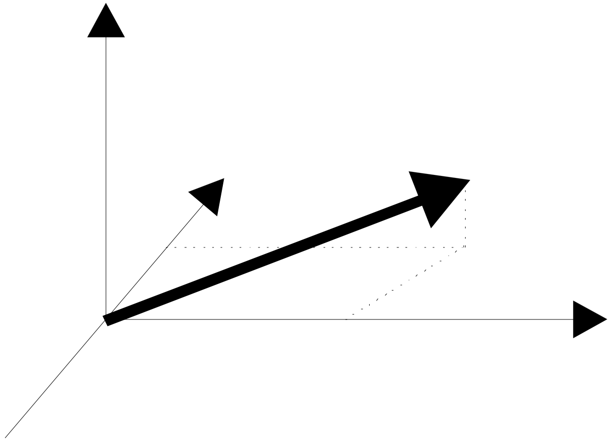
Autonomous ODE

All discussed integrators require that the domain and the dependent vector-valued variable have the same dimensionality.

$$\mathbf{v}: \mathrm{E}^{n} \rightarrow \mathbb{R}^{n}$$



2D domain with 2-vectors

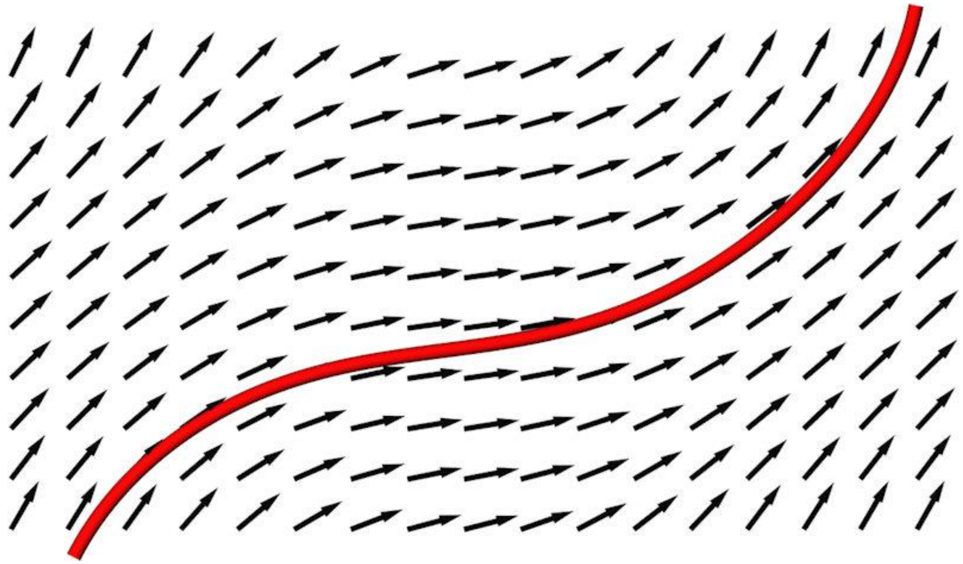


3D domain with 3-vectors

- Input
  - vector field **v**
  - seed point
- Concepts
  - forward / backward Integration
    - integrate in **v** and **–v** to get the whole tangent curve
  - stop criteria
    - number of steps
    - arc length
    - domain
    - zero or low velocity
  - arc length parameterized output
    - integration in the direction field

$$\mathbf{v}_d = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

  - output density
    - output only every *n*-th step
- Output
  - polyline approximating the tangent curve



Output Density versus Integration Step Size

integration step size

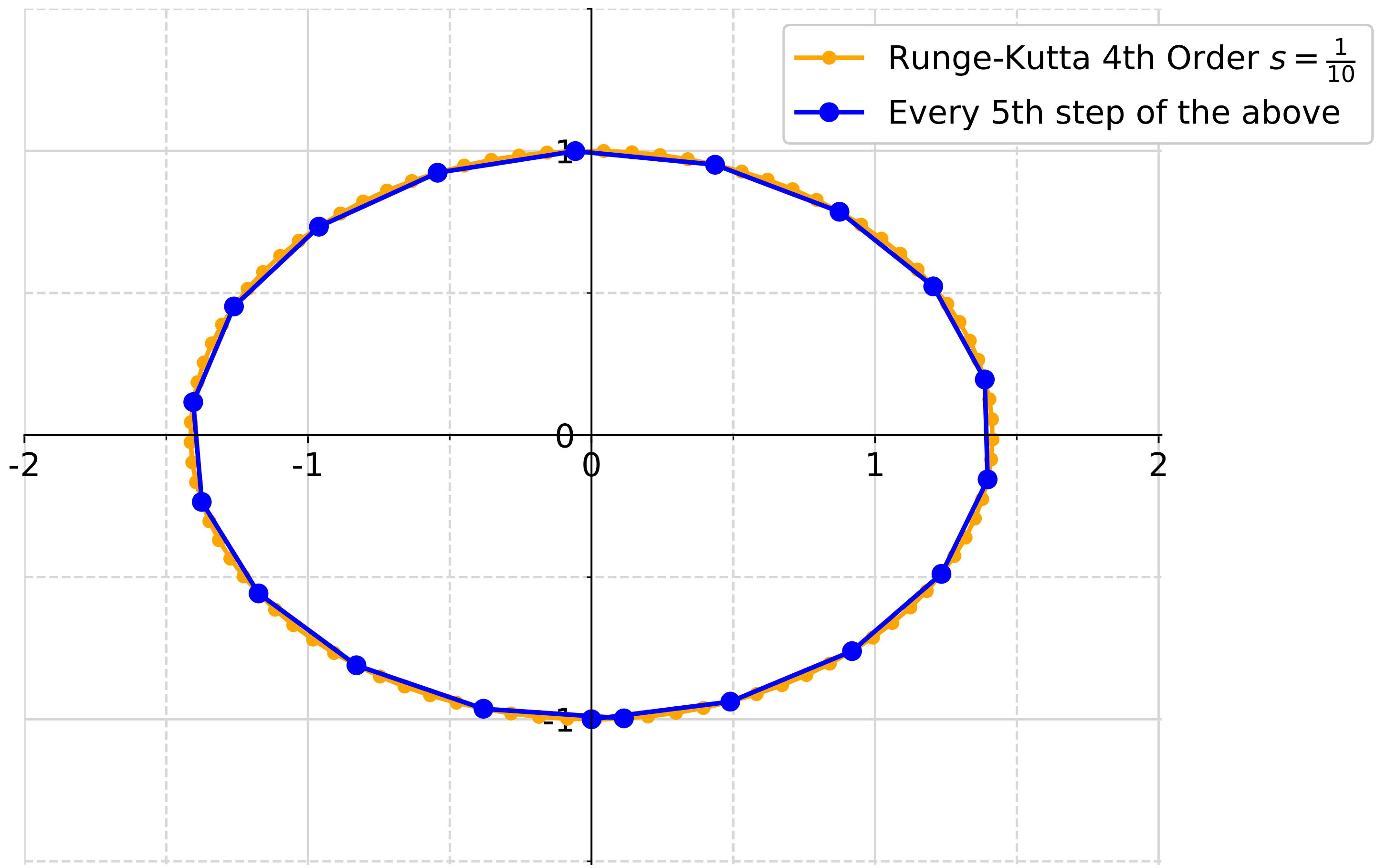
internal value

output density

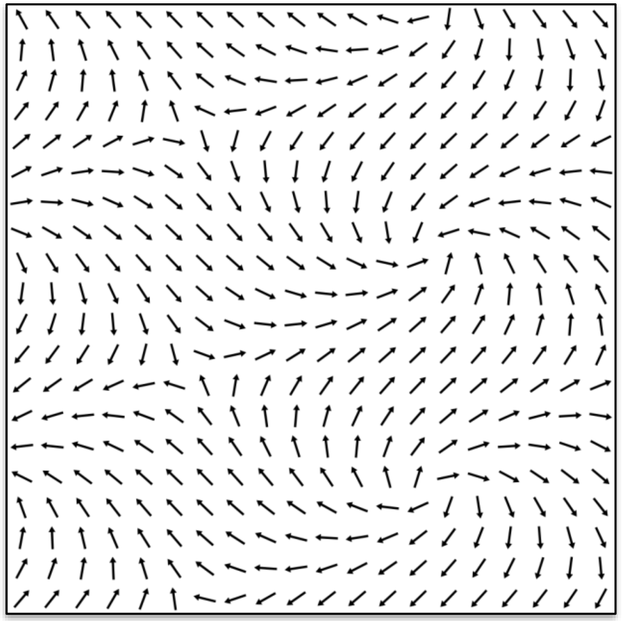
external value

high accuracy due to small integration step size

low graphical effort due to fewer output points

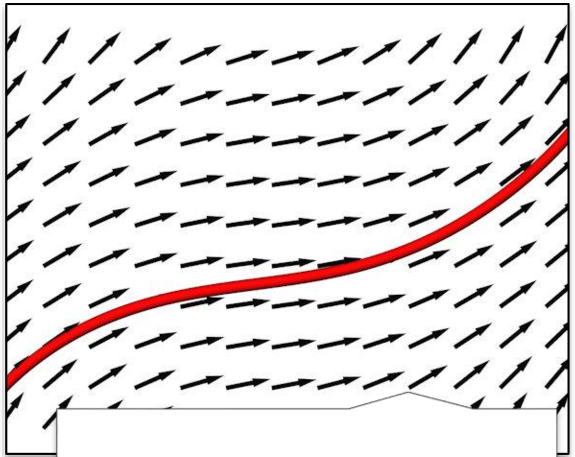


$$\mathbf{v}\colon \mathrm{E}^n \to \mathbb{R}^n$$

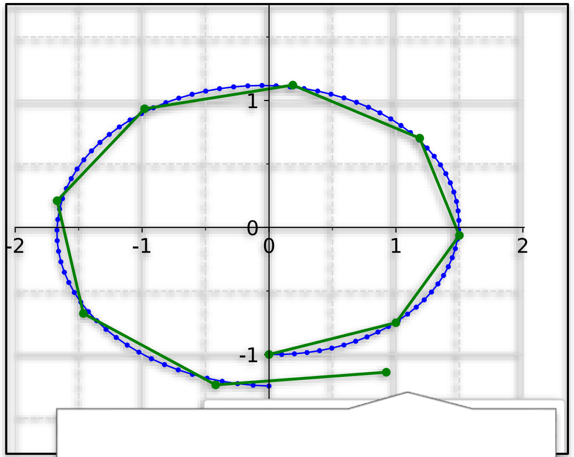


Steady Vector Fields

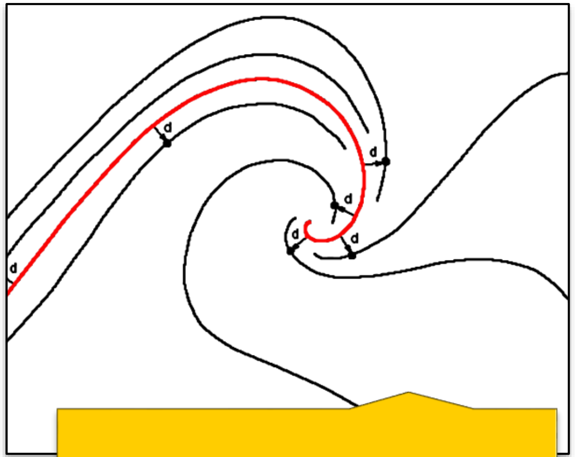
- tangent curves / stream lines
- stream surfaces



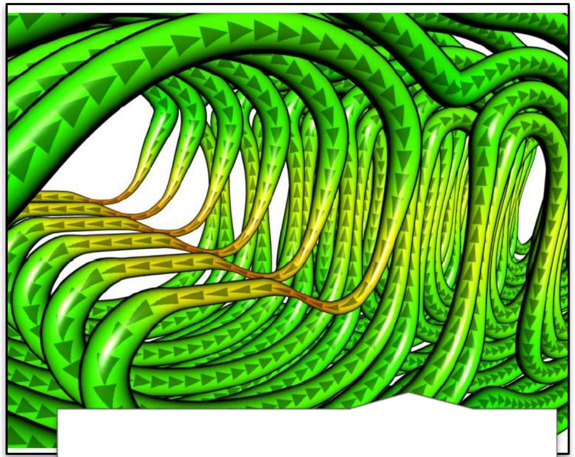
Definition



Integration



Seeding



3D impression

Seeding / 2D

Which tangent curves to visualize?

