Visualization, DD2257

Prof. Dr. Tino Weinkauf

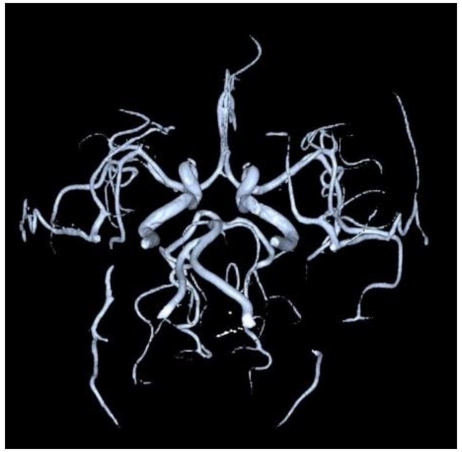
Scalar Fields

Medical Imaging

"volumetric data"

"volume data"

Computed Tomography

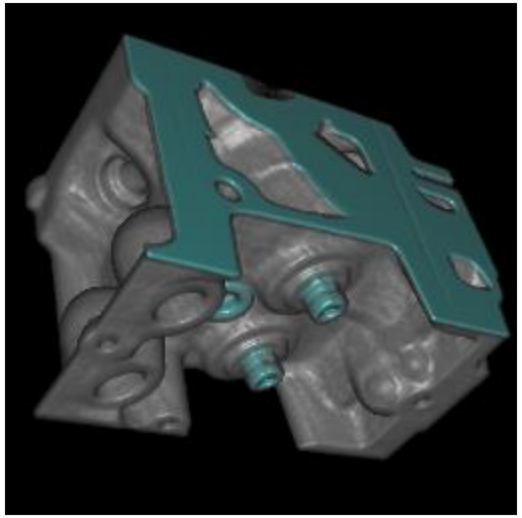


Magnetic Resonance Angiography

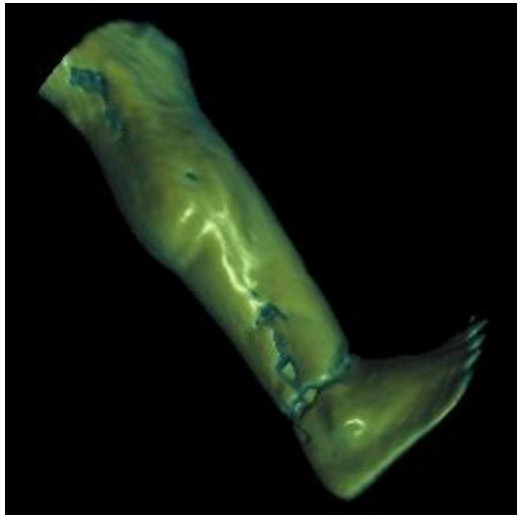
Positron Emission Tomography

3D Ultrasound

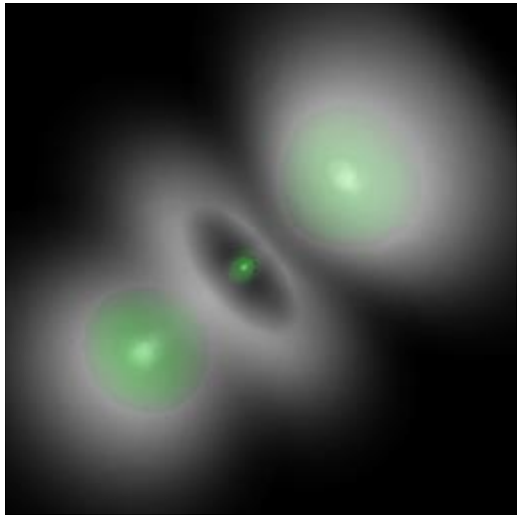
Science/Engineering



Industrial CT

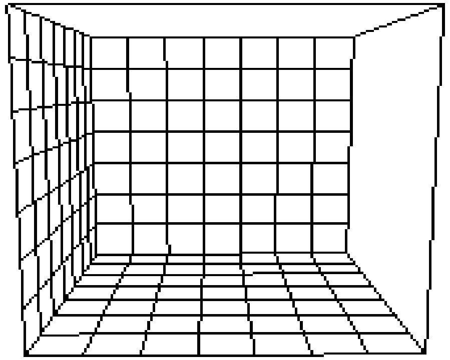


Scientific CT

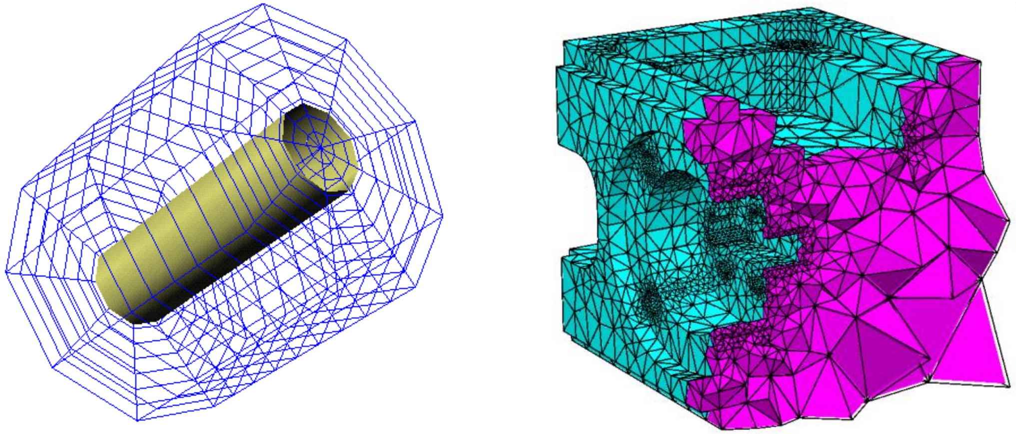


Numerical Simulation:
Hydrogen Atom

Numerical Simulation:
Fuel injection

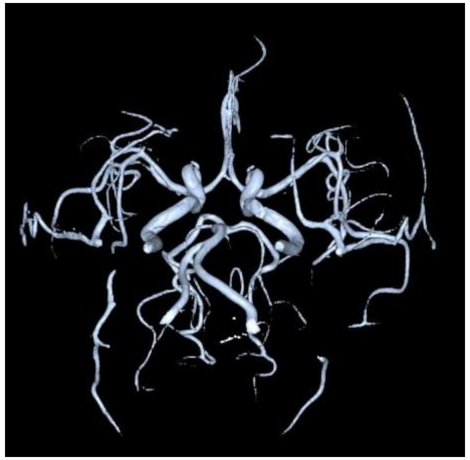


uniform grids



curvilinear grids or tetrahedral meshes

Computed Tomography



Magnetic Resonance Angiography

Numerical Simulation: Fuel injection

scalar field

$$s : \mathbb{E}^n \to \mathbb{R}$$

$$s(\mathbf{x})$$
$$\text{with } \mathbf{x} \in \mathbb{E}^n$$

$$s(x, y) = 2xy + 4y^2$$

2D scalar field

scalar field

$$s : \mathbb{E}^n \rightarrow \mathbb{R}$$

vector field

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

$$\mathbf{v}(\mathbf{x}) = \begin{pmatrix} c_1(\mathbf{x}) \\ \vdots \\ c_m(\mathbf{x}) \end{pmatrix}$$

with $\mathbf{x} \in \mathbb{E}^n$

tensor field

