

Appendix B

Vector differential operators

Rectangular coordinates (x, y, z)

$$\nabla \Phi = \hat{\mathbf{x}} \frac{\partial \Phi}{\partial x} + \hat{\mathbf{y}} \frac{\partial \Phi}{\partial y} + \hat{\mathbf{z}} \frac{\partial \Phi}{\partial z},$$

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z},$$

$$\nabla \times \mathbf{F} = \hat{\mathbf{x}} \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + \hat{\mathbf{y}} \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) + \hat{\mathbf{z}} \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right),$$

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}.$$

Cylindrical coordinates (ρ, ϕ, z)

$$\nabla \Phi = \hat{\rho} \frac{\partial \Phi}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial \Phi}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial \Phi}{\partial z},$$

$$\nabla \cdot \mathbf{F} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho F_\rho) + \frac{1}{\rho} \frac{\partial f_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z},$$

$$\nabla \times \mathbf{F} = \hat{\rho} \left(\frac{1}{\rho} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right) + \hat{\phi} \left(\frac{\partial f_\rho}{\partial z} - \frac{\partial F_z}{\partial \rho} \right) + \hat{\mathbf{z}} \frac{1}{\rho} \left(\frac{\partial}{\partial \rho} (\rho F_\phi) - \frac{\partial F_\rho}{\partial \phi} \right),$$

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \Phi}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2}.$$

Spherical coordinates (r, θ, ϕ)

$$\nabla \Phi = \hat{\mathbf{r}} \frac{\partial \Phi}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial \Phi}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial \Phi}{\partial \phi},$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (F_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi},$$

$$\begin{aligned} \nabla \times \mathbf{F} = & \hat{\mathbf{r}} \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (F_\phi \sin \theta) - \frac{\partial F_\theta}{\partial \phi} \right] \\ & + \hat{\boldsymbol{\theta}} \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial F_r}{\partial \phi} - \frac{\partial (r F_\phi)}{\partial r} \right] + \hat{\boldsymbol{\phi}} \frac{1}{r} \left[\frac{\partial (r F_\theta)}{\partial r} - \frac{\partial F_r}{\partial \theta} \right], \end{aligned}$$

$$\nabla^2 \Phi = \frac{1}{r} \frac{\partial^2}{\partial r^2} (r \Phi) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \Phi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Phi}{\partial \phi^2}.$$

Appendix C

Vector calculus identities

$$\nabla \cdot \nabla \Phi = \nabla^2 \Phi$$

$$\nabla \cdot \nabla \times \mathbf{F} = 0$$

$$\nabla \times \nabla \Phi = 0$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$$

$$\nabla(\Phi \Psi) = (\nabla \Phi) \Psi + \Phi \nabla \Psi$$

$$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla) \mathbf{G} + \mathbf{F} \times (\nabla \times \mathbf{G}) + (\mathbf{G} \cdot \nabla) \mathbf{F} + \mathbf{G} \times (\nabla \times \mathbf{F})$$

$$\nabla \cdot (\Phi \mathbf{F}) = (\nabla \Phi) \cdot \mathbf{F} + \Phi \nabla \cdot \mathbf{F}$$

$$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = (\nabla \times \mathbf{F}) \cdot \mathbf{G} - (\nabla \times \mathbf{G}) \cdot \mathbf{F}$$

$$\nabla \times (\Phi \mathbf{F}) = (\nabla \Phi) \times \mathbf{F} + \Phi \nabla \times \mathbf{F}$$

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = (\nabla \cdot \mathbf{G}) \mathbf{F} - (\nabla \cdot \mathbf{F}) \mathbf{G} + (\mathbf{G} \cdot \nabla) \mathbf{F} - (\mathbf{F} \cdot \nabla) \mathbf{G}$$

$$\int_S \mathbf{F} \cdot d\mathbf{A} = \int_V \nabla \cdot \mathbf{F} d^3x$$

$$\int_C \mathbf{F} \cdot d\boldsymbol{\ell} = \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{A}$$