define start point

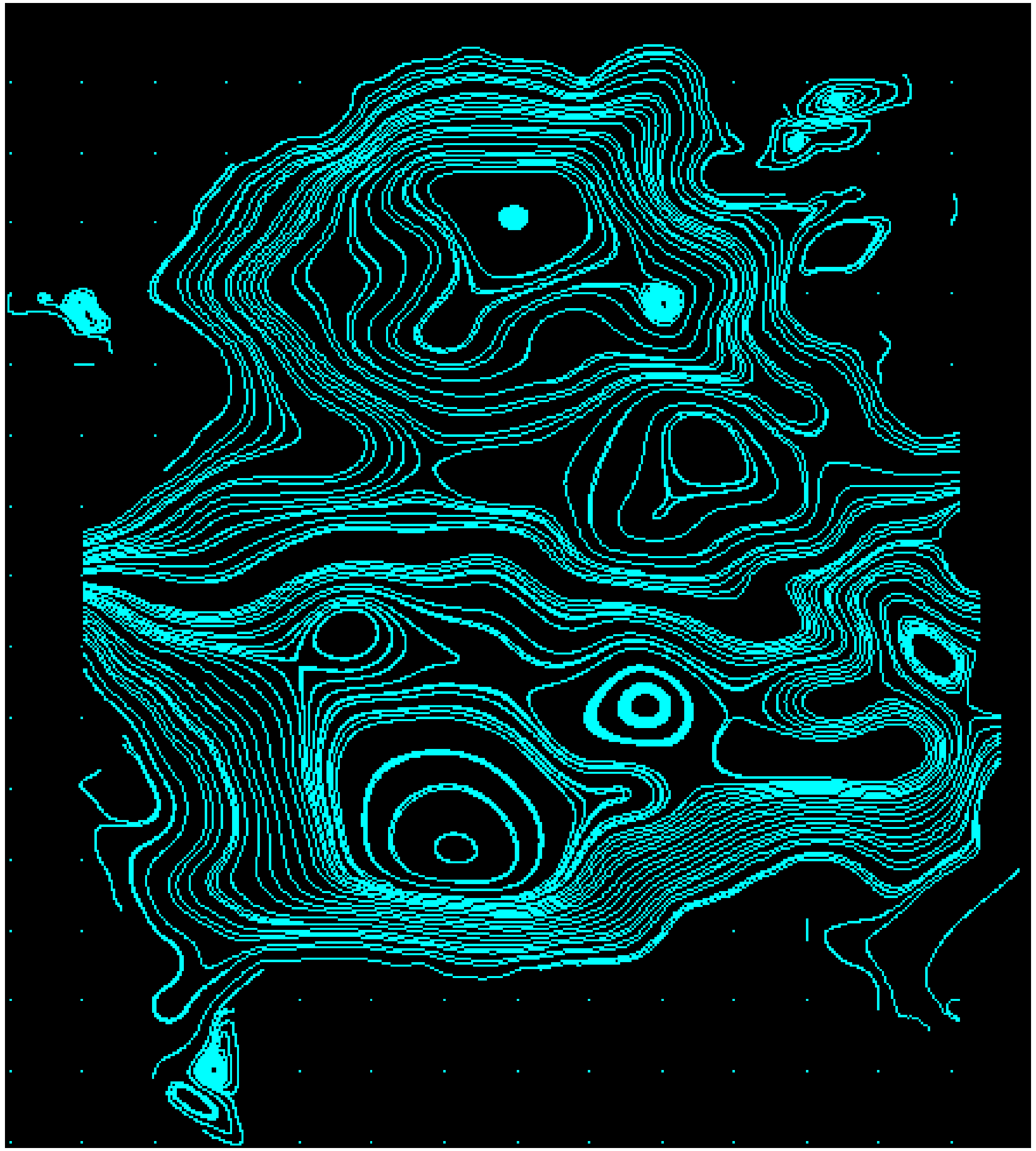
seeding point

areas with low coverage

important details may get lost

areas with too much coverage

visual overload and clutter



Seeding / 2D

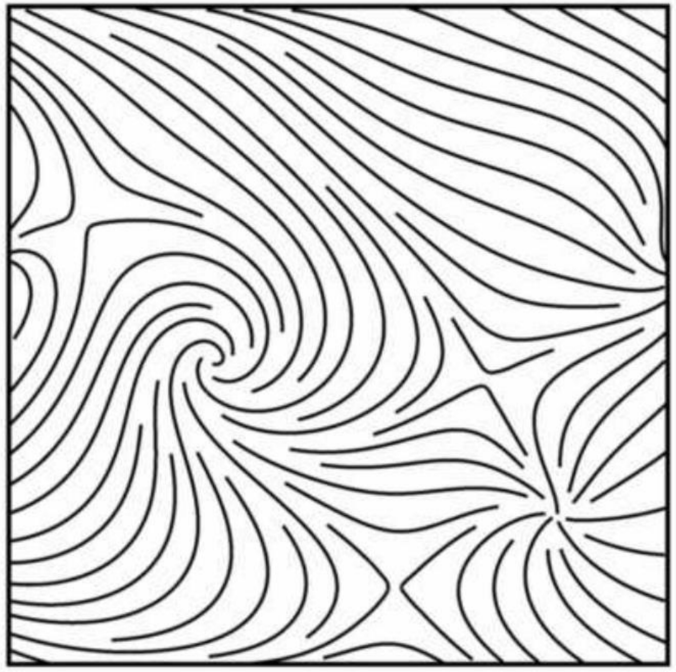
Simple approaches:

- start on regular grid points
- start randomly

Advanced approaches:

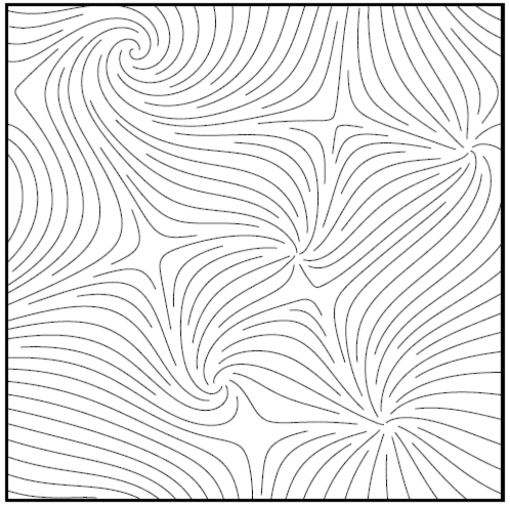
- guided by flow features
- guided by other fields
- evenly spaced

started from the vertices
of a uniform grid

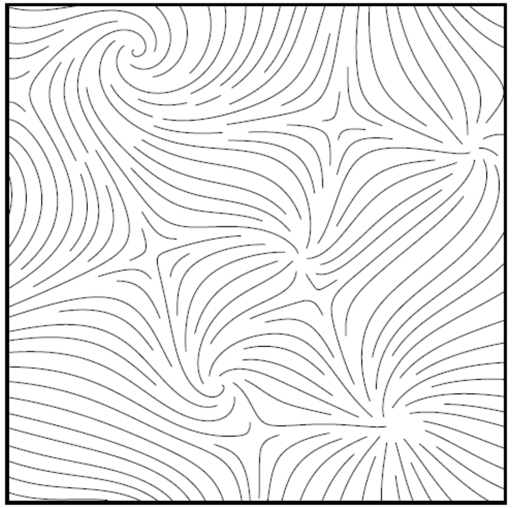


evenly spaced
using density optimization

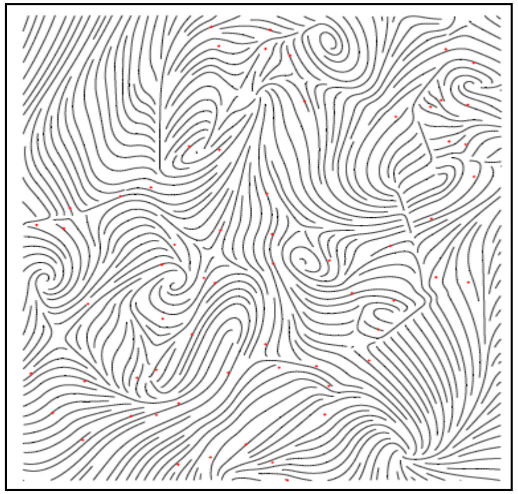
images from Turk & Banks, Image-guided streamline placement, SIGGRAPH 1996



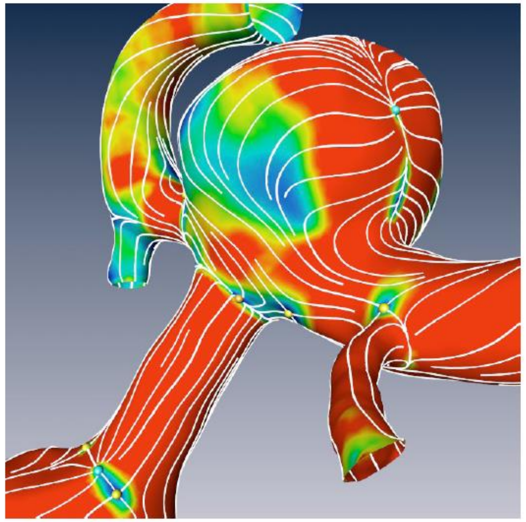
Turk and Banks, 1996



Jobard et al., 1997



Mebarki et al., 2005



Rosanwo et al., 2009

Evenly spaced tangent curves in 2D (Turk/Banks 96)

start with streamlets

very short tangent curves

apply a series of energy-decreasing elementary operations:
create, delete, move, combine, lengthen, shorten streamlets

energy: difference between low-pass filtered version of current placements and uniform grey image

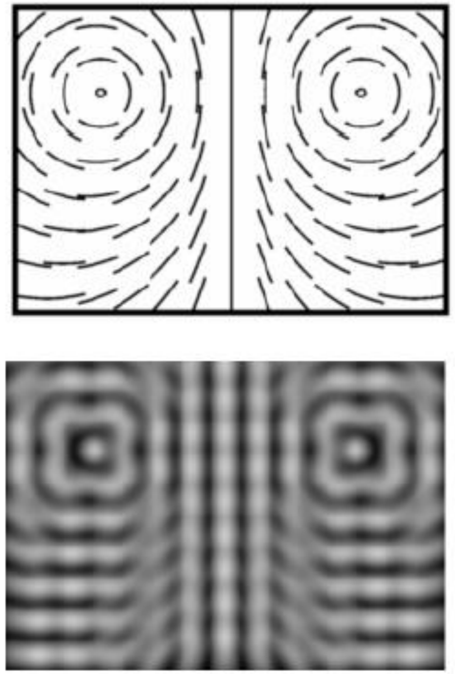


Figure 2: (a) Short streamlines with centers placed on a regular grid (top); (b) filtered version of same (bottom).

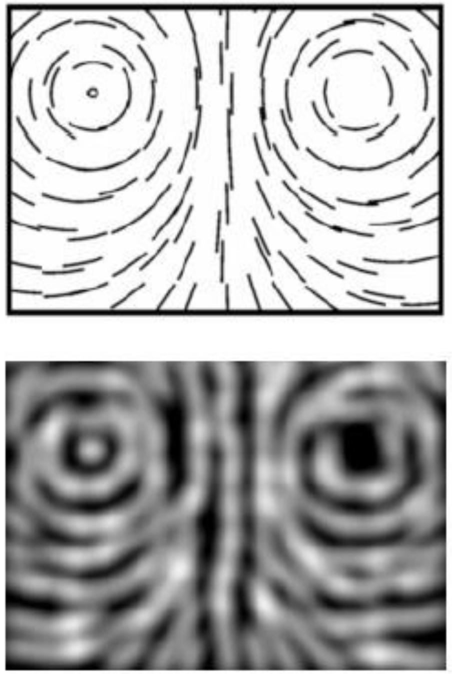


Figure 3: (a) Short streamlines with centers placed on a jittered grid (top); (b) filtered version showing bright and dark regions (bottom).

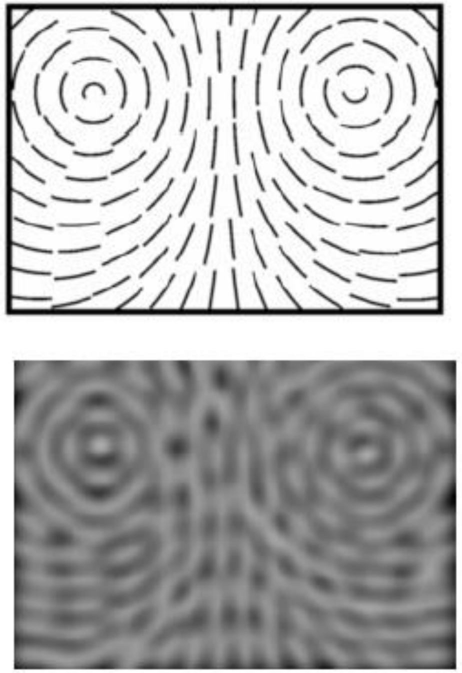


Figure 4: (a) Short streamlines placed by optimization (top); (b) filtered version showing fairly even gray value (bottom).