$$\mathbf{T} : \mathbb{E}^n \rightarrow \mathbb{R}^{m \times b}$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} c_{11}(\mathbf{x}) & \dots & c_{1b}(\mathbf{x}) \\ \vdots & & \vdots \\ c_{m1}(\mathbf{x}) & \dots & c_{mb}(\mathbf{x}) \end{pmatrix}$$

with $\mathbf{x} \in \mathbb{E}^n$

The first derivative of a scalar field is a vector field called **gradient**. It consists of the partial derivatives of the scalar function $s(\mathbf{x})$ for each dimension of the observation space.

$$s(x, y)$$

2D scalar field

$$\nabla s(x, y) = \begin{pmatrix} \frac{\partial s}{\partial x} \\ \frac{\partial s}{\partial y} \end{pmatrix} = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

gradient

scalar field

$$s : \mathbb{E}^n \rightarrow \mathbb{R}$$

$$s(\mathbf{x})$$
with $\mathbf{x} \in \mathbb{E}^n$

vector field

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

$$\begin{pmatrix} c_1(\mathbf{x}) \\ \vdots \end{pmatrix}$$

tensor field

$$\mathbf{T} : \mathbb{E}^n \rightarrow \mathbb{R}^{m \times b}$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} c_{11}(\mathbf{x}) & \dots & c_{1b}(\mathbf{x}) \\ \vdots & & \vdots \\ c_{m1}(\mathbf{x}) & \dots & c_{mb}(\mathbf{x}) \end{pmatrix}$$
with $\mathbf{x} \in \mathbb{E}^n$

