

Theory of PDE MM8008/SF2739 Homework.

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Due date: 12th December at 23:59.¹ **Do not forget to add your name, email and Swedish personal id number** (if you have one) to your solutions.

Marks: Maximum 9 marks. To pass you need 7 or more marks.

1. Let $S : \mathcal{B} \mapsto \mathcal{B}$ be a linear and continuous operator on the Banach space \mathcal{B} .

a) Show that

$$\text{Ker}(S) = \{x \in \mathcal{B}; Sx = 0\}$$

is a closed subspace.

[2 marks]

b) If \mathcal{B} is a Hilbert space show that the orthogonal complement of $\text{Ker}(S)$ is also closed.

[2 marks]

2. Let $T : L^2([0, 1]) \mapsto L^2([0, 1])$ be defined by

$$Tf(x) = \int_0^1 f(x)dx.$$

a) Find the dual of T .

[1 mark]

b) Show that T is compact.

[1 mark]

c) For which $\lambda \in \mathbb{R}$ is $I - \lambda T$ invertable?

[1 mark]

d) For each $\lambda \in \mathbb{R}$ such that $I - \lambda T$ is not invertable, find the range and kernel of $I - \lambda T$.

[2 marks]

¹If you email your solutions email me a PDF file that is either computer written or a scan of your handwritten solutions. Do not send me photos of your solution since they are usually very difficult to read.