Evenly spaced tangent curves in 2D (Turk/Banks 96)

Evenly spaced tangent curves in 2D (Jobard/Lefer 97)

greedy placement of new tangent curves near already present ones

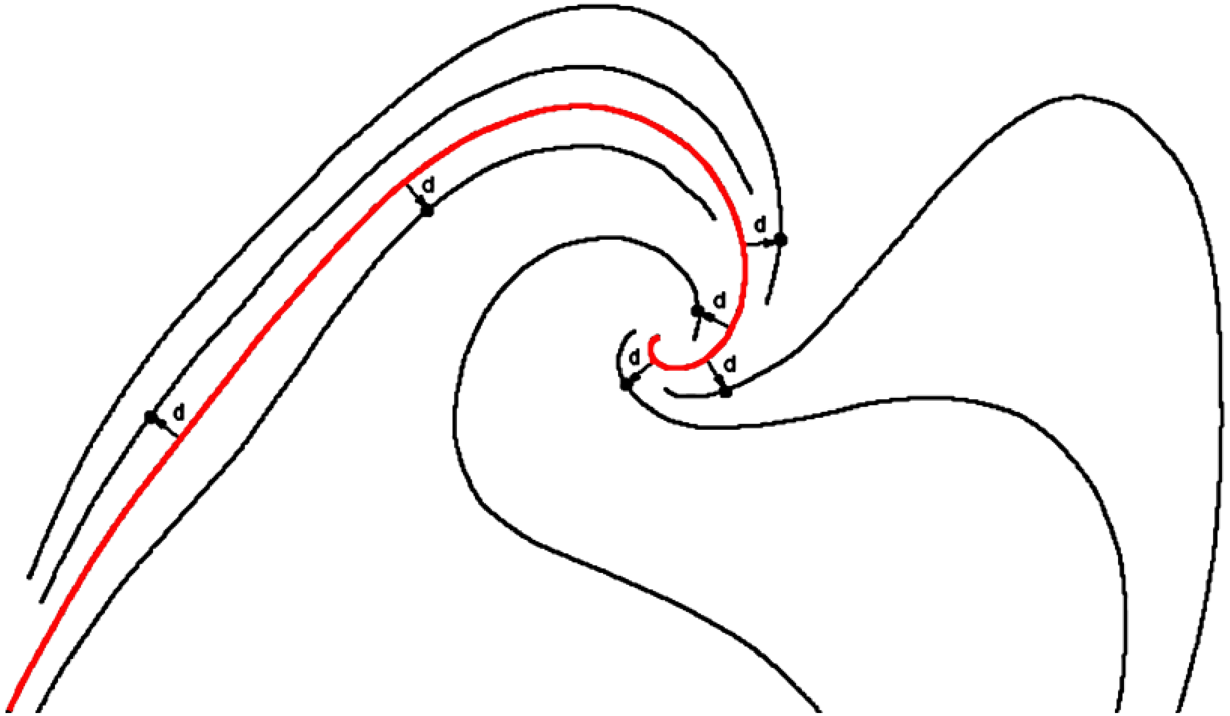
2 steering parameters d_{sep}, d_{test} with $d_{test} < d_{sep}$

Start integration of an arbitrary tangent curve in forward and backward direction until it

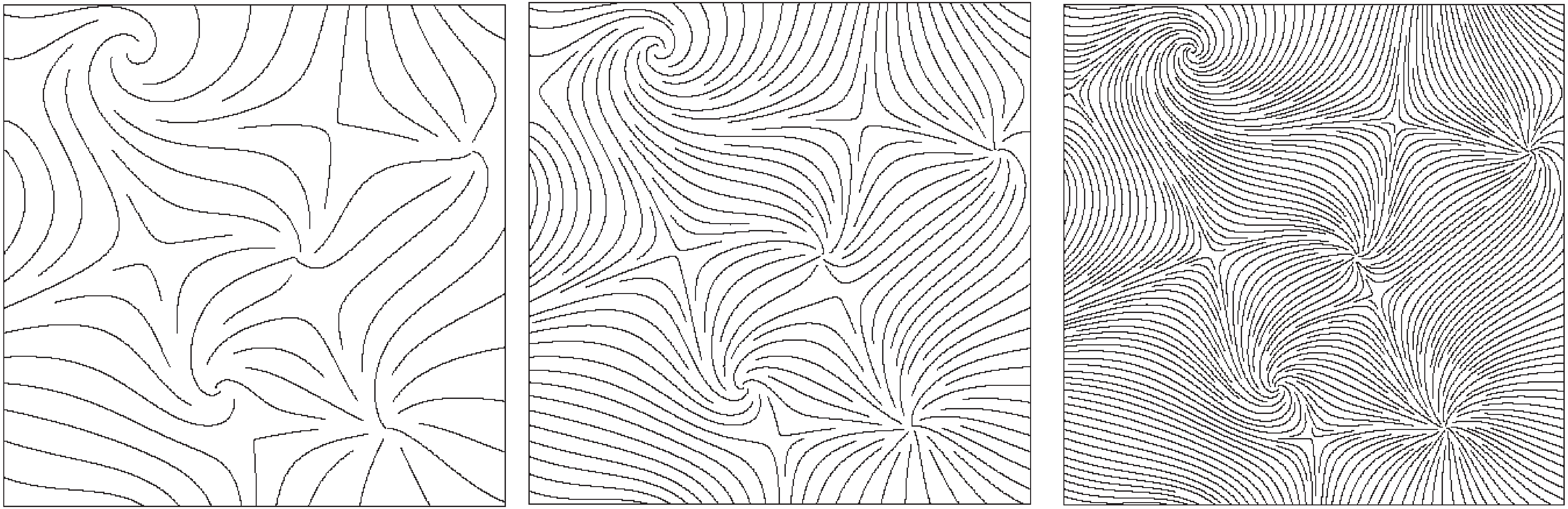
- Leaves the domain
- Ends in a critical point
- Comes closer than d_{test} to another drawn tangent curve

Next tangent curve: starts in a point with distance d_{sep} to first tangent curve

Ends when no further tangent curves are possible



Evenly spaced tangent curves in 2D (Jobard/Lefer 97)



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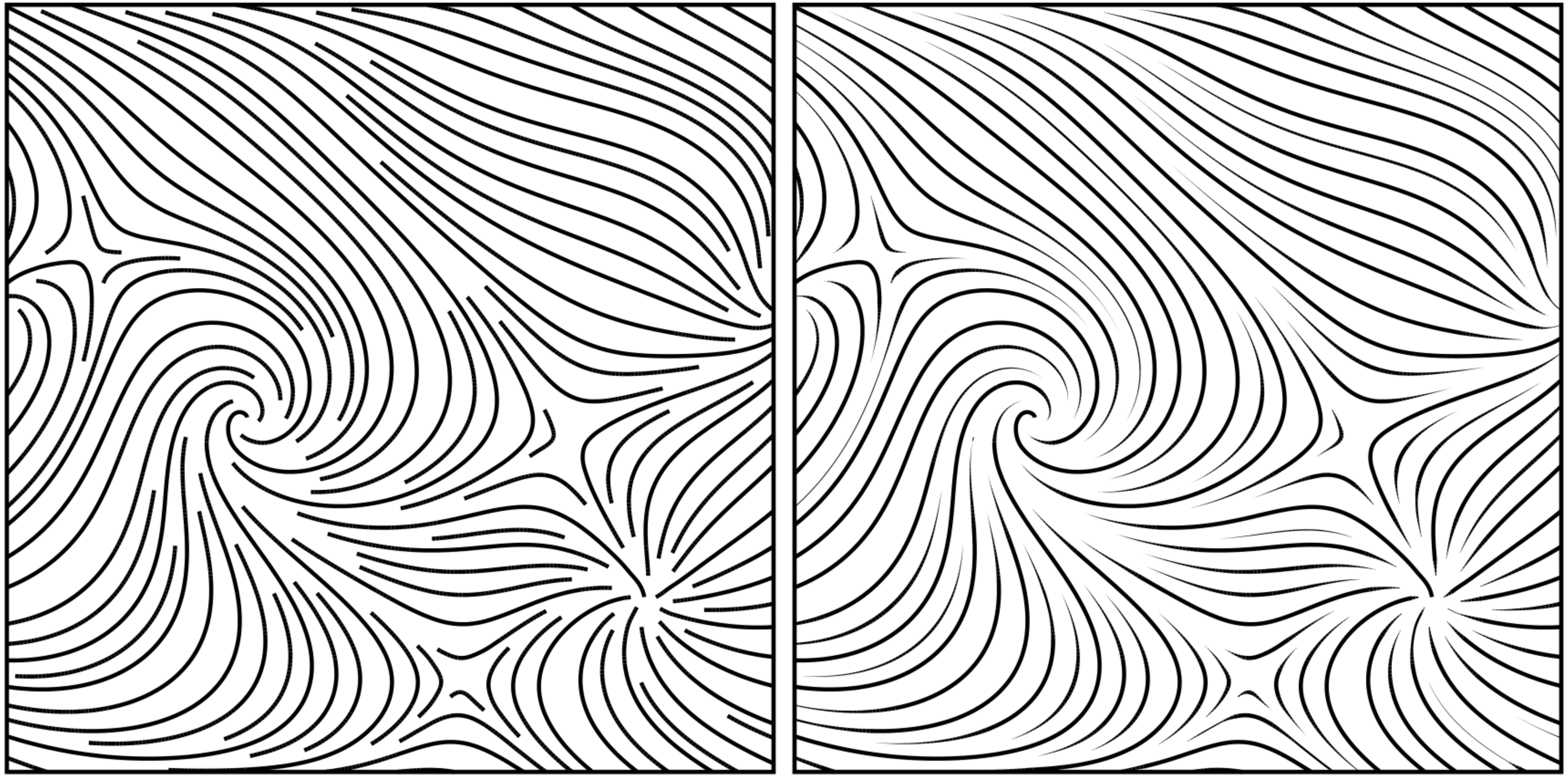
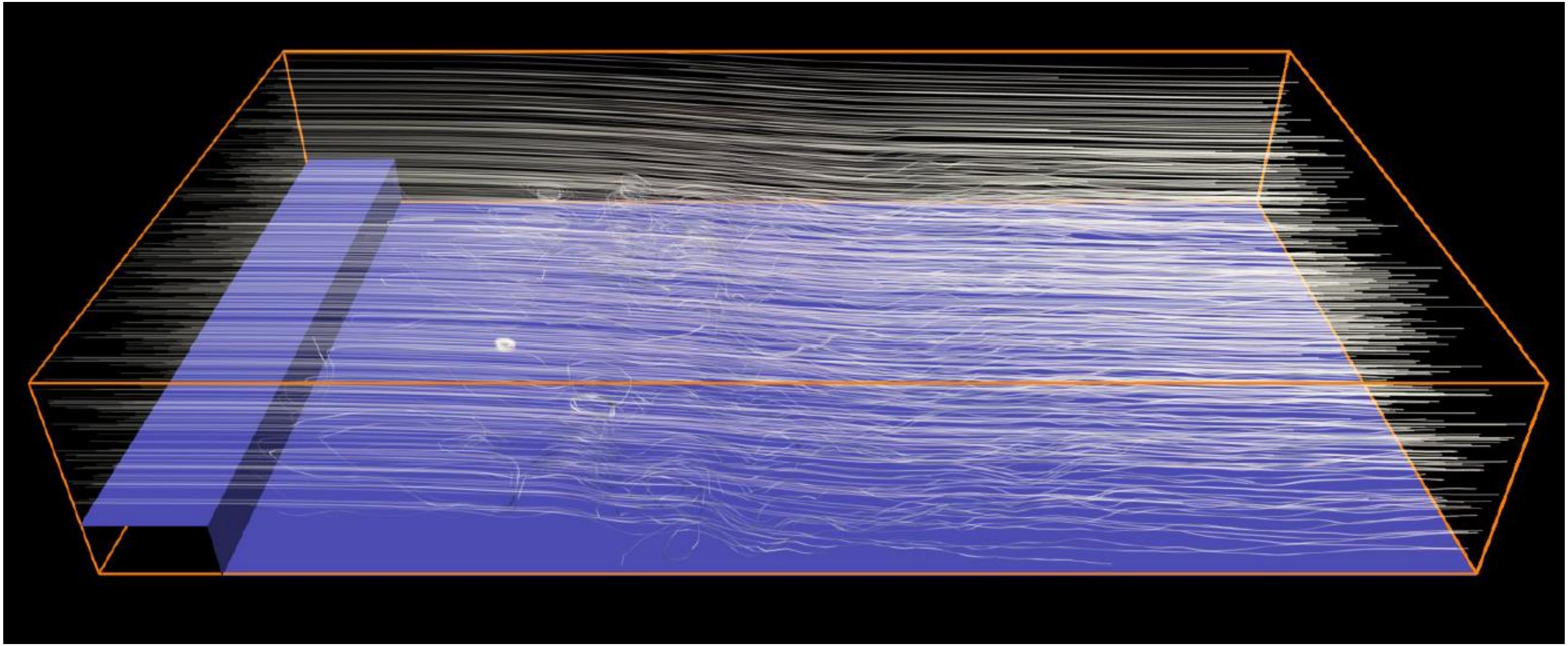