The second derivative of a scalar field is a tensor field called **Hessian**. It consists of the partial derivatives of $s(\mathbf{x})$ derived twice for each dimension of the observation space.

$$s(x, y)$$

2D scalar field

$$\nabla s(x, y) = \begin{pmatrix} \frac{\partial s}{\partial x} \\ \frac{\partial s}{\partial y} \end{pmatrix} = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

gradient

$$\nabla^2 s(x, y) = \begin{pmatrix} s_{xx} & s_{xy} \\ s_{yx} & s_{yy} \end{pmatrix}$$

Hessian

very important

Contours in Scalar Fields

given:
scalar function $f\colon \mathbb{R}^n \to \mathbb{R}$
isovalue $c \in \mathbb{R}$

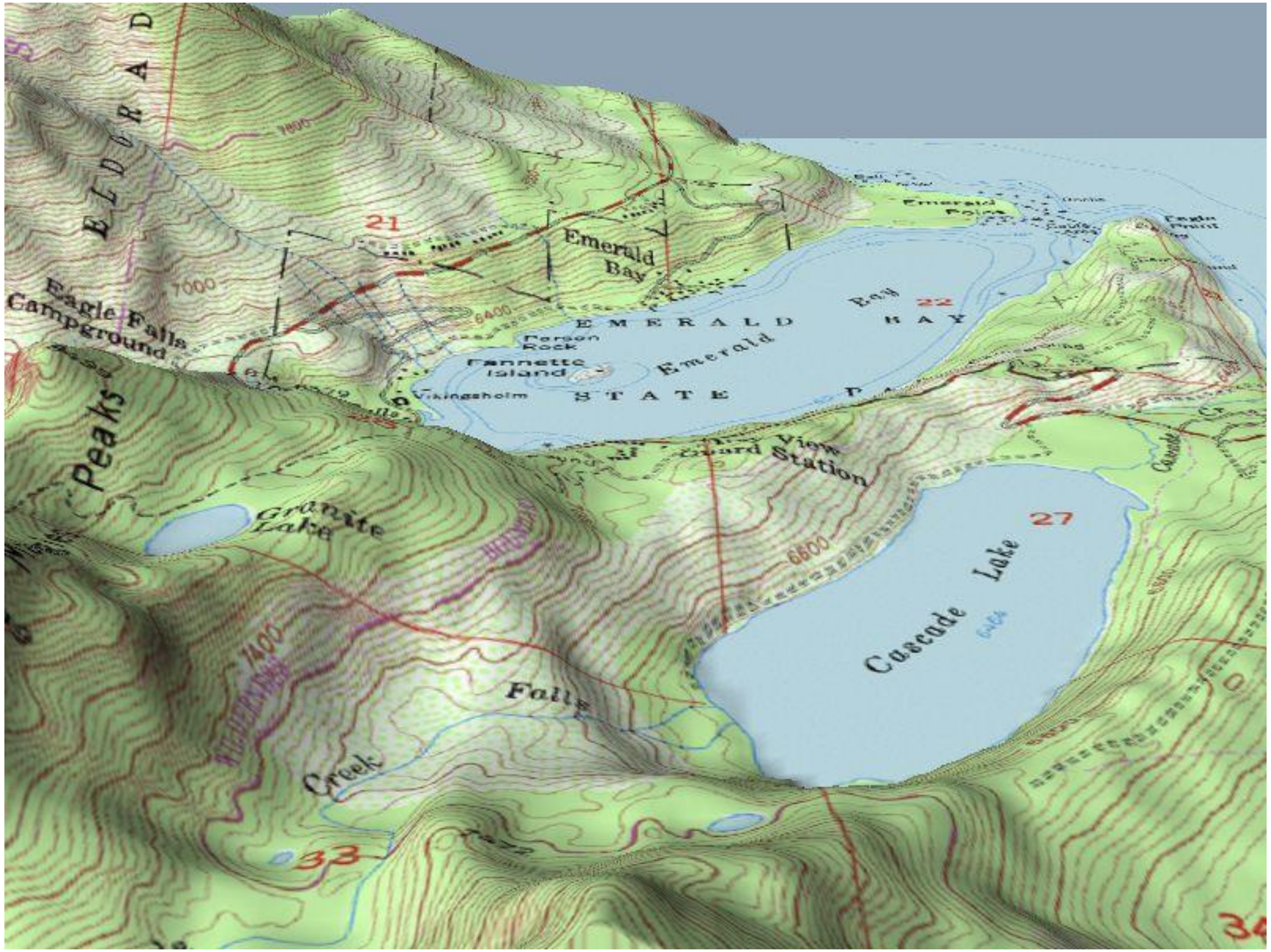
definition of **a contour**:

$$\{(\mathbf{x}) \mid f(\mathbf{x}) = c\}$$

2D contours are curves

3D contours are surfaces

common names:
isolines/isosurfaces



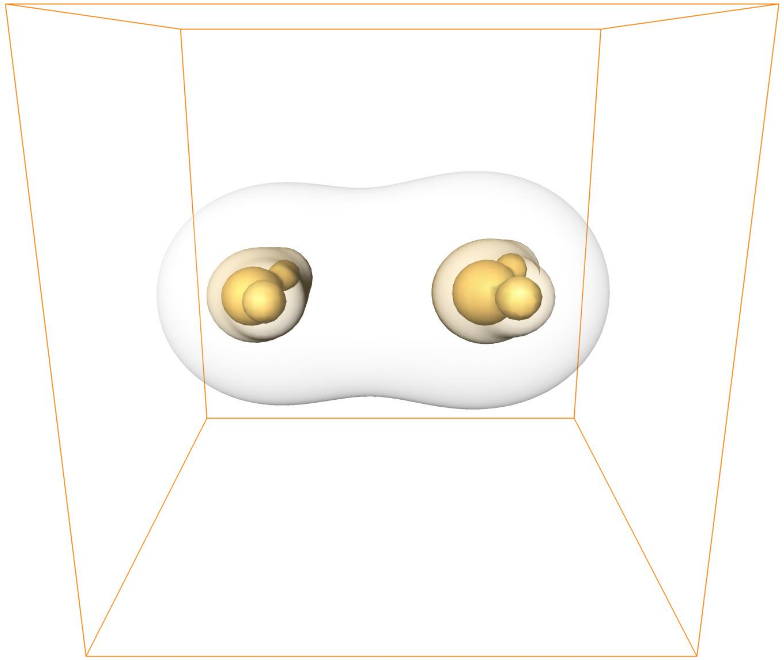