Fig. 6. Hand-drawing style images computed without and with the tapering effect

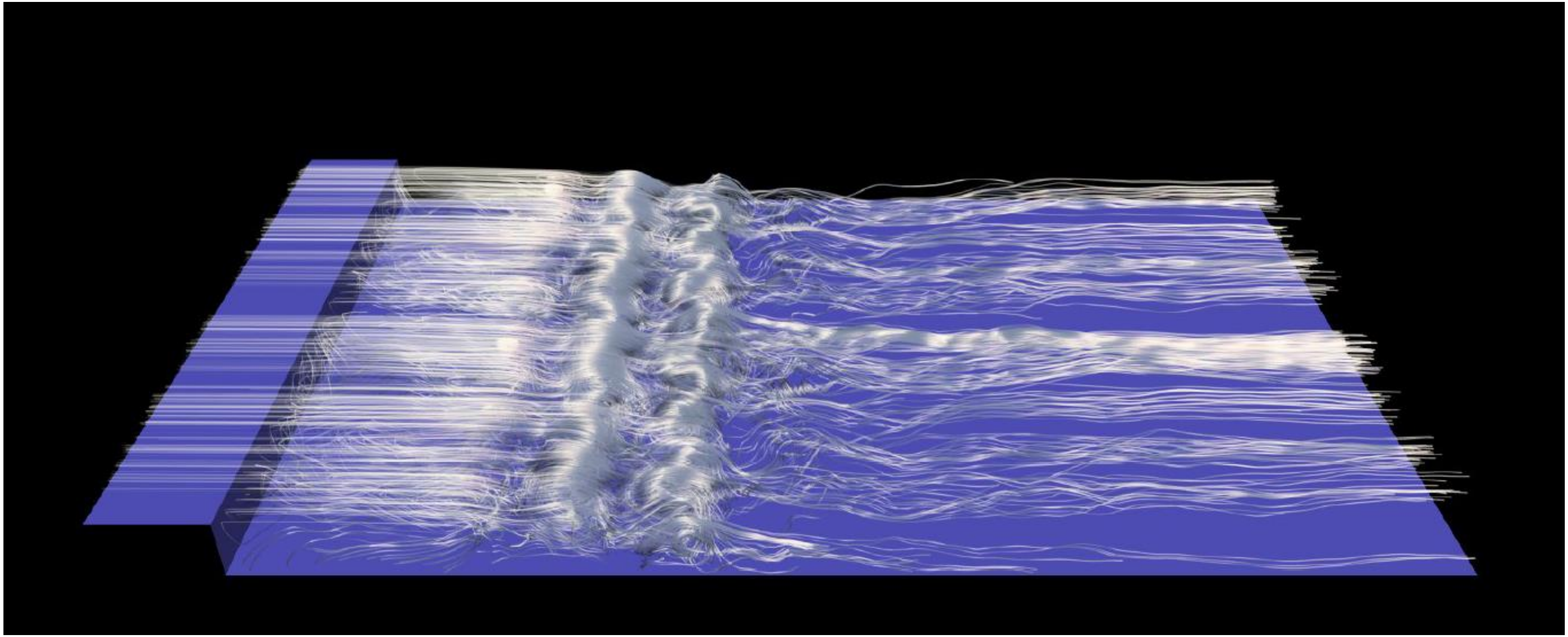
Seeding / 3D



Evenly-spaced placement does not make much sense. Here, seeds were placed uniformly.

image from Weinkauf et al., Coherent Structures in a Transitional Flow around a Backward-Facing Step, Physics of Fluids 15(9), September 2003

Seeding / 3D



Starting in regions of high vector field curvature (e.g., close to critical points).

image from Weinkauf et al., Coherent Structures in a Transitional Flow around a Backward-Facing Step, Physics of Fluids 15(9), September 2003

Opacity Optimization

integrate many tangent curves

evaluate their importance

length of curve

curvature

application-dependent

modulate opacity based on importance

view-dependent

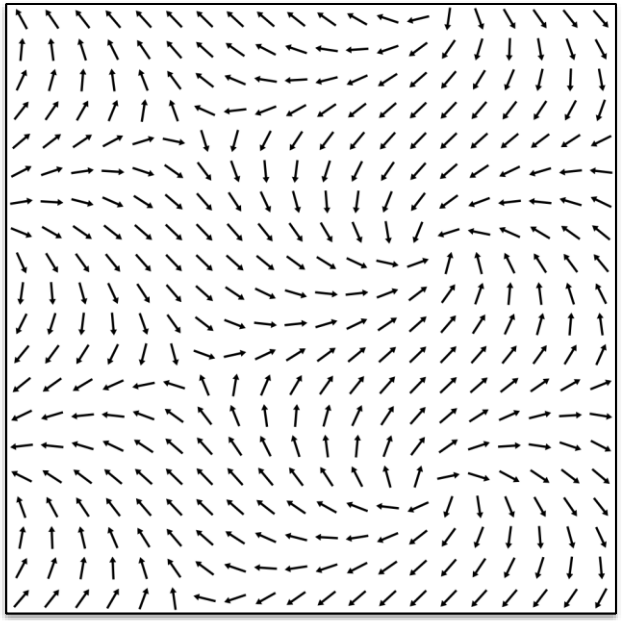
frame coherent

video from Tobias Günther, Christian Rössl, Holger Theisel, Opacity Optimization for 3D Line Fields, SIGGRAPH 2013

Opacity Optimization

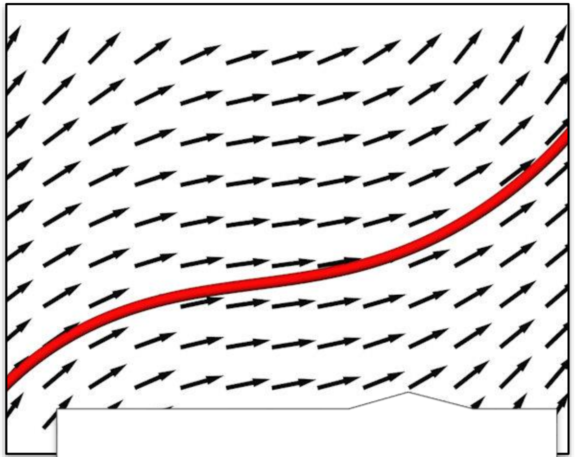
video from Irene Baeza Rojo, Markus Gross, Tobias Günther, Fourier Opacity Optimization for Scalable Exploration, TVCG 2019

$$\mathbf{v}\colon \mathrm{E}^n \to \mathbb{R}^n$$

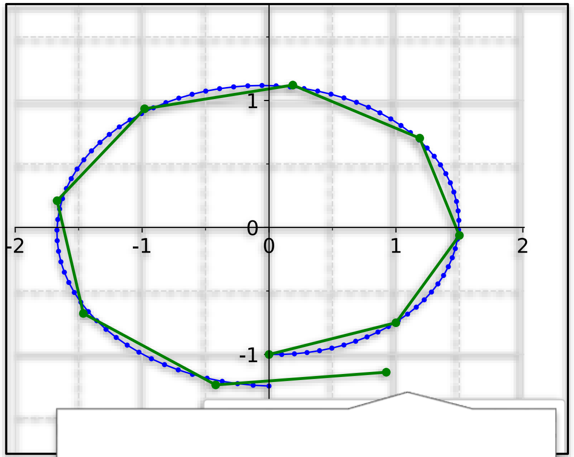


Steady Vector Fields

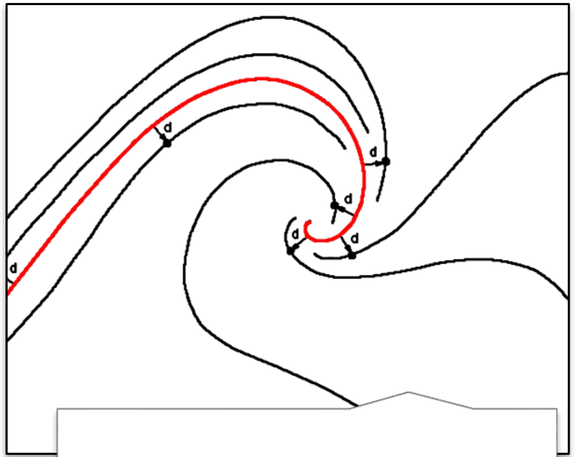
- tangent curves / stream lines
- stream surfaces



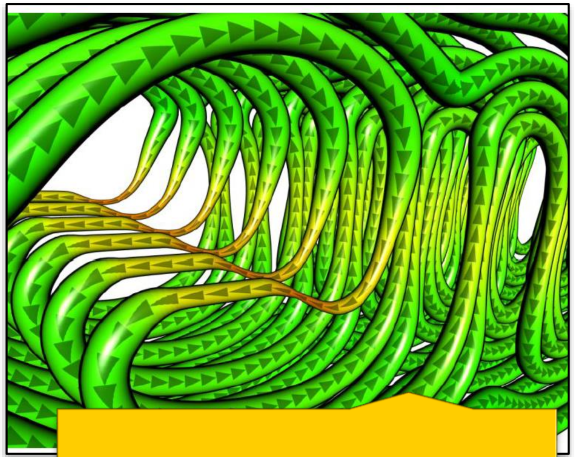
Definition



Integration



Seeding



3D impression

3D Impression of 3D Curves

depth perception

lighting

normal vector

normal vector is not uniquely defined by curve geometry

instead: 2D normal space with infinitely many normals

How to choose a preferred normal vector?

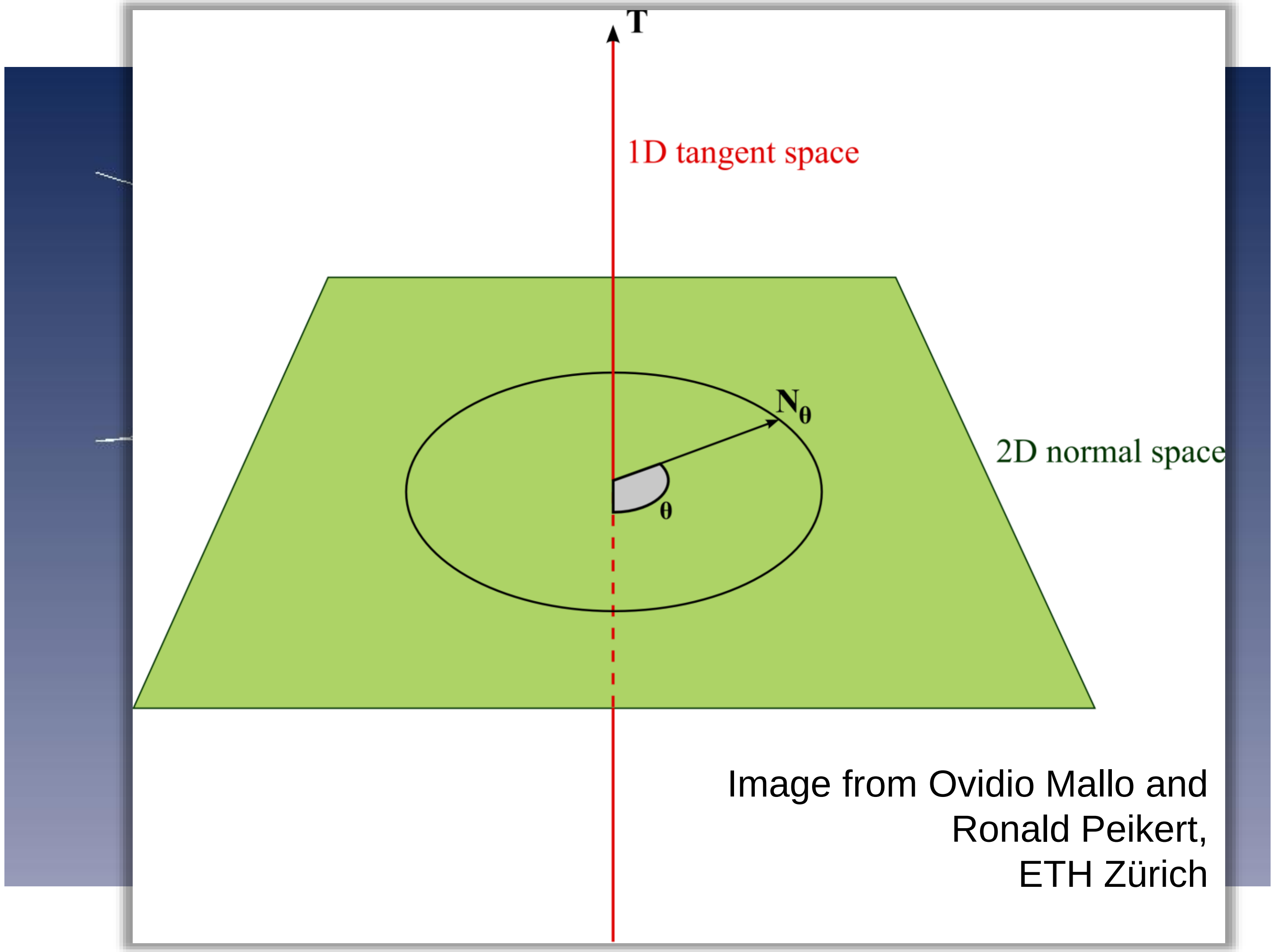


Image from Ovidio Mallo and Ronald Peikert, ETH Zürich

Illuminated Field Lines (Zöckler, Stalling, Hege, IEEE Vis 1996)

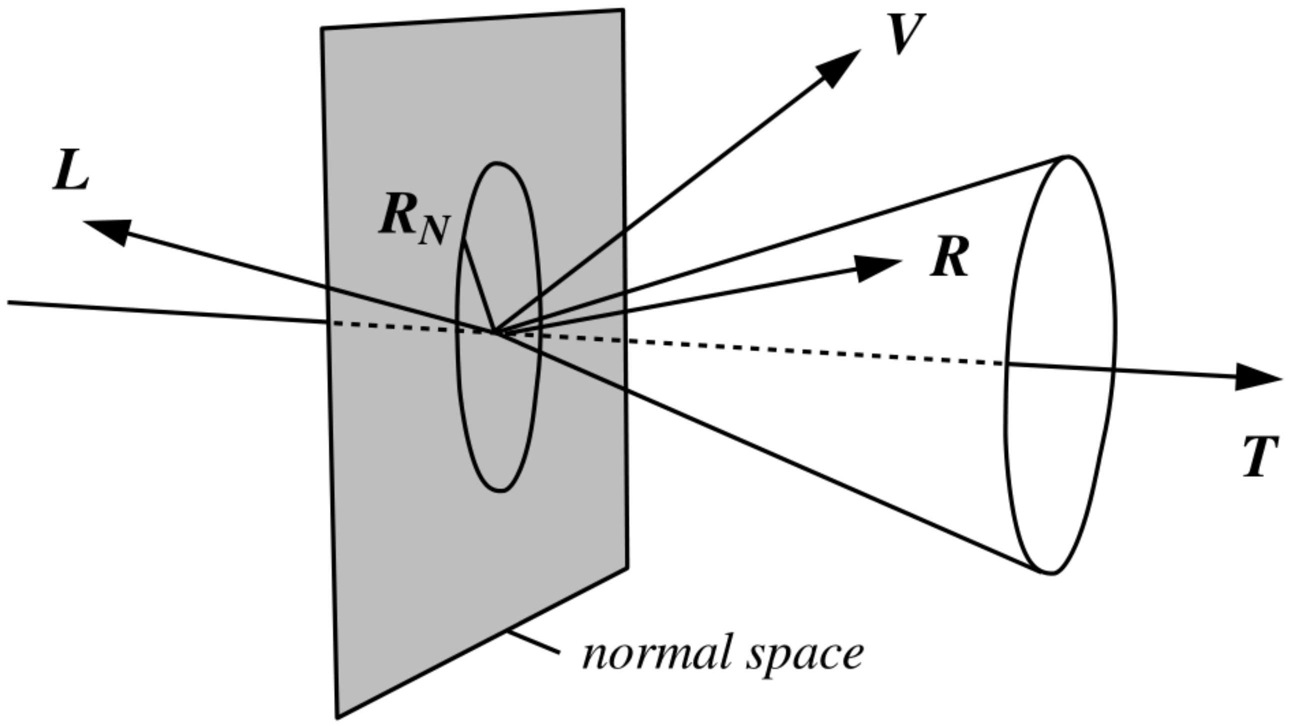
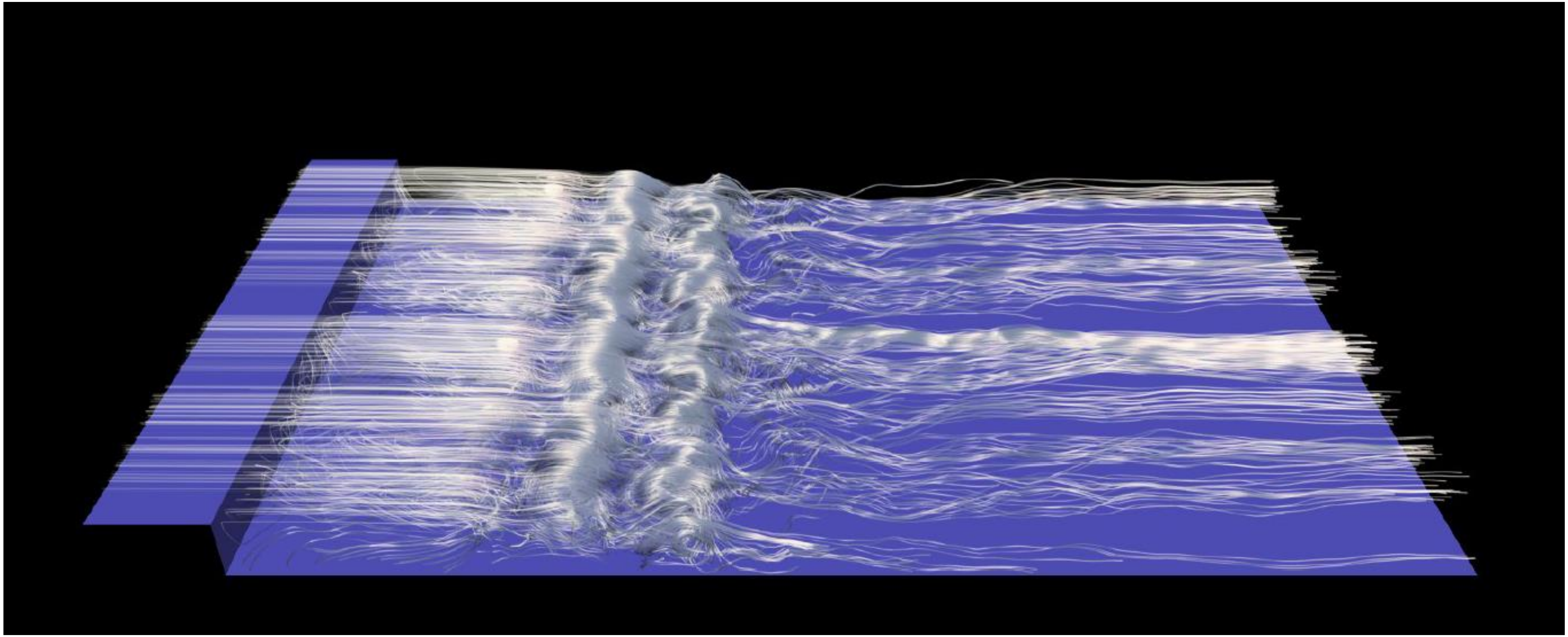


Fig. 1. For line primitives there are infinitely many possible reflection vectors $\boldsymbol{R}$ lying on a cone around $\boldsymbol{T}$. For the actual lighting calculation we choose the one contained in the $\boldsymbol{L}$-$\boldsymbol{T}$-plane.

Illuminated Field Lines (Zöckler, Stalling, Hege, IEEE Vis 1996)



Illuminated stream lines in a backward-facing step.

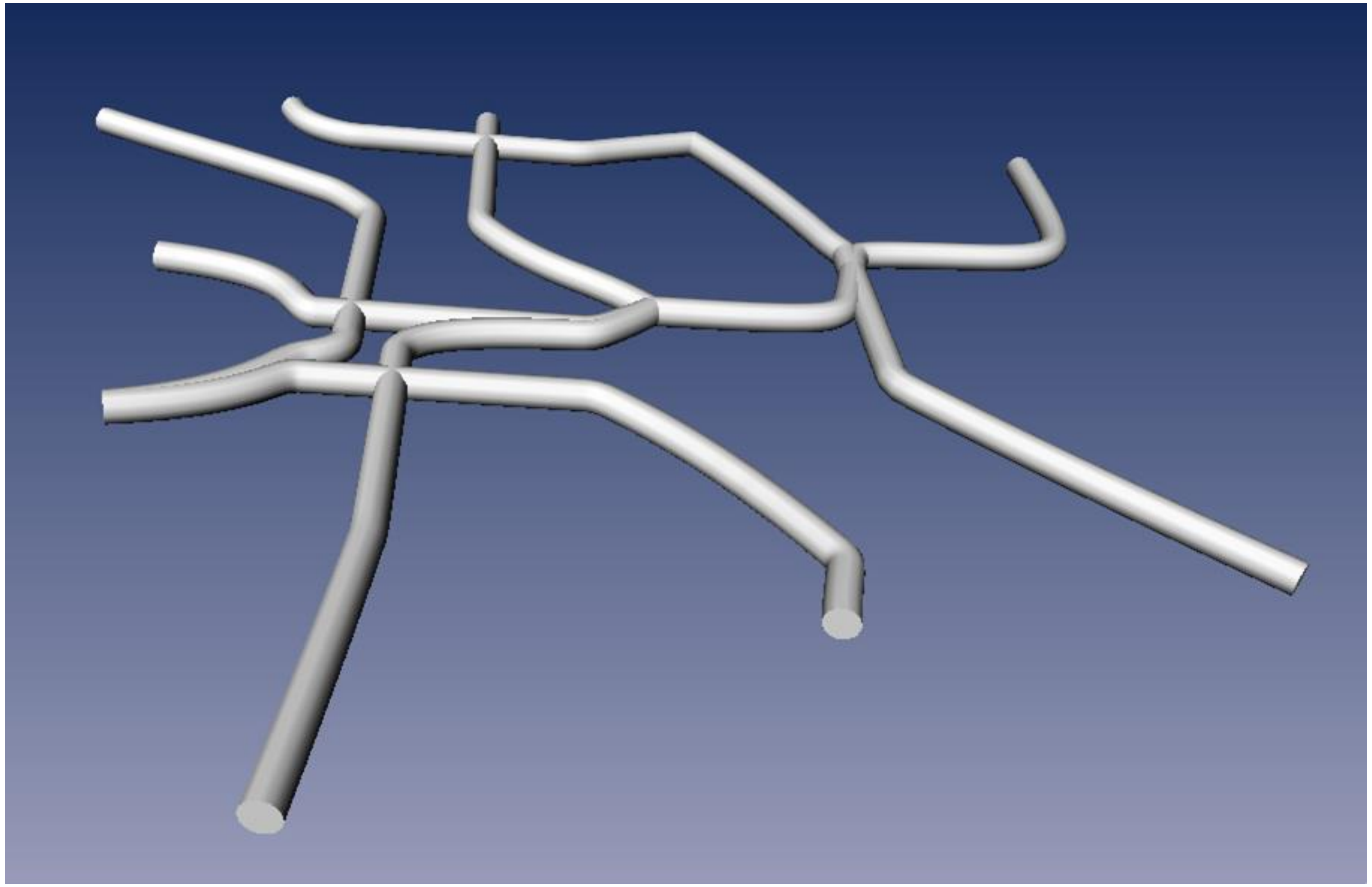
image from Weinkauf et al., Coherent Structures in a Transitional Flow around a Backward-Facing Step, Physics of Fluids 15(9), September 2003