Properties of Contours

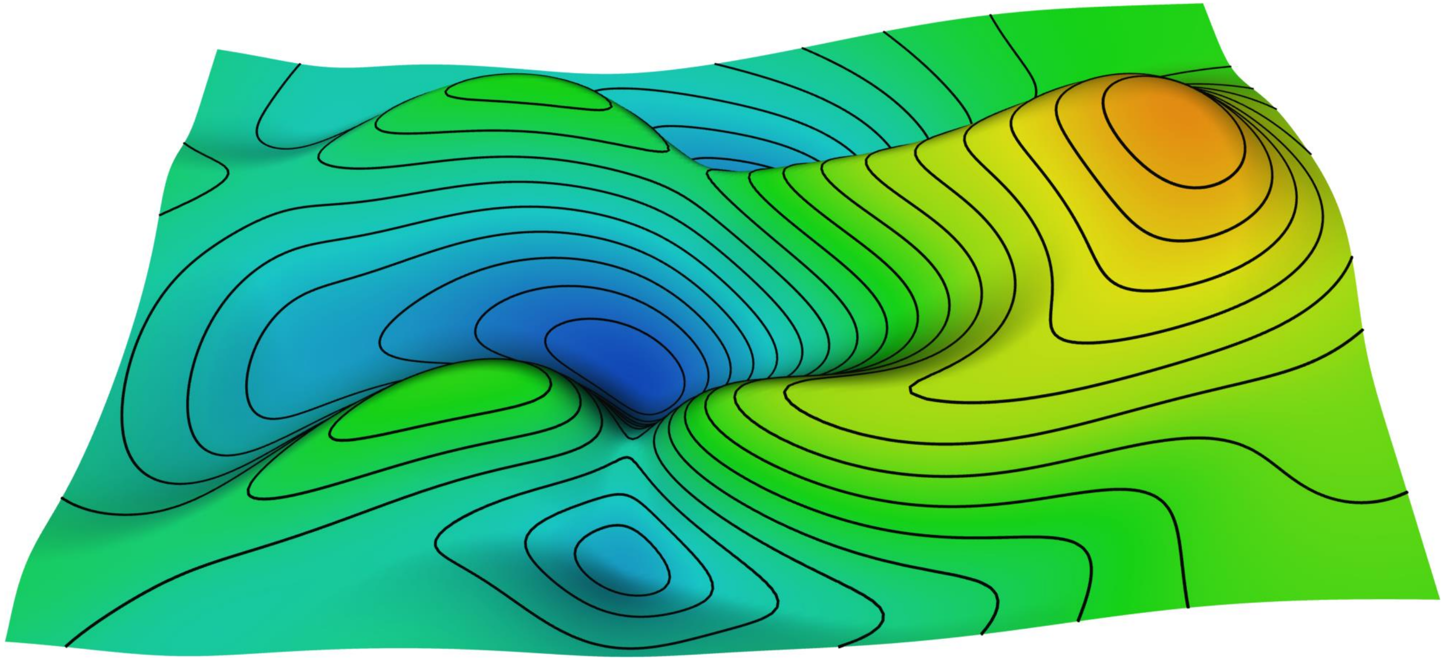
closed

unless exiting the domain

cannot intersect each other

nested