3D Impression of 3D Curves

depth perception

lighting

normal vector



cylinders around the lines

surface with normals

many triangles

more triangles = smoother shape

3D Impression of 3D Curves

depth perception

lighting

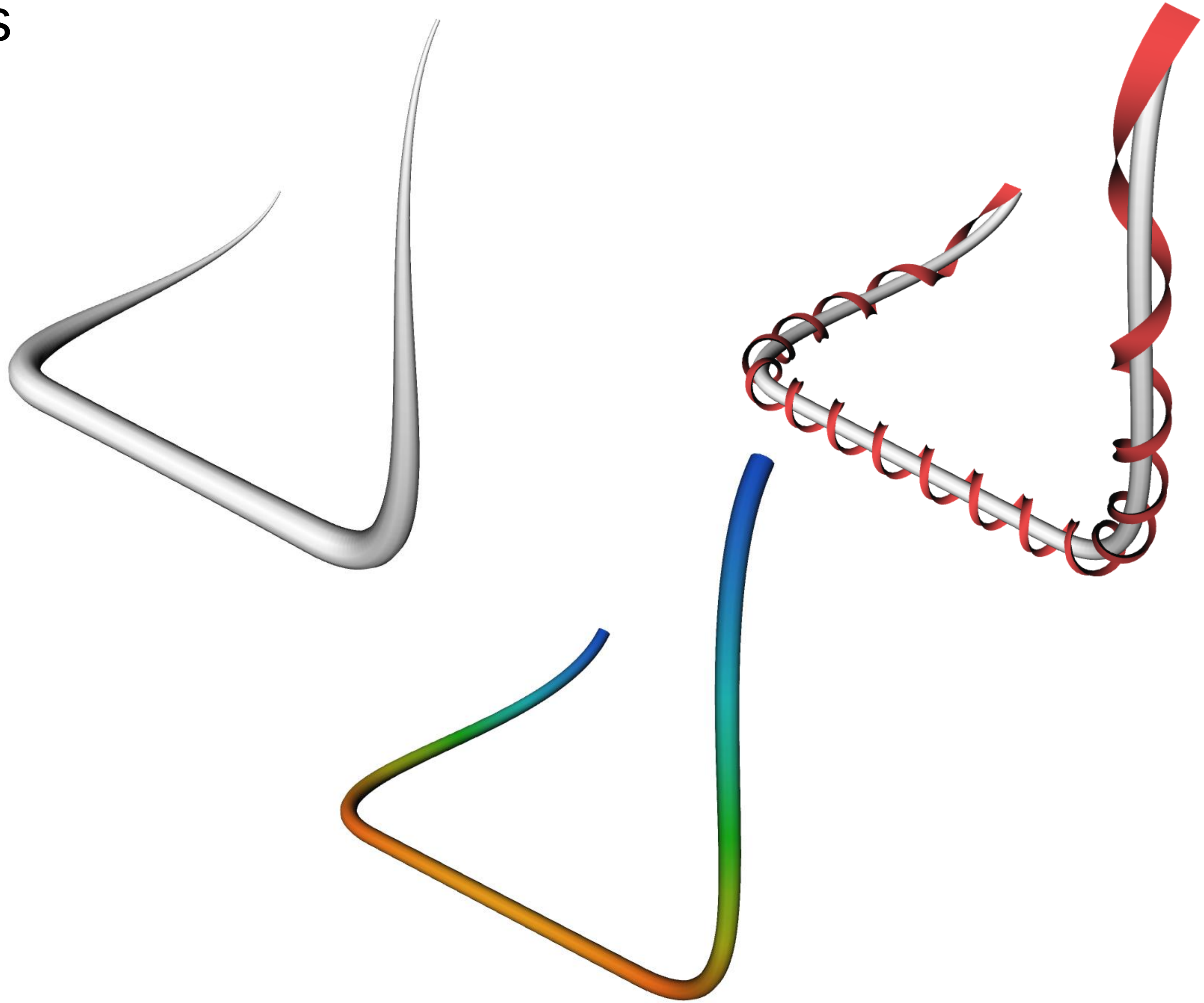
normal vector

cylinders around the lines

surface with normals

many triangles

more triangles = smoother shape



3D Impression of 3D Curves

depth perception

lighting

normal vector

modern approach: shaders

send polyline to GPU

create triangle strip in the viewing plane

geometry shader

compute cylinder-like shading & depth

fragment shader

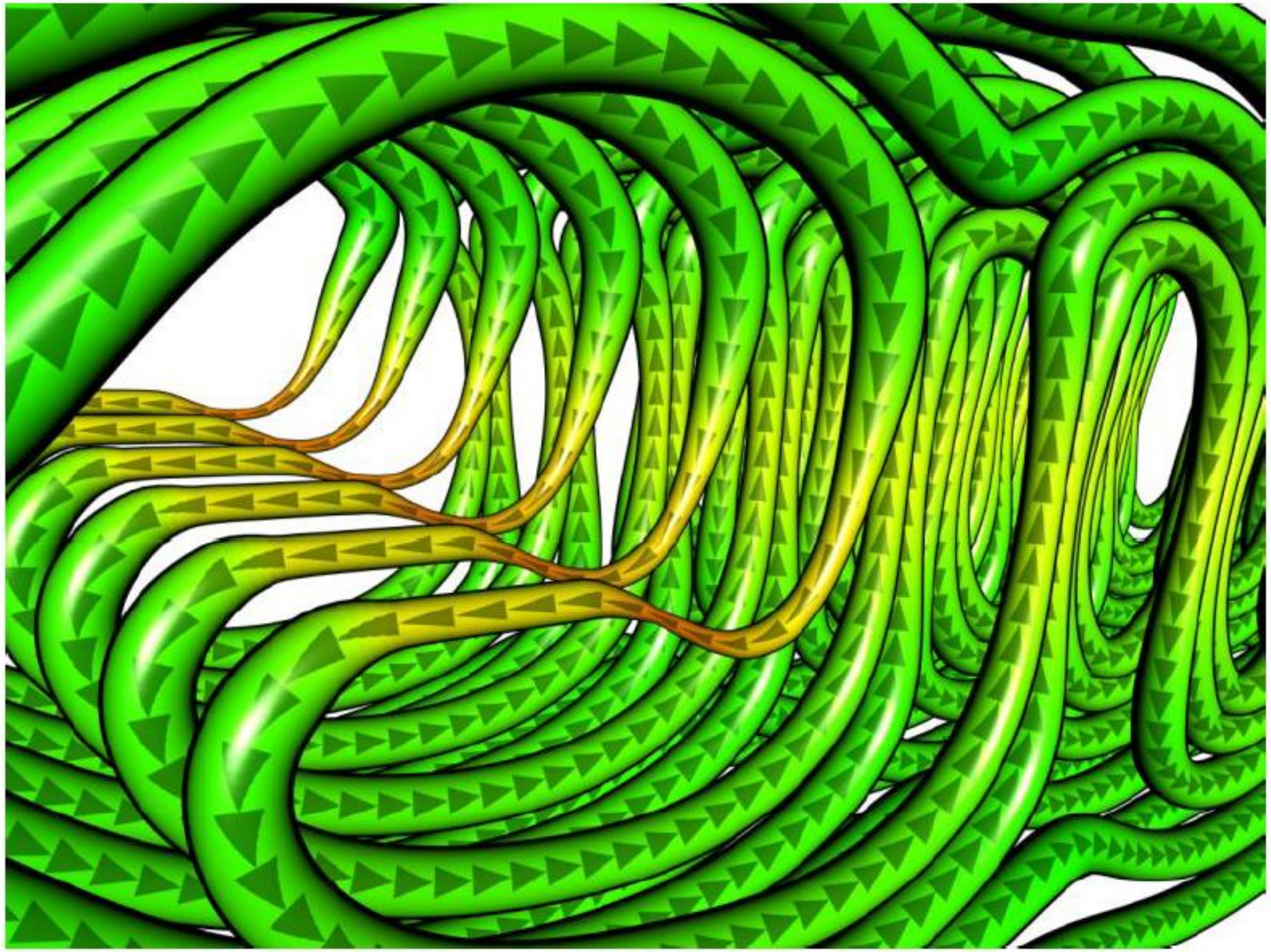
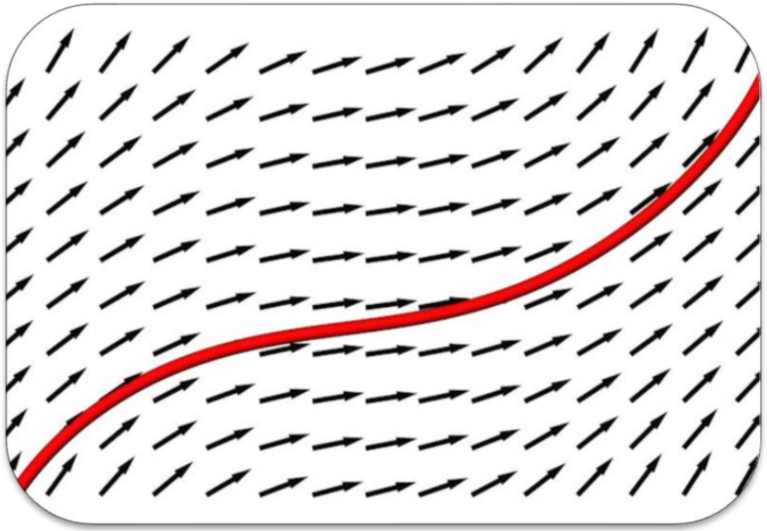
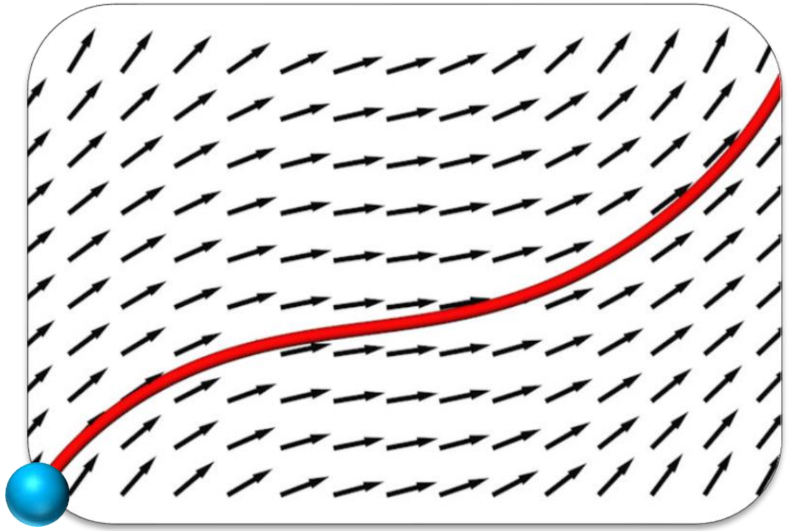


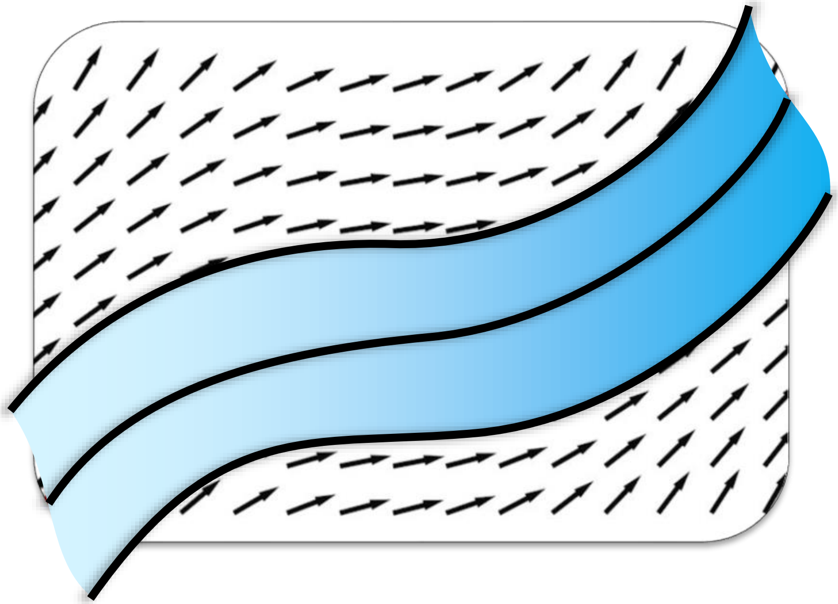
image from Carsten Stoll & Stefan Gumhold, Visualization with stylized line primitives, IEEE Visualization 2005



tangent curves / stream lines



particles



stream surfaces

Particle Tracer

show current integration step

instead of entire tangent curve

several parameters

transparency

number of initial particles

seed particles

box, sphere, line, ...

enhance with additional information

pressure, temperature, ...

maximum age of a particle

remove particles after some time

integrate particles

RK4, ...

particle tails

show previous integration steps

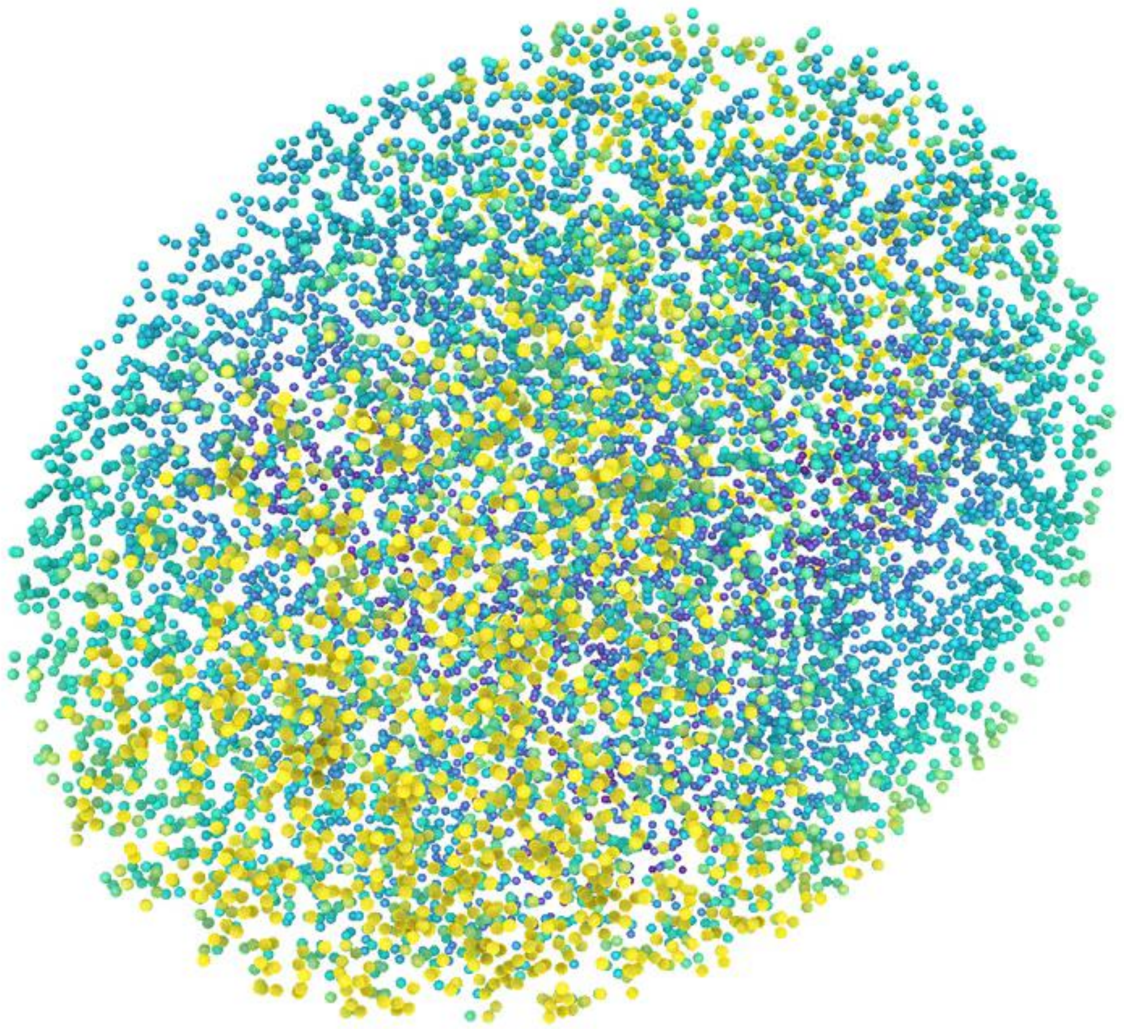
using a small curve

highlight current position

number of particles seeded in every n-th time step

render particles after every integration step

points, spheres



Stream Surfaces

seeding line

polyline

apply one integration step

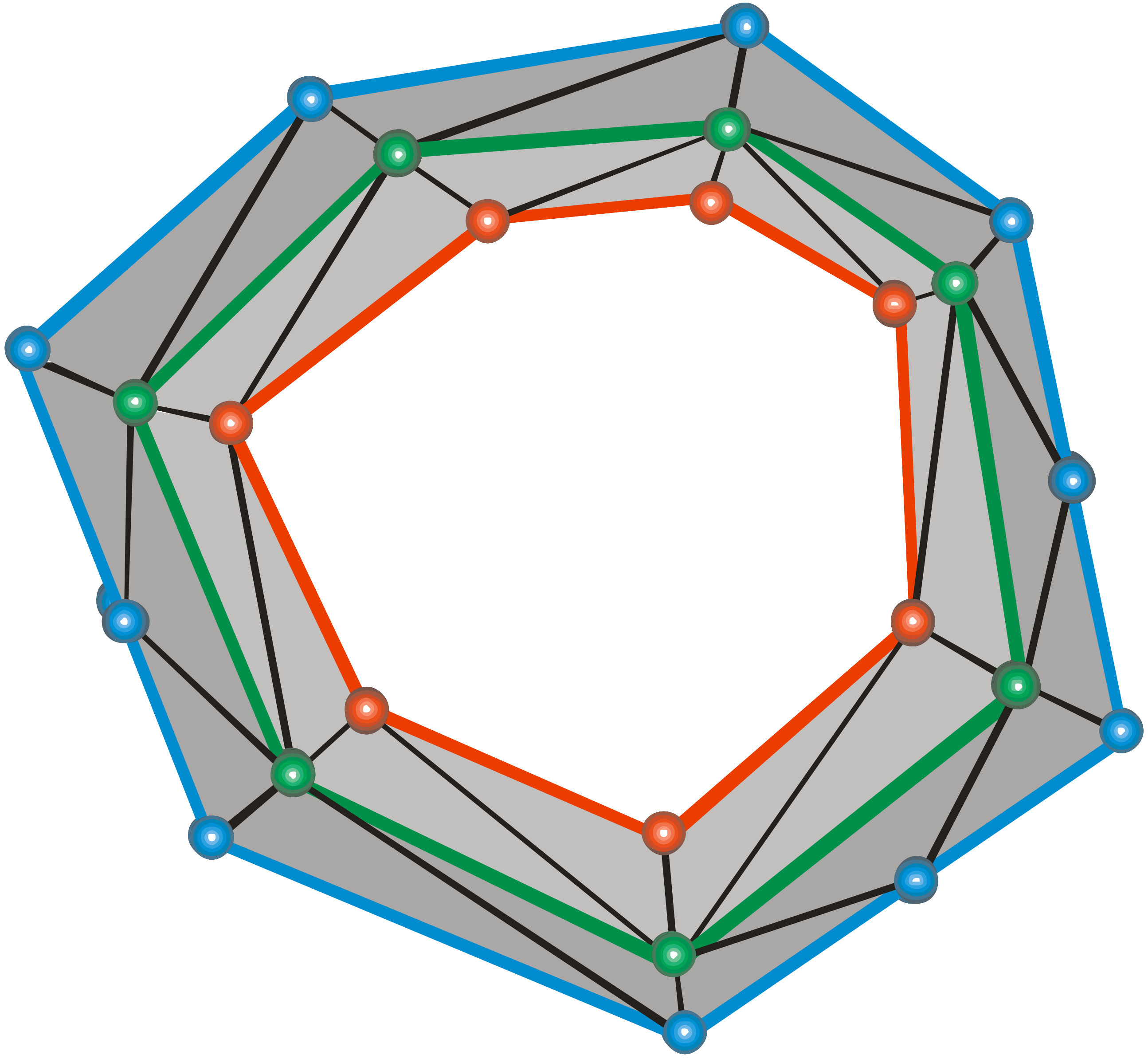
to each vertex

➔ *creates new polyline*

triangulate between the two polylines

adaptively remove/insert vertices

Euclidian distance, angle



Stream Surfaces

seeding line

polyline

apply one integration step

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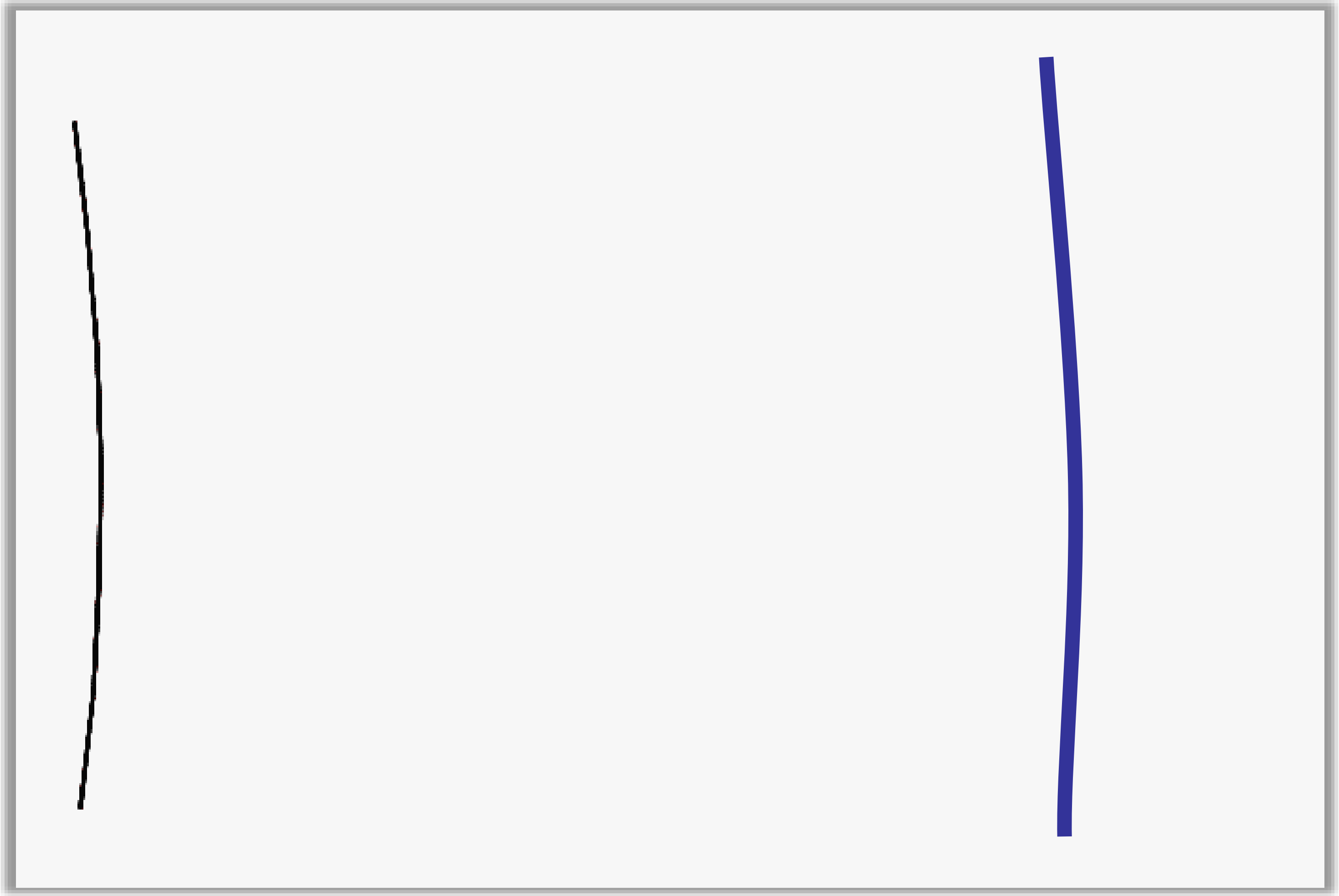
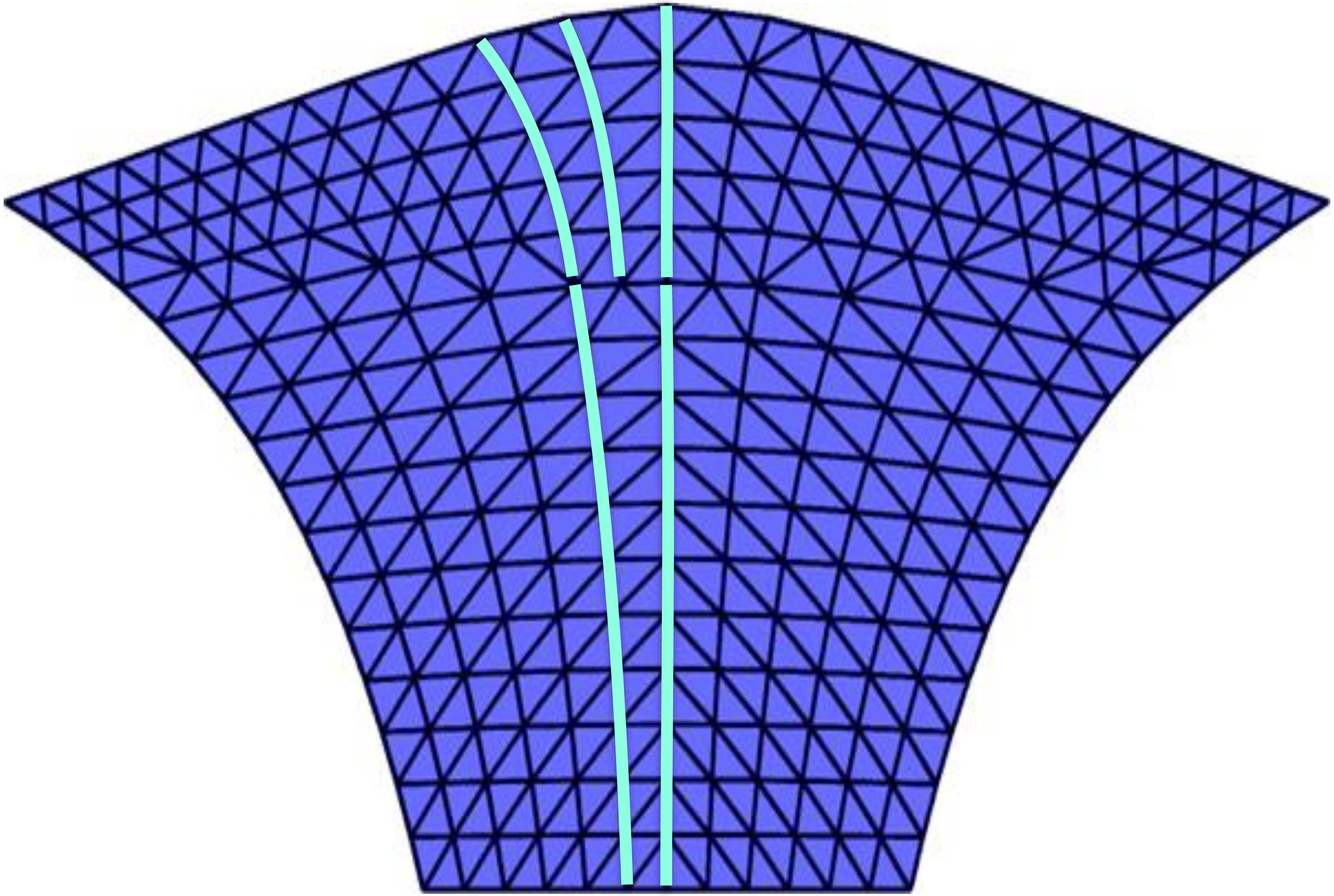
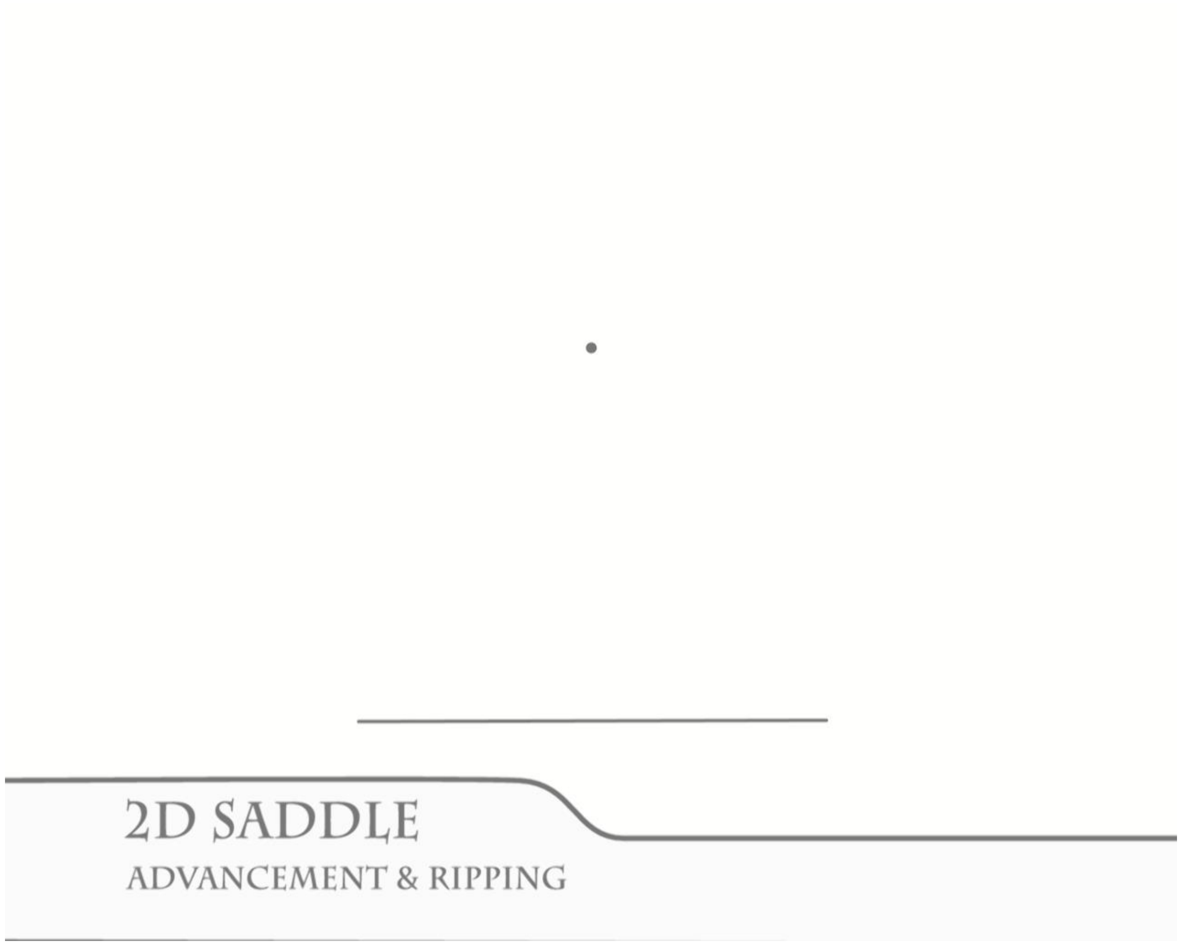