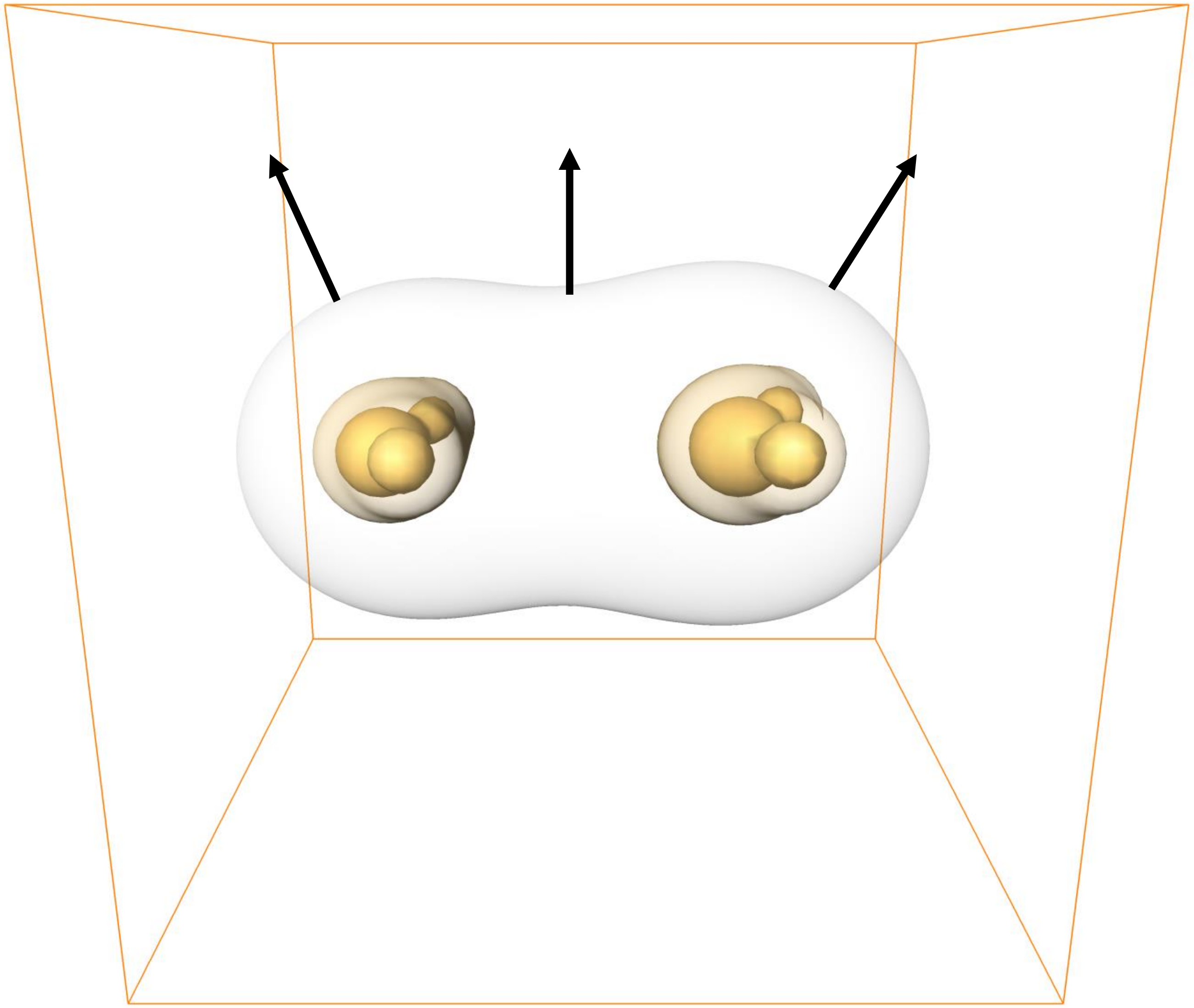




Properties of Contours

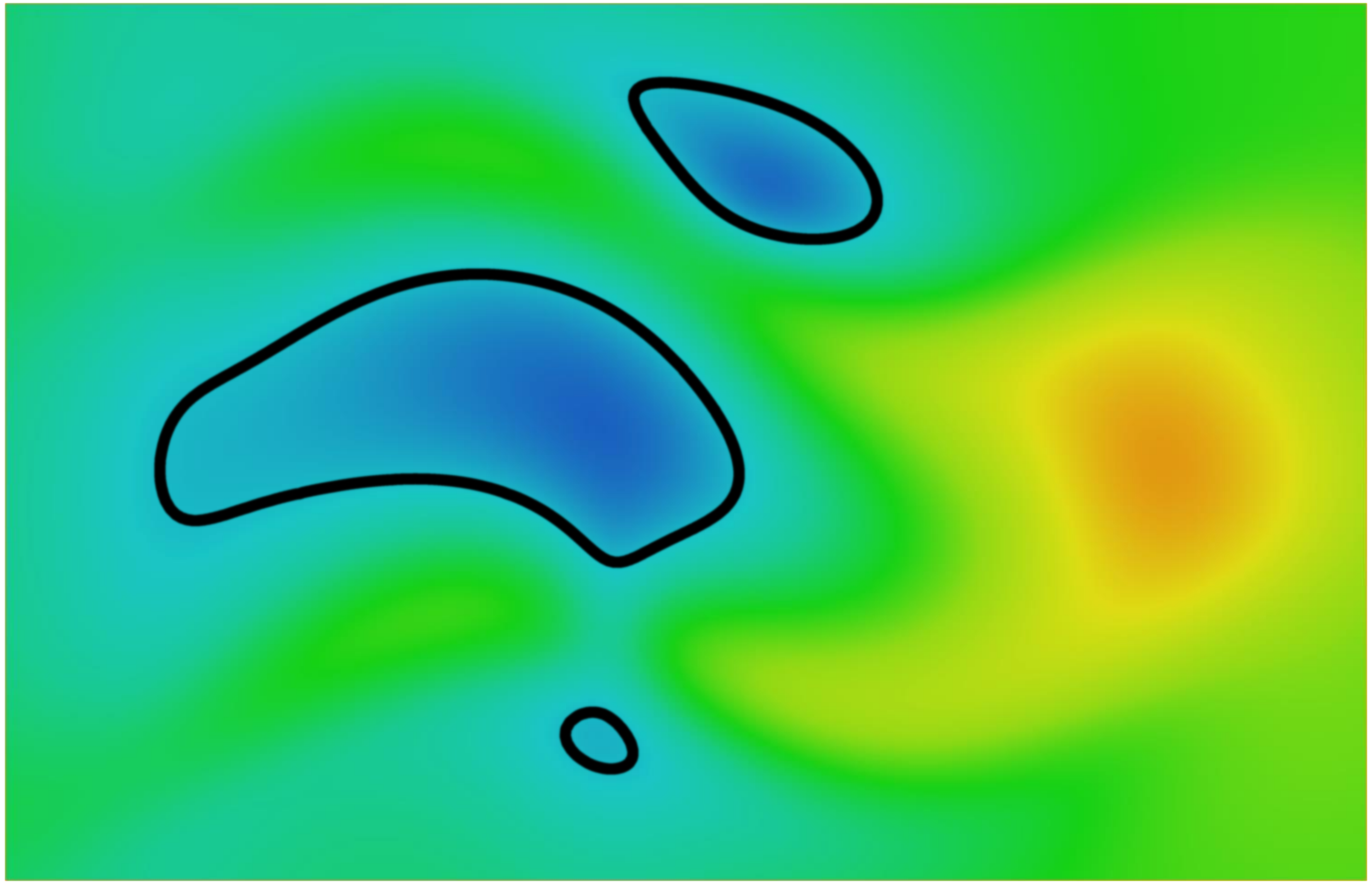
gradient is perpendicular to the contour