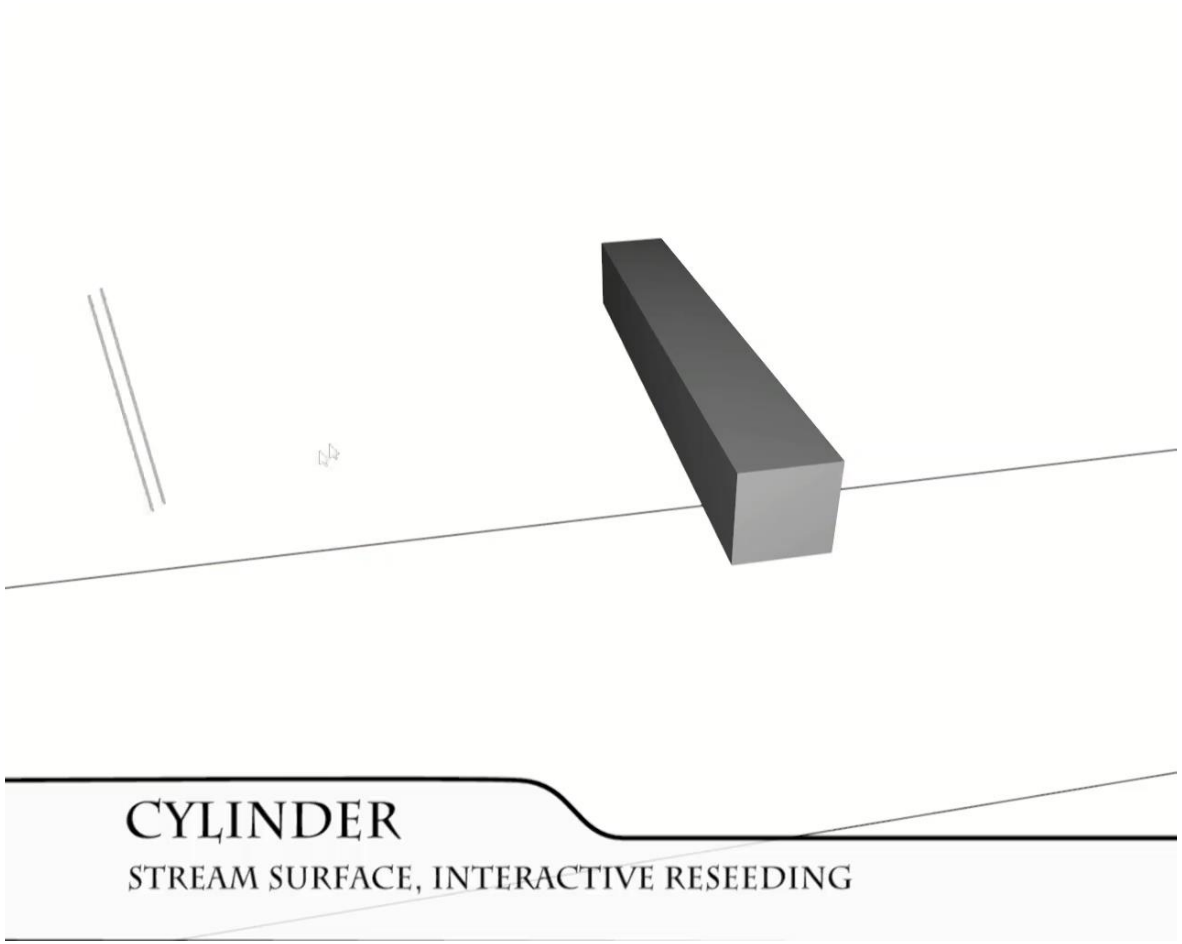
General stream surfaces
[Hultquist, Vis 1992]

Topological considerations
[Stalling, Phd thesis, 1998]

High accuracy
[Garth et al., Vis 2008]
[Schulze et al., SGP 2012]

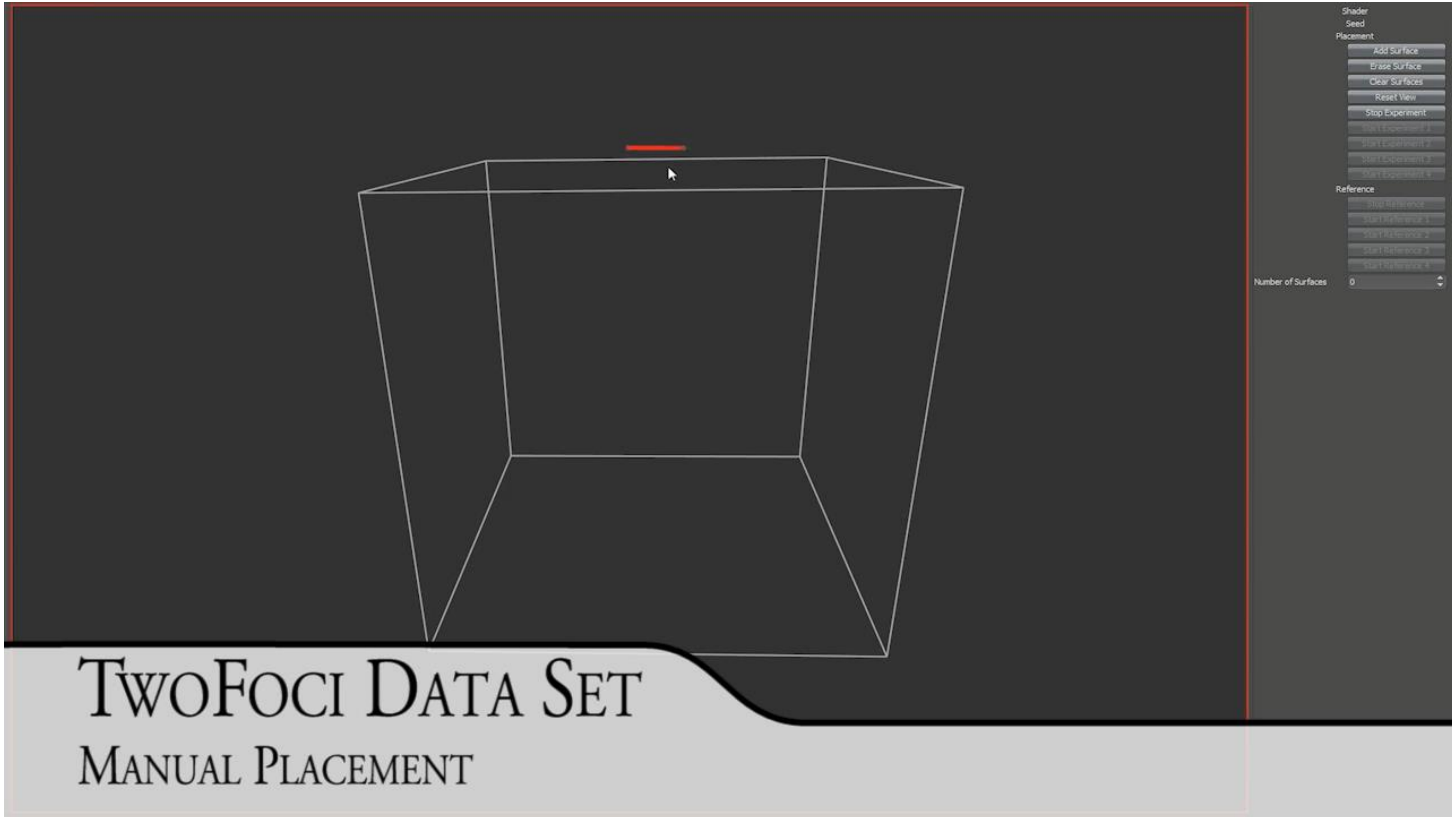






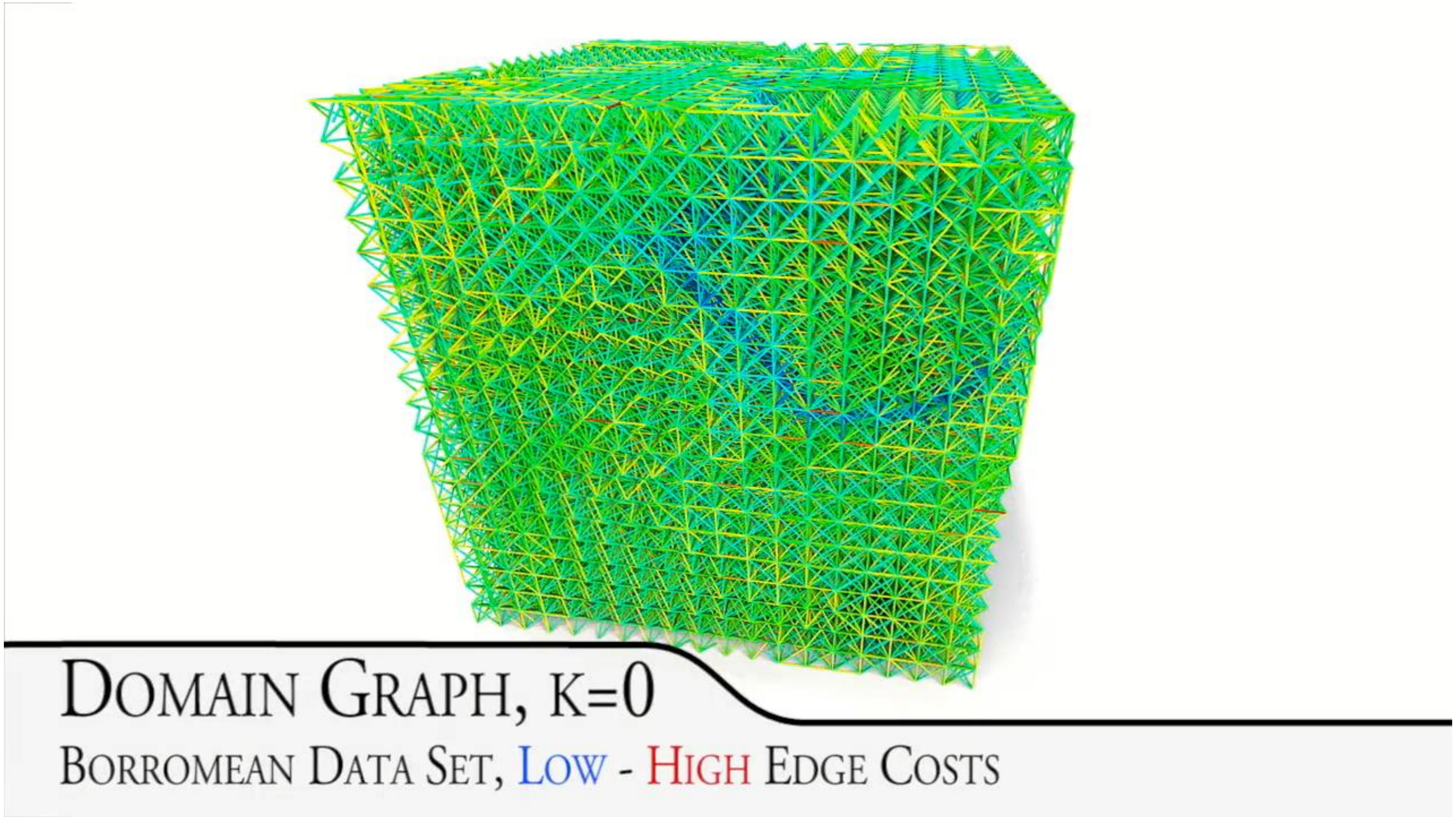
videos from Schulze, Germer, Rössl, Theisel, Stream Surface Parametrization by Flow-Orthogonal Front Lines, SGP 2012

Manual Placement of Stream Surfaces



video from Schulze, Martinez Esturo, Günther, Rössl, Seidel, Weinkauf, Theisel, Sets of Globally Optimal Stream Surfaces for Flow Visualization, EuroVis 2014

Automated Placement of Stream Surfaces



video from Schulze, Martinez Esturo, Günther, Rössl, Seidel, Weinkauf, Theisel, Sets of Globally Optimal Stream Surfaces for Flow Visualization, EuroVis 2014

Summary

- Tangent curves are tangential to the vectors in a vector field
  - paths of massless particles
  - alternative name: stream lines
- Properties
  - cannot intersect each other
  - there is one and only one tangent curve through every point of a vector field
- Computation using numerical integration
  - Euler, Runge-Kutta 2nd-order
  - Runge-Kutta 4th-order method
- Placement of tangent curves
  - 2D: evenly spaced
  - 3D: importance-driven
- Particle tracer
  - shows massless particles in vector field animated along tangent curves
- Stream surfaces
  - family of tangent curves
  - adaptive removal/insertion of tracers at the frontline
  - manual / automatic placement