rate of change is zero along any isocontour



Properties of Contours

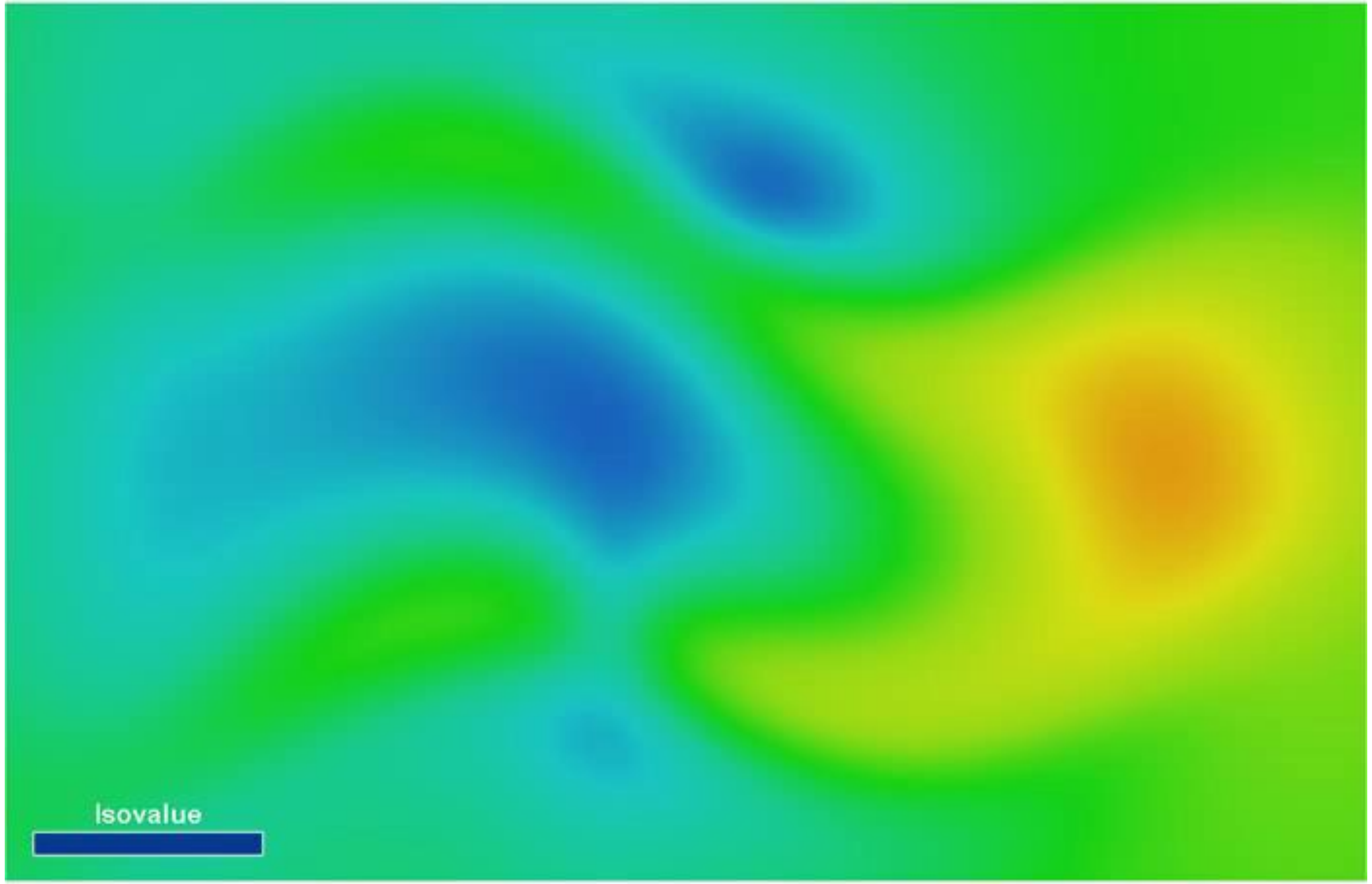
connected component:
a given isovalue produces one isocontour often consisting of several separate lines/surfaces



three connected components making up one isocontour

Properties of Contours

connected component merge and split when considering an increasing isovalue



Properties of Contours

connected component merge and split when considering an increasing isovalue

topology

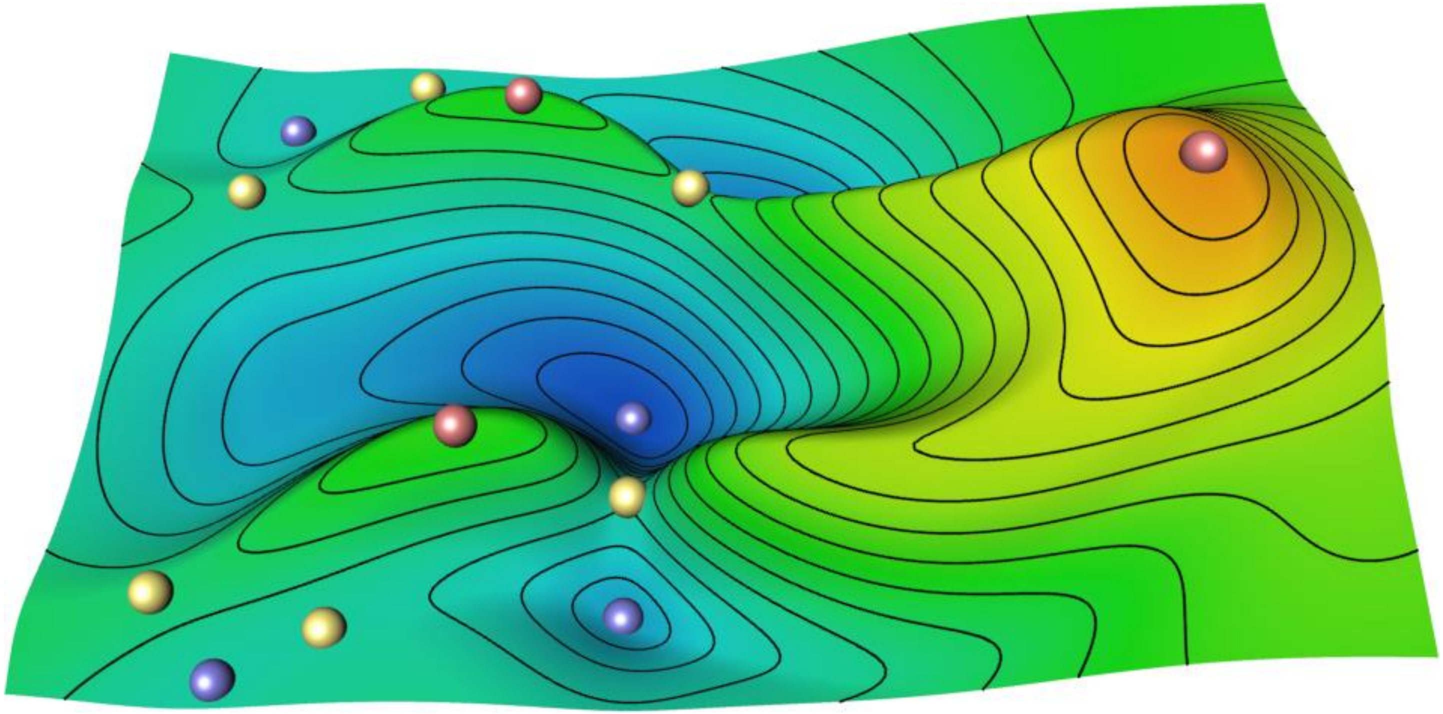
behavior of connected components

critical points

maxima

minima

saddle points



Overview of Visualization Methods for Scalar Fields

Color

- Color mapping (2D)
- Slicing (3D ➔ 2D)
- Direct Volume Rendering (3D)

Geometry

- Contours
- Height Plot

Features

- Morse-Smale Complex
- Contour tree, Merge trees
- Extremal lines and surfaces